Local generalised method of moments: an application to point process-based rainfall models

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Main aim

The simulation of realistic artificial rainfall series at a given site

Requirements:

- reproduce statistical properties of real data (mean, variance, skewness, autocorrelations; wet-dry properties; extreme values) at a range of temporal scales
- sub-daily, perhaps sub-hourly, resolution
- can reflect potential impacts of climate change

Such series are required for hydrological studies, e.g. urban drainage design needs sub-hourly data

Statistical downscaling

GCMs: main tools for predicting future climate impacts from the increase of greenhouse gases in the atmosphere.

Problem: cannot reliably model local precipitation:

Solution: “statistical downscaling”. Model relationships between large-scale climate variables of the GCM (or RCM) and local climate

Problem: Structure depends on specific division of time scale, usually daily (and not suitable at subdaily scales)

Solution: Models based on point processes operate in continuous time, allowing aggregation to any timescales

Poisson-cluster process rainfall models

Represent the rainfall process in a simplified way with interpretable parameters

Problem: Models are stationary.

Current practice:

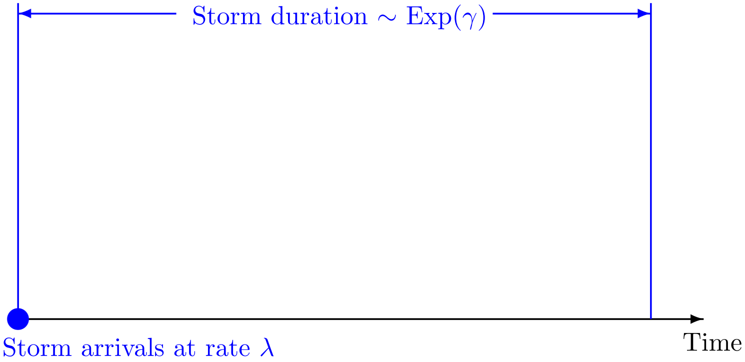
- Seasonality: fit model separately within each month
- Month just a proxy for climate variables. Seasonal patterns changing?

This work:

- allow parameters to vary smoothly with covariates
- ... month, GCM variables: surface temperature, sea-level pressure (slp), zonal (west-east) wind, ...
- expect complicated relationships: use local modelling
- enables simulation of future rainfall for use in climate impact studies

Bartlett-Lewis Rectangular Pulse model

Bartlett-Lewis Poisson cluster process of cell origins:



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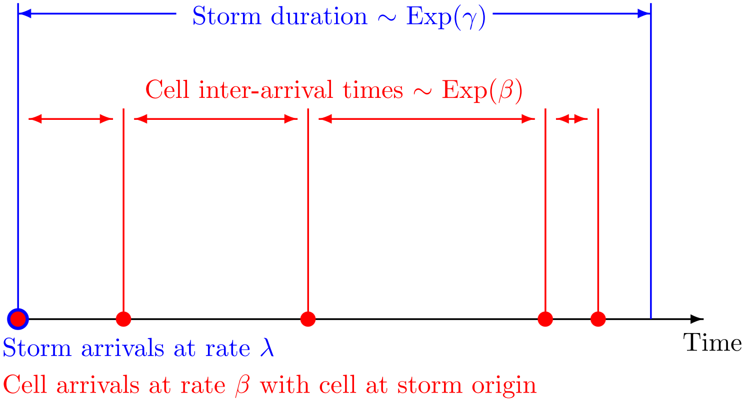
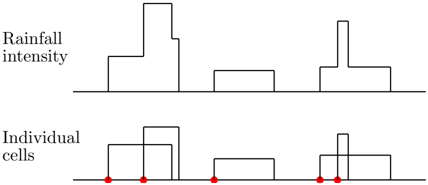


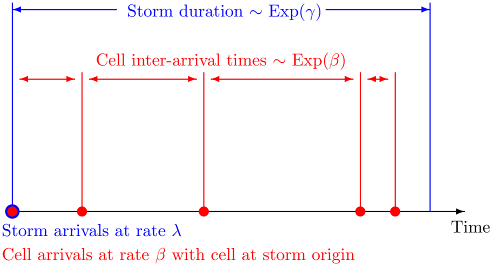
Bartlett-Lewis Rectangular Pulse model

The cells have random durations and intensities.

