Modelling non-stationary spatially-dependent extremes

Paul Northrop
University College London
paul@stats.ucl.ac.uk

Newcastle University
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This is joint work with Philip Jonathan (Shell Research)

Outline

- Wave height data $\rightarrow$ design of safe marine structures.
- Extreme value modelling
- Modelling issues
    - Spatial non-stationarity and dependence
    - Thresholds for non-stationary extremes
    - Model parameterisation
- Theoretical and simulation studies
- Wave height data

Wave heights from the Gulf of Mexico

- Hindcasts of Y storm peak significant wave height (in metres) in the Gulf of Mexico.
  - wave height : trough to the crest of the wave.
  - significant wave height : the average of the largest 1/3 wave heights. A measure of sea surface roughness.
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  - storm peak: largest value from each (hurricane-induced) storm.
- a 6×12 grid of 72 sites ($\approx$ 14 km apart).
- Sep 1900 to Sep 2005 : 315 storms .
- average of 3 observations (storms) per year, at each site.

Hurricane Katrina : Aug 2005

Hurricane damage

Hurricane damage

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- Extrapolation !

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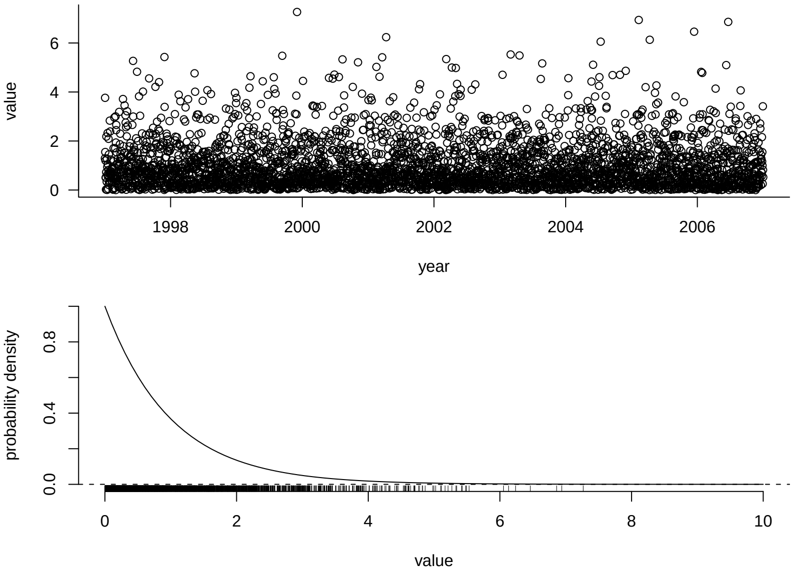
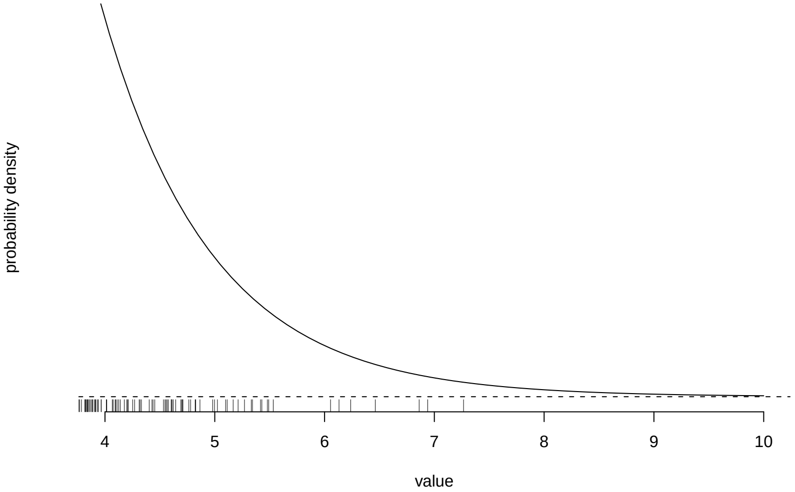
- Base inferences about future extreme behaviour on extreme data.

  ...but how should we define 'extreme data', and what models should we fit to these data?

Simulated data [i.i.d. exp(1)]



Upper tail



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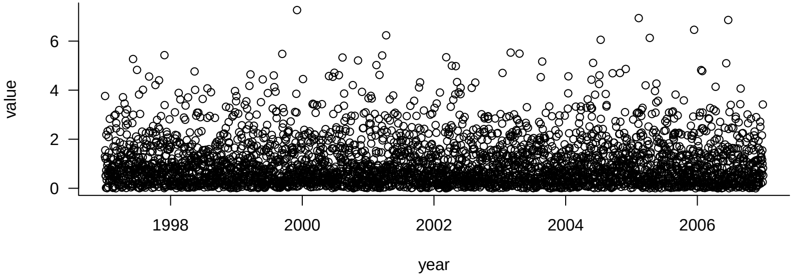
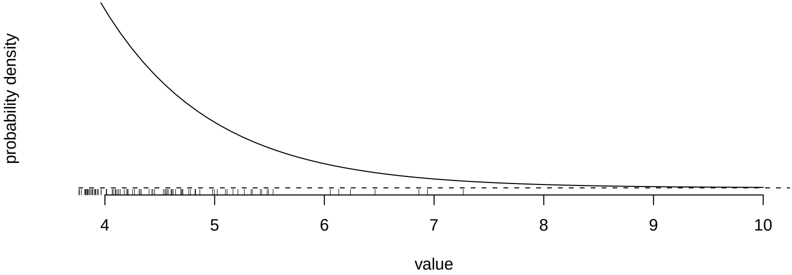
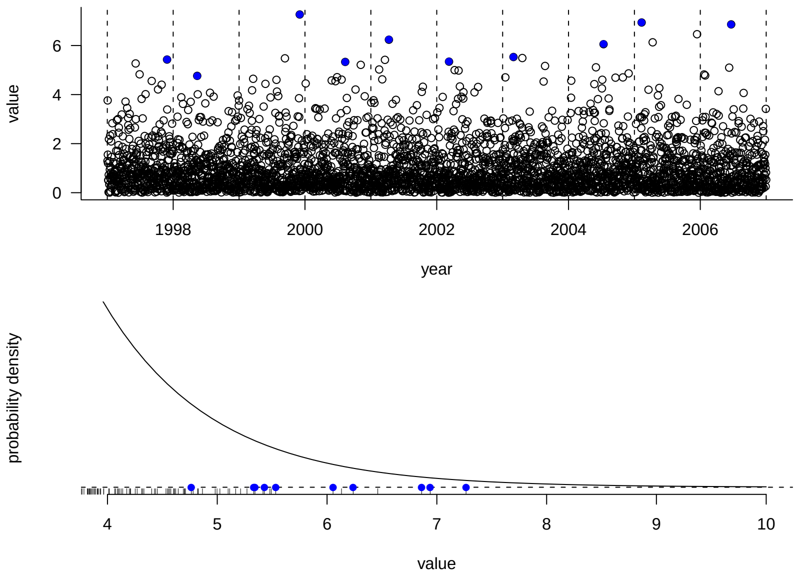
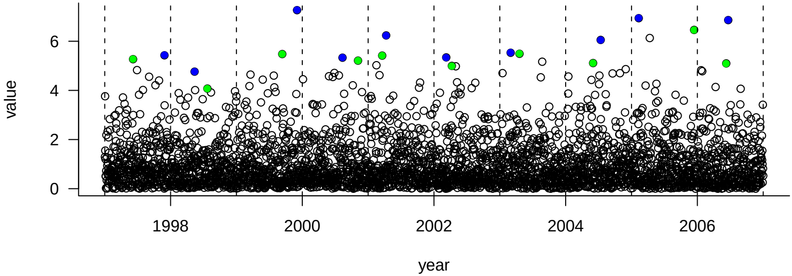


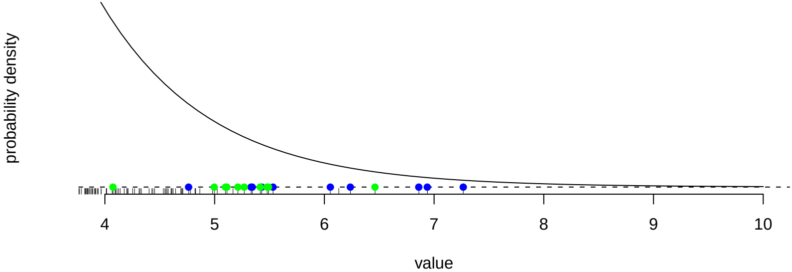


Block (annual) maxima

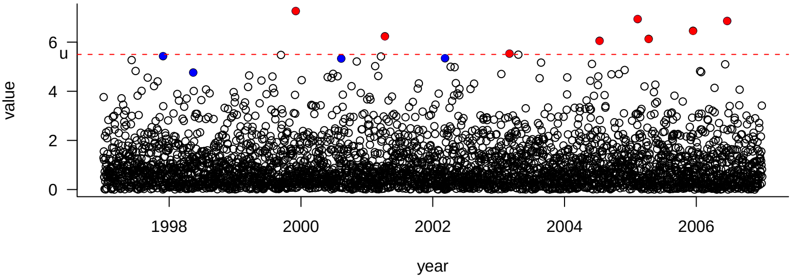


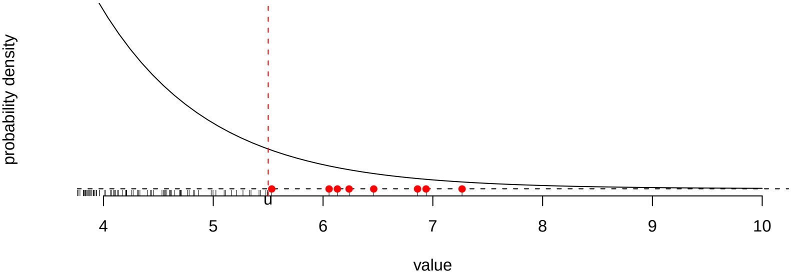
Maxima and second largest





Threshold exceedances, u=5.5





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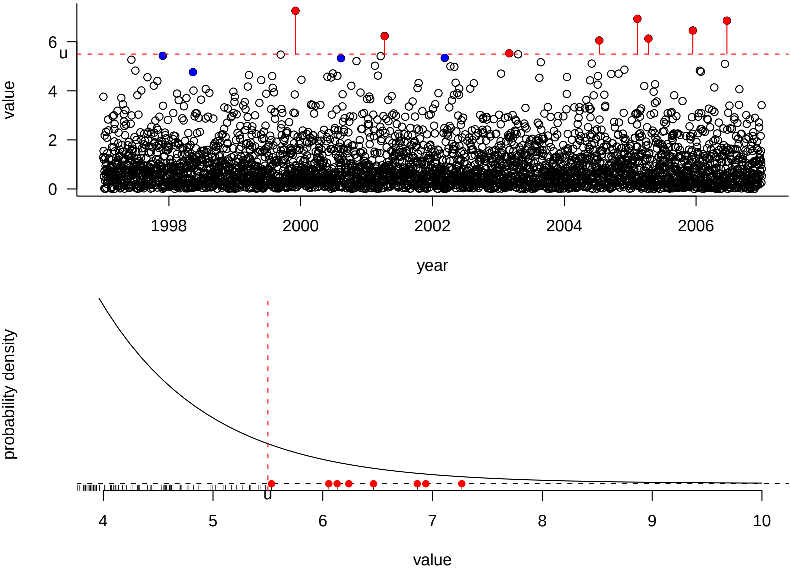

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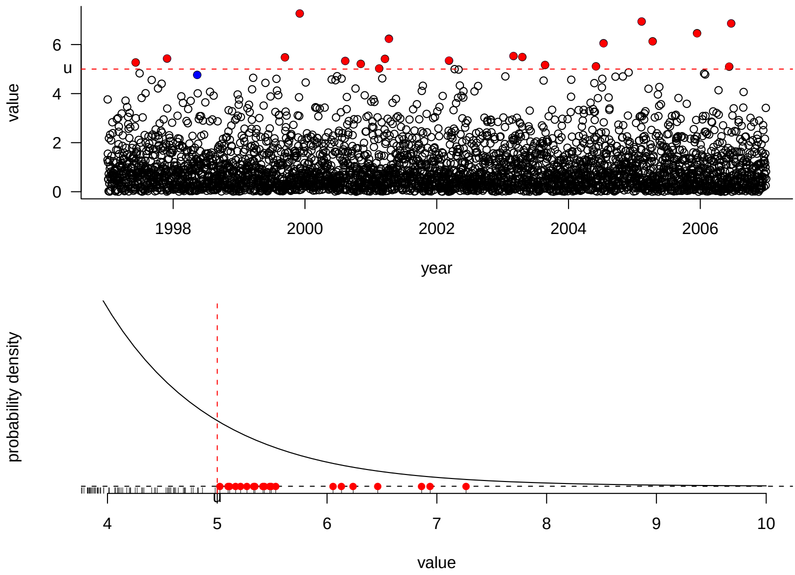