η rate of cell death

μ_X mean cell intensity





λ rate of storm arrival

γ rate of storm death

β rate of cell arrival

$$\theta = (\lambda, \gamma, \beta, \eta, \mu_X)$$

Data

- 5-min rainfall totals from Bochum, Germany, 1931–1999
- Y_t: data in month t, $t = 1, \ldots, 624$
- From Jan 1948 : monthly NCEP reanalysis data (52.5N, 7.5E) plus NAO index
- 624 monthly fitting properties and covariate values
  - $T(Y_t) = (T_1(Y_t), \ldots, T_k(Y_t))$ sample fitting properties
    [1h mean; 5 min, (1, 6, 24)h coeff. of variation, lag 1 autocorr., skewness]
  - $X_t = (X_{t_1}, X_{t_2}, \ldots, m_t)$: monthly temperature, slp . . . , month

Generalised Method of Moments (GMM)

- $\widehat{\theta}$ minimises a (quadratic form) measure of discrepancy between sample $T(Y_t)$ and model $\boldsymbol{\tau}(\theta)$ values
- Weight sample properties to account for their (co)variances
- Optimal weight matrix W is $\{\mathrm{cov}[T(Y_t)]\}^{-1}$, but in practice $\mathrm{diag}\{\mathrm{var}[T_i(Y_t)]^{-1}\}$ is close to optimal and easier to estimate reliably (Jesus and Chandler, 2011)

Local mean GMM

Estimator for month m

$$\widehat{\theta}_m = \text{argmin}_{\theta_m} \left[\left\{ \frac{1}{\sum_{t=1}^{n} I(m_t = m)} \sum_{t=1}^{n} I(m_t = m) \left[T(Y_t) - \tau(\theta_m) \right] \right\}^{\mathrm{T}} \right.$$

$$\left. \widehat{W}_m \left\{ \frac{1}{\sum_{t=1}^{n} I(m_t = m)} \sum_{t=1}^{n} I(m_t = m) \left[T(Y_t) - \tau(\theta_m) \right] \right\} \right]$$

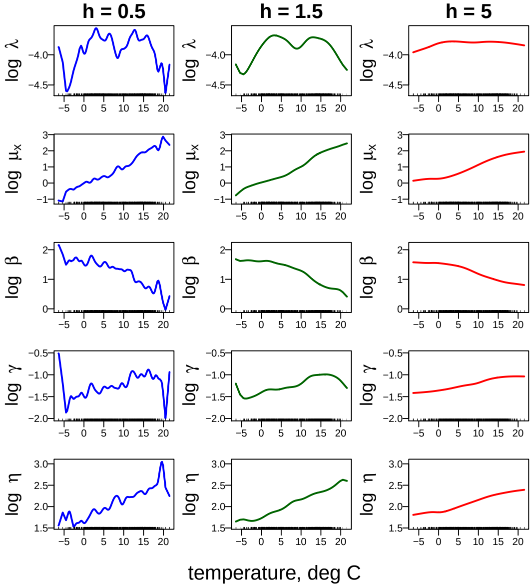
Estimator at covariate $X = x_0$

$$\widehat{\theta}(x_0) = \text{argmin}_{\theta_{x_0}} \left[\left\{ \frac{1}{n} \sum_{t=1}^{n} K_h(X_t - x_0) \left[T(Y_t) - \tau(\theta_{x0}) \right] \right\}^{\mathrm{T}} \right.$$

$$\left. \widehat{W}_{x_0} \left\{ \frac{1}{n} \sum_{t=1}^{n} K_h(X_t - x_0) \left[T(Y_t) - \tau(\theta_{x0}) \right] \right\} \right]$$

- $K_h(X_t - x_0) = \dfrac{1}{h} K\left(\dfrac{X_t - x_0}{h} \right)$, for a kernel function $K()$
- bandwidth h