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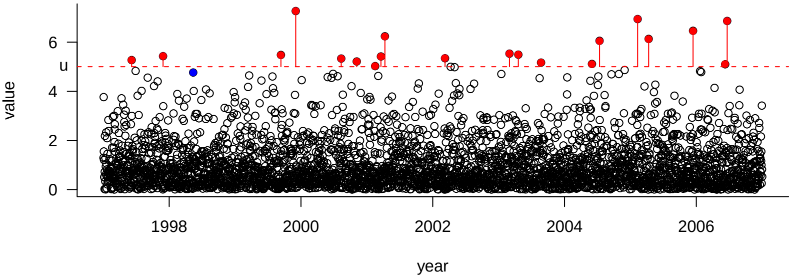

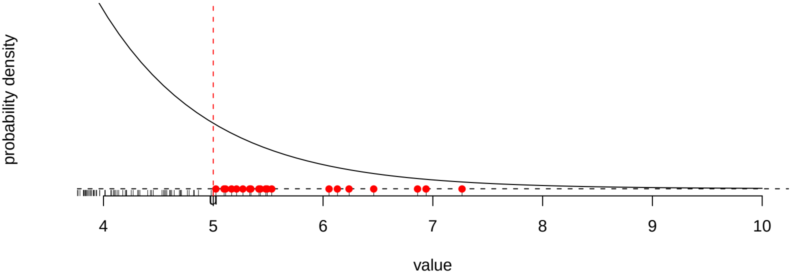

Modelling block maxima

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- Any possible (non-degenerate) distribution of M_n as $n \to \infty$ is in the GEV (Generalised Extreme Value) family, with c.d.f.

$$G_{GEV}(z; \mu, \sigma, \xi) = \exp\left\{ -\left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]^{-1/\xi} \right\},$$

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  where $1 + \xi(z - \mu)/\sigma > 0$ and $\sigma > 0$.
- Suggests GEV distn as a model for, say, annual maxima .
- Upper end point is finite for $\xi < 0$ and infinite for $\xi \geqslant 0$.
- Related asymptotic model for r-largest order statistics.

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If $Y_1, Y_2, \ldots$ are independent then we get a binomial-GP model.

Modelling threshold exceedances II

The GEV parameters (μ, σ, ξ) and the binomial-GP parameters (p_u, σ_u, ξ) are related …

- $\sigma_u = \sigma + \xi(u - \mu)$;
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- Poisson process $\approx$ binomial process.

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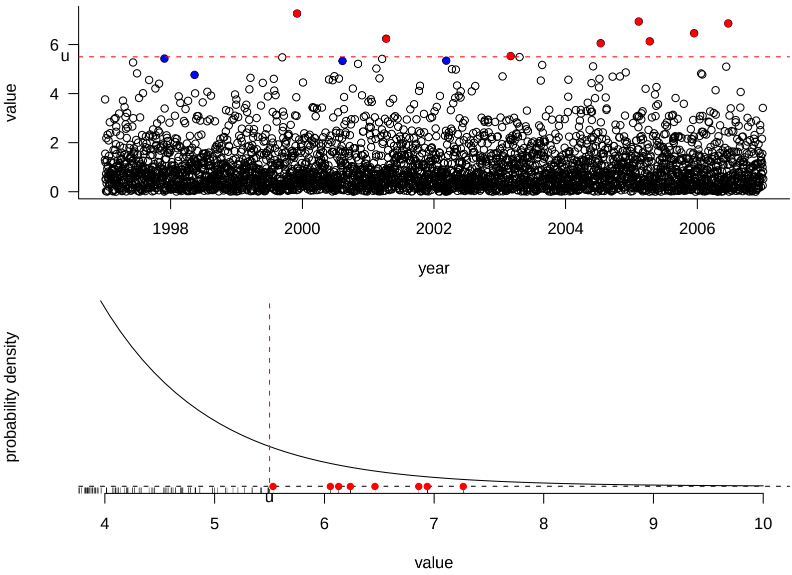

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- Appeal to standard theory conditional on the covariates.
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- The PP model has the advantage (over the GP model) that its parameters are invariant to u.

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Not necessary for the waves data because separate cluster (storm) are provided.

. . . but we will use 3. to adjust for spatial dependence .

Quantities of interest

- (Conditional) return levels.

  Suppose that annual maxima are GEV$(\mu(x), \sigma(x), \xi(x))$

  The *m*-year return level y_m satisfies

$$G_{GEV}(y_m; \mu(x), \sigma(x), \xi(x)) = 1 - \frac{1}{m},$$

  so that

$$y_m = \mu(x) + \sigma(x)\left\{\left[-\log\left(1 - \frac{1}{m}\right)\right]^{-\xi(x)} - 1\right\}.$$

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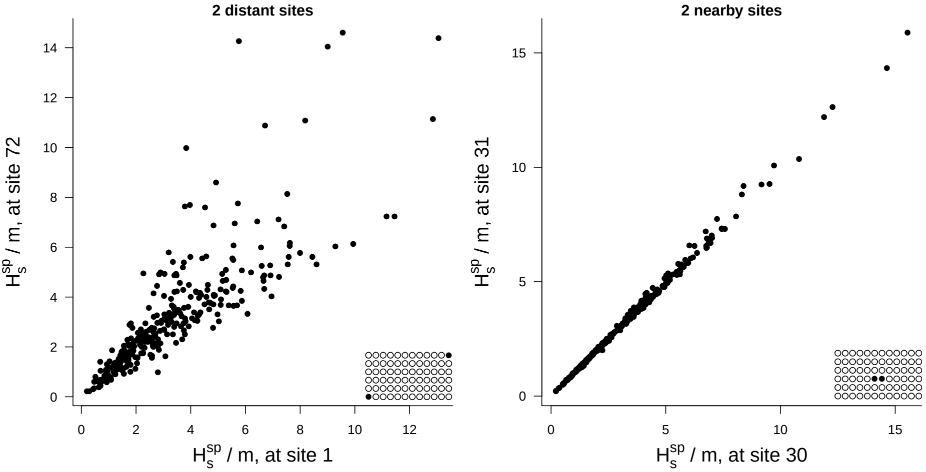
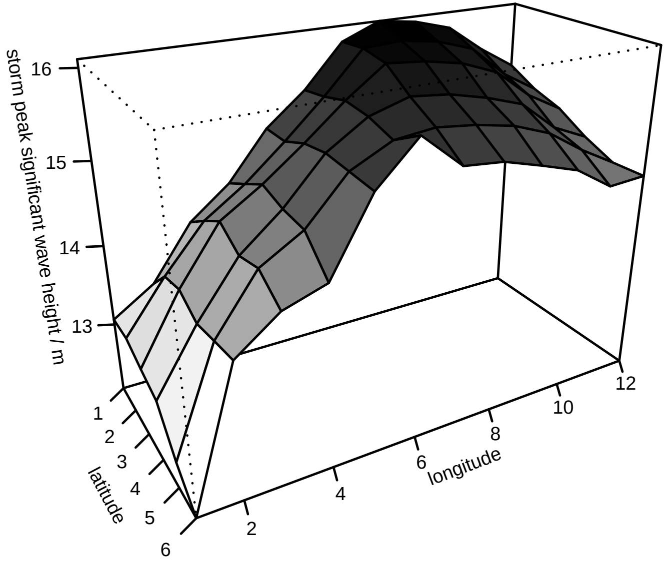
- More specific quantities, e.g. probability that a certain value is exceeded during the next m years.

Spatial dependence



Spatial non-stationarity



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- Spatial dependence not modelled explicitly: effect on marine structure is at one location.
- Storm peak from separate storms approximately independent.
- Time trends not apparent.
- Estimate return levels at centre of the region.

Modelling approach

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- Spatial dependence .
  - Estimate parameters assuming conditional independence of responses given covariate values.
  - Adjust standard errors etc. for spatial dependence.

Point process regression model

Conditional on covariates $\boldsymbol{x}_{ij}$ exceedances over a high threshold $u(\boldsymbol{x}_{ij})$ follow a **2-dimensional non-homogeneous Poisson process.**

If responses $Y_{ij}, i = 1, \ldots, 72$ (space), $j = 1, \ldots, 315$ (storm) are conditionally independent:

$$L(\theta) = \prod_{j=1}^{315} \prod_{i=1}^{72} \exp\left\{ -\frac{1}{\lambda} \left[1 + \xi(\boldsymbol{x}_{ij}) \left(\frac{u(\boldsymbol{x}_{ij}) - \mu(\boldsymbol{x}_{ij})}{\sigma(\boldsymbol{x}_{ij})} \right) \right]_+^{-1/\xi(\boldsymbol{x}_{ij})} \right\}$$

$$\times \prod_{j=1}^{315} \prod_{i: y_{ij} > u(\boldsymbol{x}_{ij})} \frac{1}{\sigma(\boldsymbol{x}_{ij})} \left[1 + \xi(\boldsymbol{x}_{ij}) \left(\frac{y_{ij} - \mu(\boldsymbol{x}_{ij})}{\sigma(\boldsymbol{x}_{ij})} \right) \right]_+^{-1/\xi(\boldsymbol{x}_{ij})-1} .$$

λ : mean number of observations per year;
$\mu(\boldsymbol{x}_{ij}), \sigma(\boldsymbol{x}_{ij}), \xi(\boldsymbol{x}_{ij})$: GEV parameters of annual maxima at $\boldsymbol{x}_{ij}$;
θ : vector of all model parameters:

Covariate-dependent thresholds

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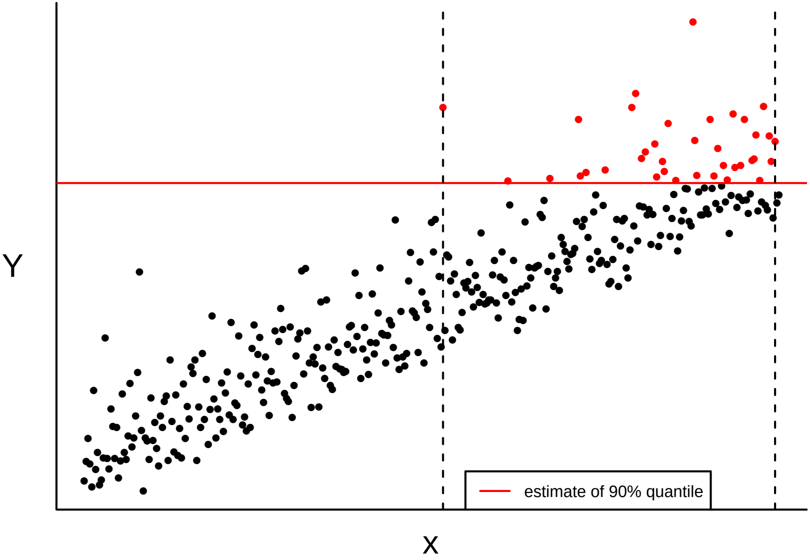
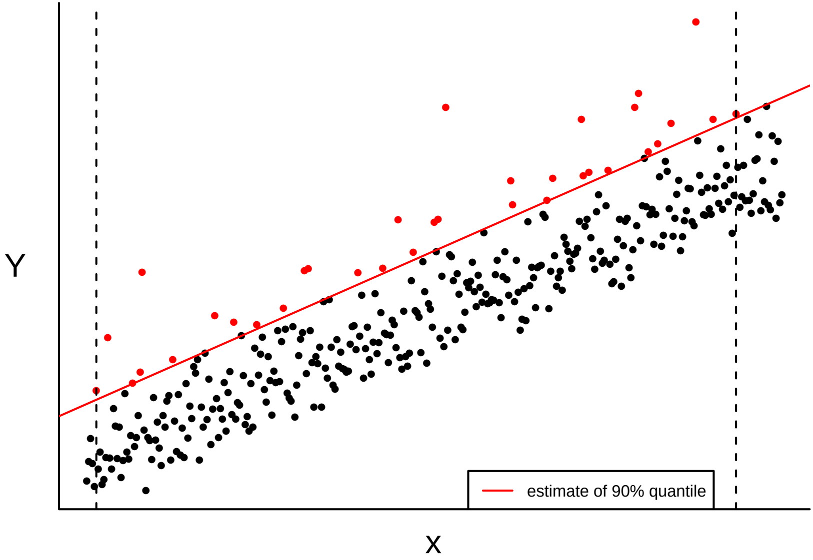
- Set $u(\boldsymbol{x}_{ij})$ by trial-and-error or by discretising $\boldsymbol{x}_{ij}$, e.g. different threshold for different locations, months etc.
- Quantile regression (QR) : model quantiles of Y as a function of covariates.