Single covariate: effect of bandwidth



Bias-variance trade-off: a larger *h* gives a smoother, flatter curve, with lower variance, but higher bias

We use a *global* bandwidth: i.e. the same *h* across the whole curve

Choice of h; multiple covariates

Cross-validation (CV), using repeated random sub-sampling

For each covariate

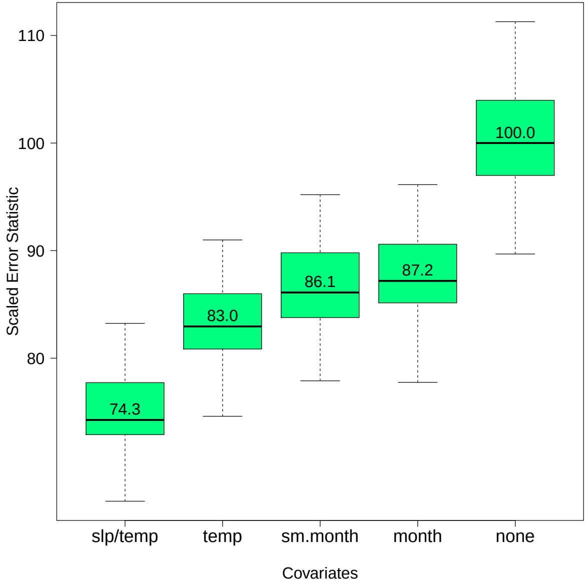
- For each of a set of candidate values of h ...
- Split 624 months into 399 (training) and 225 (test)
- Fit using training set then calculate mean weighted squared error (MWSE) on test set
- Repeat 25 times; find the best h (lowest MWSE) each time

Set 'optimal' h as an average of the best values

Multiple covariates

- use diagonal bandwidth matrix H: product kernels
- use CV to find choose an optimal multiple of a diagonal matrix of the optimal univariate bandwidths
- curse of dimensionality restricts the number of covariates