Constant threshold



Quantile regression



Model parameterisation

Let $p(\boldsymbol{x}_{ij}) = P(Y_{ij} > u(\boldsymbol{x}_{ij}))$. Then, if $\xi(\boldsymbol{x}_{ij}) = \xi$ is constant,

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The form of $u(\boldsymbol{x}_{ij})$ is determined by the extreme value model:

- if $\mu(\boldsymbol{x}_{ij})$ and/or $\sigma(\boldsymbol{x}_{ij})$ are linear in $\boldsymbol{x}_{ij}$: linear QR ;
- if $\log(\mu(\boldsymbol{x}_{ij})$ and/or $\log(\sigma(\boldsymbol{x}_{ij})$ is linear in $\boldsymbol{x}_{ij}$: non-linear QR .

Theoretical study (with Nicolas Attalides)

Data-generating process: for covariate values $x_1, \ldots, x_n$

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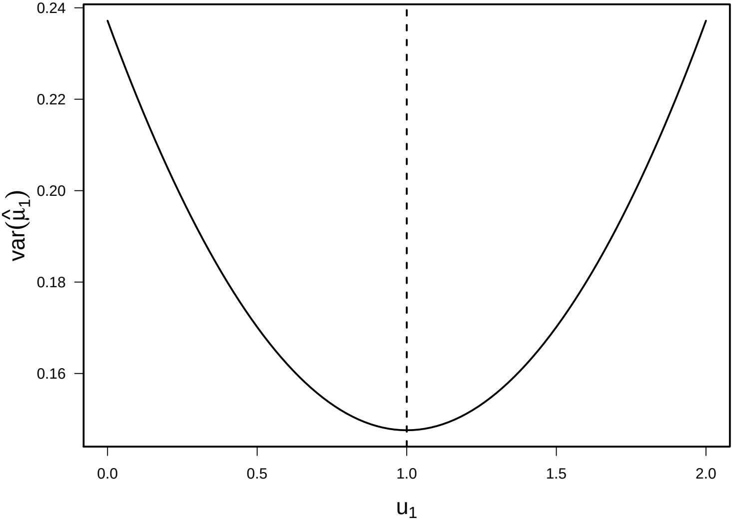
- Calculate Fisher expected information for $(\mu_0, \mu_1, \sigma, \xi)$.
- Invert to find asymptotic V-C of MLEs $\widehat{\mu}_0, \widehat{\mu}_1, \widehat{\sigma}, \widehat{\xi}$ and hence var($\widehat{\mu}_1$).
- Find the value of u_1 that minimises var($\widehat{\mu}_1$).

Preliminary findings

Let $\tilde{u}_1$ be the value of u_1 that minimises $\mathrm{var}(\widehat{\mu}_1)$.

- If covariate values $x_1, \ldots, x_n$ are symmetrically distributed then $\tilde{u}_1 = \mu_1$ (c.f. quantile regression).

$\mu_1 = 1$: symmetric *x*



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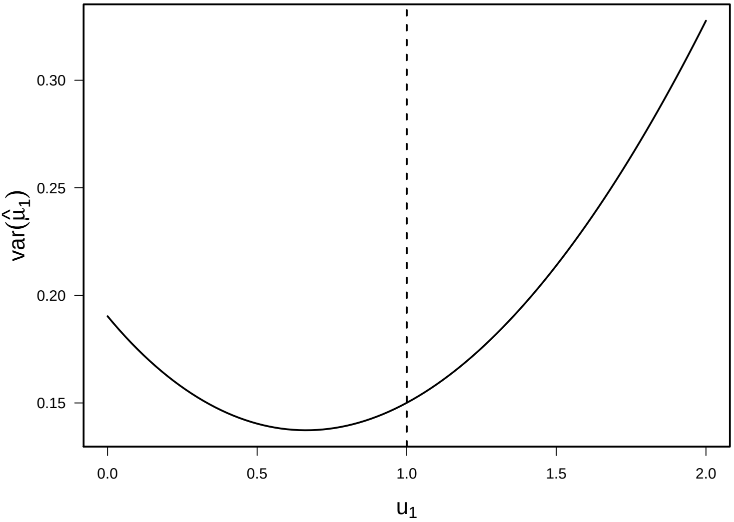
- If $x_1, \ldots, x_n$ are positive (negative) skew then $\tilde{u}_1 < \mu_1$ ($\tilde{u}_1 > \mu_1$).

$\mu_1 = 1$: positive skew *x* (skewness = 1)



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Extensions:

- More general models.
- Effect of model mis-specification due to low threshold.

Adjustment for spatial dependence

Independence log-likelihood:

$$l_{IND}(\theta) = \underbrace{\sum_{j=1}^{k}}_{\text{(storms)}} \underbrace{\sum_{i=1}^{72}}_{\text{(space)}} \log f_{ij}(y_{ij};\theta) = \sum_{j=1}^{k} l_j(\theta).$$

In regular problems, as $k \to \infty$,

$$\widehat{\theta} \to N(\theta_0, H^{-1}\, V\, H^{-1}),$$

- $H =$ expected Hessian: $\mathrm{E}\left(\frac{\partial^2}{\partial\theta^2}\, l_{IND}(\theta_0)\right)$;
- $V = \mathrm{var}\left(\frac{\partial}{\partial\theta}\, l_{IND}(\theta)\right)$

Adjustment of $l_{IND}(\theta)$

- $\widehat{H}$ = observed Hessian, at $\widehat{\theta}$;
- $\widehat{V} = \sum_{j=1}^{k} U_j\left(\widehat{\theta}\right)^T U_j\left(\widehat{\theta}\right), \qquad U_j(\theta) = \frac{\partial l_j(\theta)}{\partial \theta}$.

Let $\widehat{H}_A = \left(-\widehat{H}^{-1}\,\widehat{V}\,\widehat{H}^{-1}\right)^{-1}$.

Chandler and Bate (2007):

$$l_{ADJ}(\theta) = l_{IND}(\widehat{\theta}) + \frac{(\theta-\widehat{\theta})'\,\widehat{H}_A\,(\theta-\widehat{\theta})}{(\theta-\widehat{\theta})'\,\widehat{H}\,(\theta-\widehat{\theta})}\left(l_{IND}(\theta) - l_{IND}(\widehat{\theta})\right),$$

UCL

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- Adjust $l_{IND}(\theta)$ so that its Hessian is H_A at $\widehat{\theta}$ rather than H.
- Preserves the usual asymptotic distribution of the likelihood ratio statistic.

Findings of a simulation study

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- Slight underestimation of standard errors : uncertainty in threshold ignored.