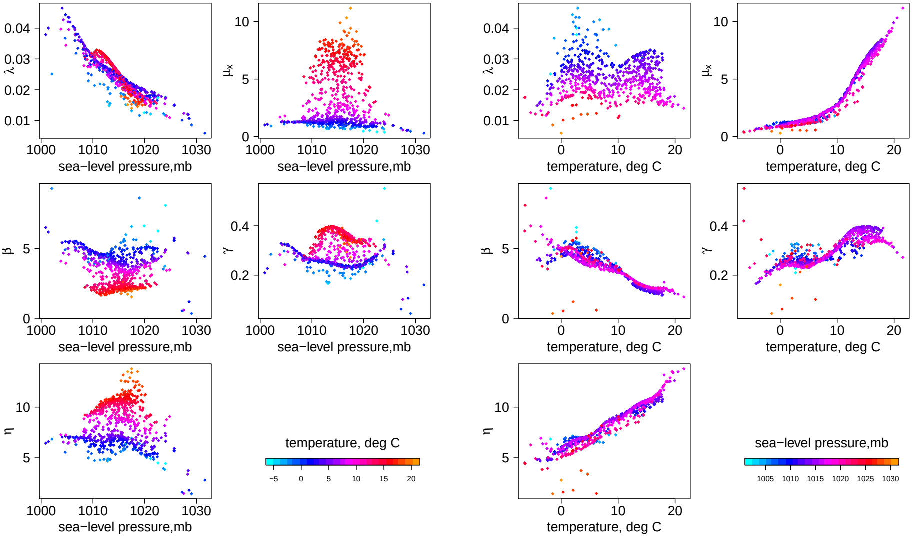
Model Comparison: optimal covariates



We compare prediction errors with different covariates

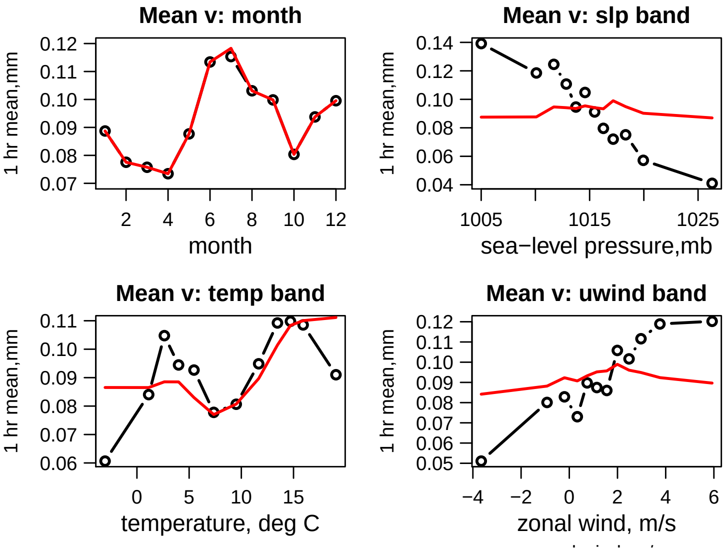
sm.month : local kernel smoothing based on month

Optimal pair of covariates: slp and temp



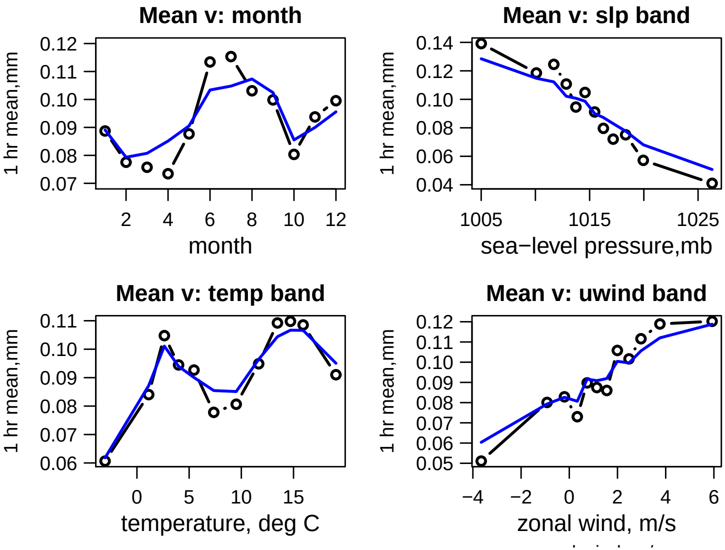
Bandwidths: sea-level pressure: 2.0; temperature: 1.75

Comparison of fit v current approach



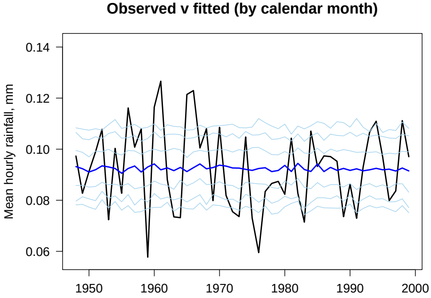
Covariate: Month

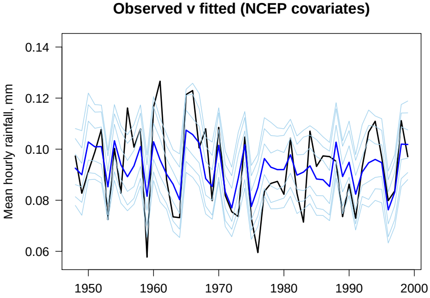
Comparison of fit v current approach



Covariates: Temperature, sea-level pressure and zonal wind

Interannual variability at Bochum





Simulated distributions of mean annual rainfall

- blue bands: (5, 10, 25, 50, 75, 90, 95) percentiles
- black line: sample values

Using covariate information has improved the representation of interannual variability

Concluding remarks

Local smoothing used to relate Poisson-cluster rainfall model parameters to covariates

Advantages

- covariates can replace, and improve, on month : once temperature is included, month adds no further benefit
- improved representation of interannual variability
- simulations can reflect future climate change scenarios

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Thank you for your attention

References

Research presented here

- Kaczmarska, J. M., Isham, V. S. & Northrop, P. J. (2015), 'Local generalised method of moments: an application to point process-based rainfall models', *Environmetrics* **26 (4)**, 312-325
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