Findings of a simulation study

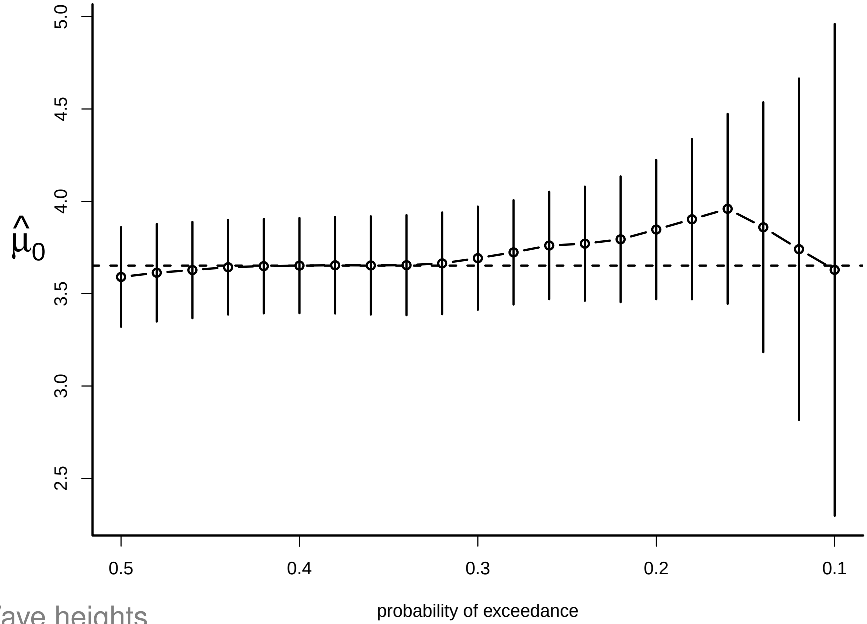
- Data simulated with spatial and temporal dependence and with spatial variation.
- Slight underestimation of standard errors : uncertainty in threshold ignored.
- Uncertainties in covariate effects of threshold are negligible compared to the uncertainty in the level of the threshold.
- Estimates of regression effects from QR and EV models are very close : both estimate extreme quantiles from the same data.
- To a large extent fitting the EV model accounts for uncertainty in the covariate effects at the level of the threshold.

Wave height modelling

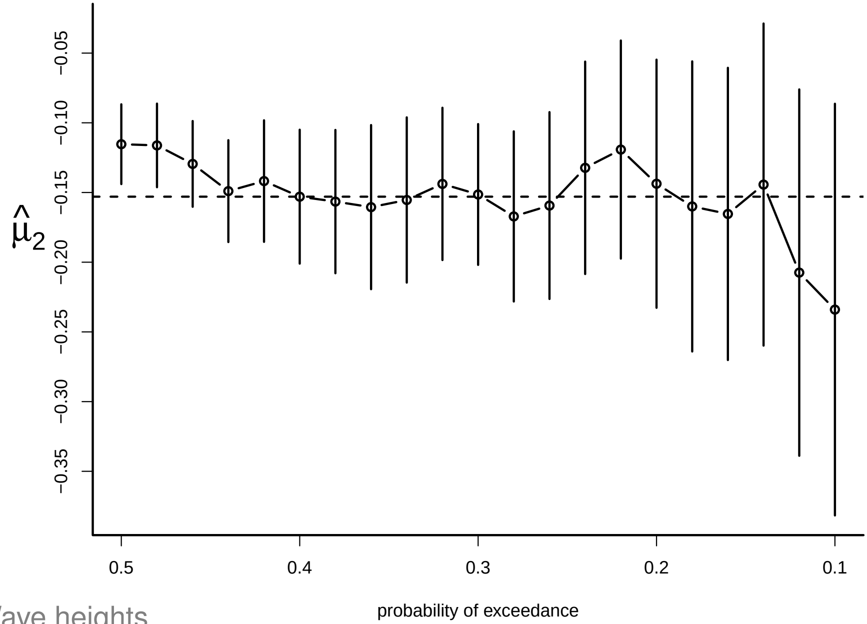
Threshold selection:

- Iterative: form of threshold depends on model.
- For given EV model set threshold using appropriate QR model.
- Choice of exceedance probability p: look for stability in parameter estimates.
- Based on μ (and u) quadratic in longtiude and latitude, σ and ξ constant ...

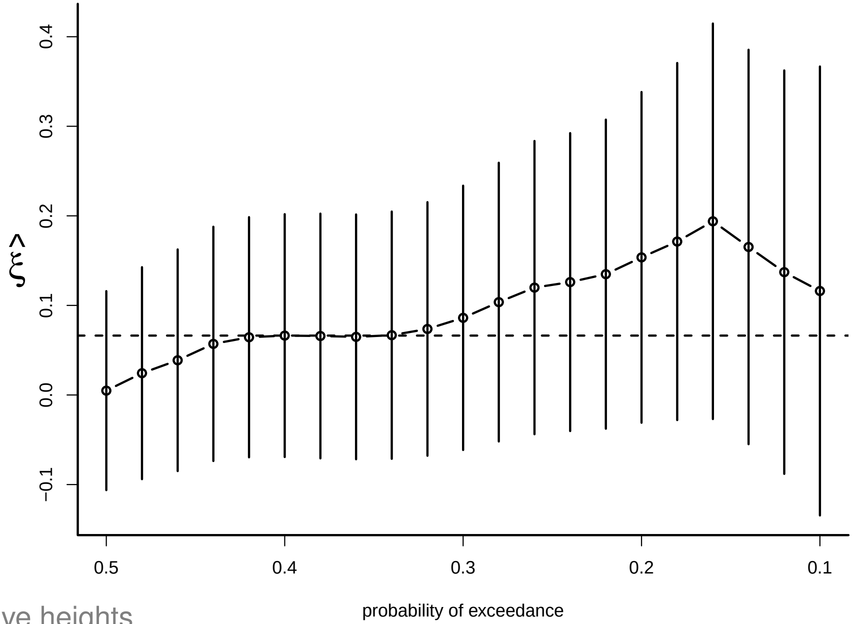
Threshold selection : μ intercept



Threshold selection : μ coeff of latitude



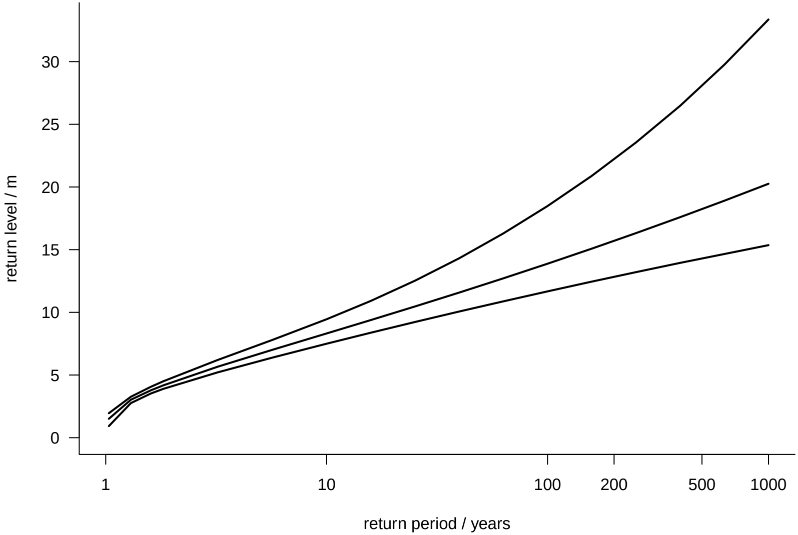
Threshold selection : ξ



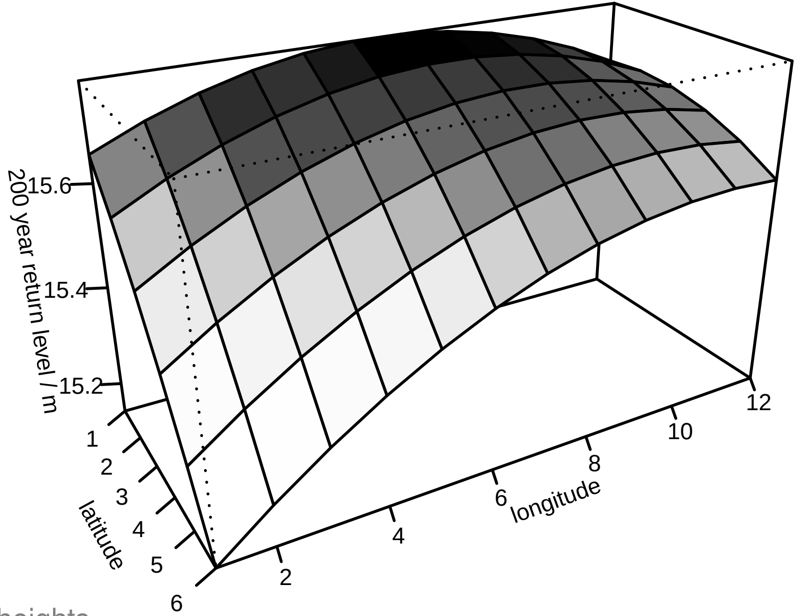
Summary of results

- Choice of p: look for stability in parameter estimates. Use $p = 0.4$.
- Model diagnostics : slight underestimation at very high levels, but consistent with estimated sampling variability.
- QR model and EV model agree closely.
- $\widehat{\xi} = 0.066$, with 95% confidence interval $(-0.052, 0.223)$.
- Estimated 200 year return level at centre of region is 15.70m with 95% confidence interval $(12.82, 22.20)$m.

Fitted return levels at centre + 95% CIs



At-site 200 year return levels



Discussion

Quantile regression

- a simple and effective strategy to set thresholds for non-stationary EV models;
- supported by simulation study;
- theoretical work is on-going;

Kyselý *et al.* (2010): QR for time-dependent thresholds (GP).

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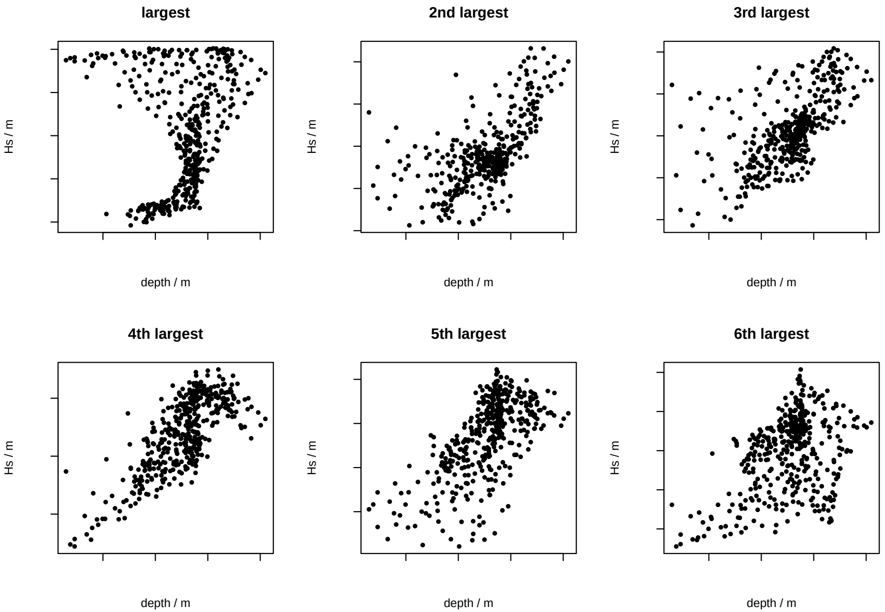
Comments

- Simpler r-largest order statistic analysis preferable for these data - but not for irregularly-spaced covariates.
- Other covariates: water depth; wave direction; 'hurricane alley'?.
- $(\widehat{U}\,\widehat{H}^{-1})^T\widehat{U}\,\widehat{H}^{-1}$ more comp. stable than $\widehat{H}^{-1}\widehat{V}\,\widehat{H}^{-1}$.

At-site order stats of H_s vs. water depth



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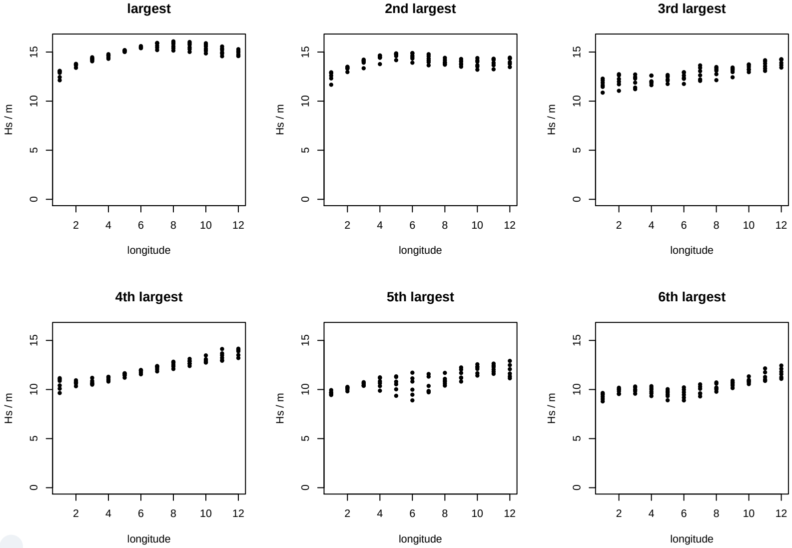
Northrop, P. J. and Jonathan, P. Threshold modelling of spatially-dependent non-stationary extremes with application to hurricane-induced wave heights. Published online in *Environmetrics*. (With discussion.)

Thank you for your attention.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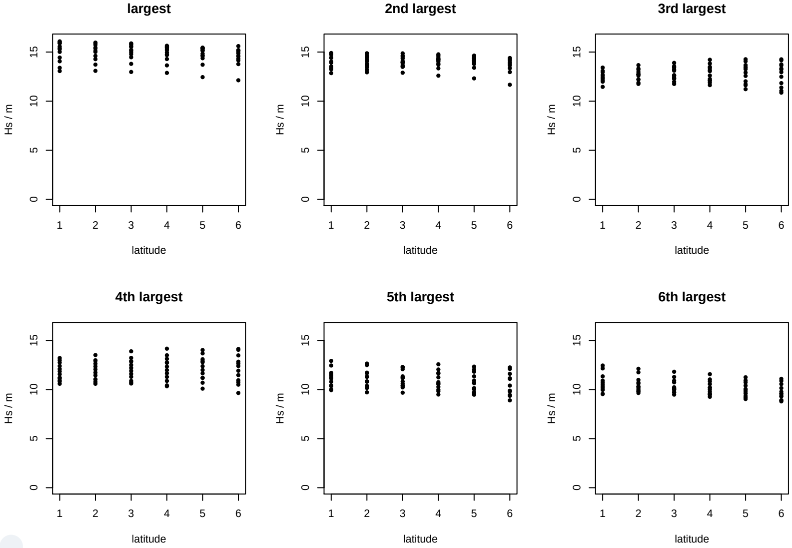
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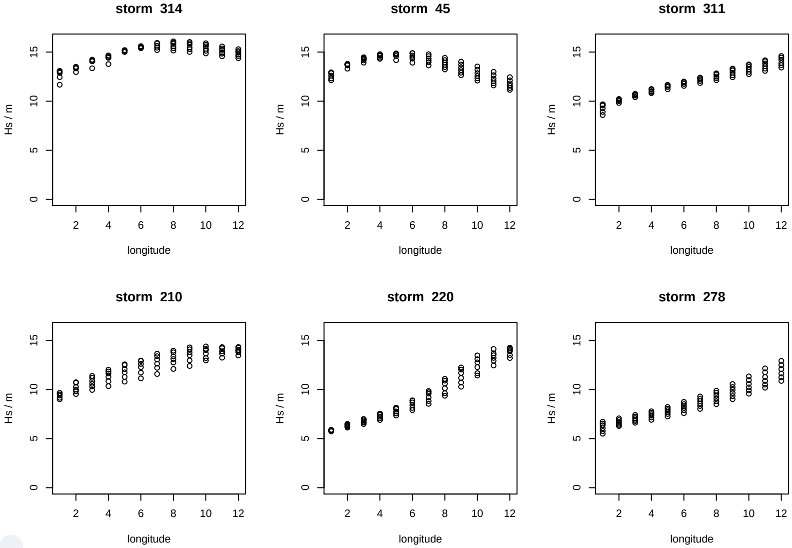
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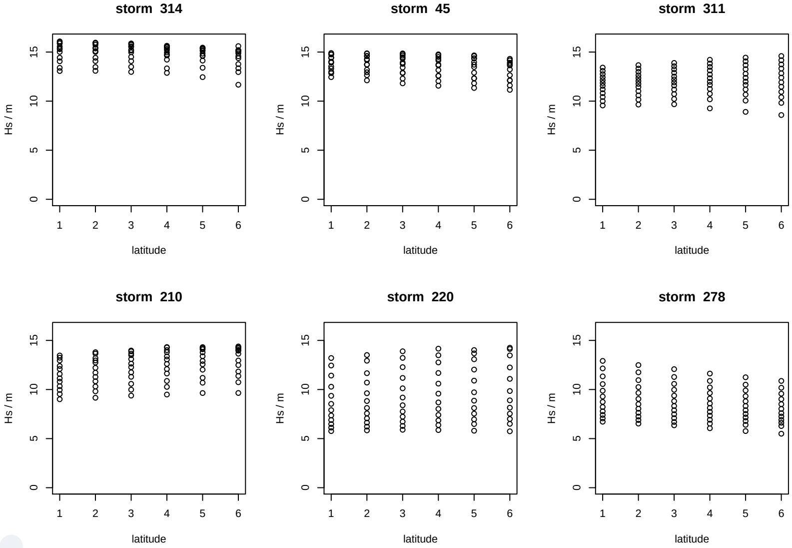
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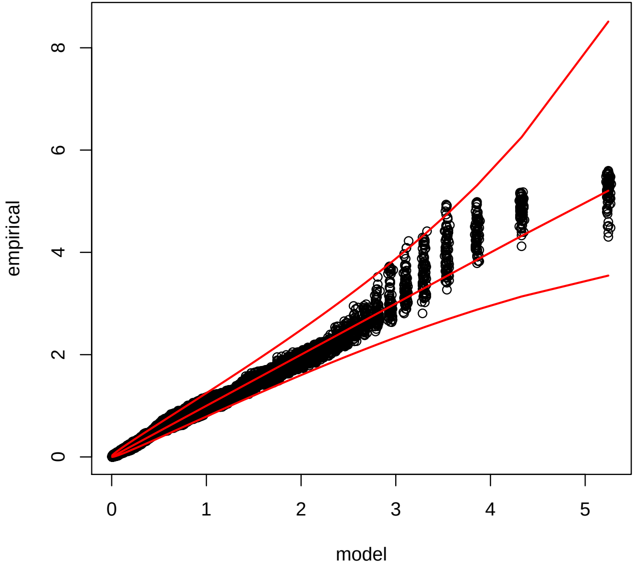
empirical
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