Project IV (MATH4072) Plan 2018/19

Title

Random walks and electrical networks

Supervisors

Chak Hei Lo, room CM225, `c.h.lo@durham.ac.uk`
Andrew Wade, room CM302, `andrew.wade@durham.ac.uk`

Abstract

Random walks are basic models of dynamics subject to random fluctuations, with wide-ranging applications in, for instance, physics (Brownian motion), finance (market models), and biology (microbe locomotion). In simple symmetric random walk on the d-dimensional integer lattice, a particle moves at each step from its current position to one of its $2d$ neighbouring sites, with equal probability of each.

A celebrated theorem of Polya says that the walk will return to its starting point again and again when d is 1 or 2, but not when d is 3 or more. One way to prove this theorem is to exploit a beautiful connection between the simple random walk problem and the theory of electrical networks. The random walk can be studied by considering the resistor network formed by wiring a 1 Ohm resistor along each of the unit-length line segments of the lattice. Roughly speaking, the random walk returns to its starting point infinitely often if and only if the corresponding resistor network has infinite effective resistance (calculated using the usual Kirchhoff laws).

The project will involve investigating aspects of random walks (with scope for simulation), including the connection to the theory of electrical networks, and applications, for example, to gambling problems.

There will also be scope for simulation.

Main aims

- Understand the basic theory of simple random walks on finite or infinite graphs, with particular emphasis on simple random walk on the integers.
- Explain the link between electrical networks and random walks on graphs, and illustrate the link via the study of some specific examples.
- Consider in detail the recurrence problem for one-dimensional random walks.

Other aims could include:

- Investigate simulations of some random walks.
- Investigate (by simulation) some random walks in random environments.

How to get started

- Revise material on random walks and Markov chains from previous courses.
- Look at some of the recommended literature (or other literature you find) to see which look most helpful. Look at other resources, e.g. on the internet.
- Read the introductory material in Doyle and Snell, and look at Chapters 3 and 14 of Feller.

General remarks

- We will have a regular (weekly) meeting to discuss your progress. It would help if the day before the meeting you send me material you have generated during the week. Each week we will review progress and decide what to do next week.
- We can of course meet at other times if needed. Just come to one of our offices or email to arrange a meeting.
- Try to write something each week. This will help to organize your work and make the final writing process easier. You should think about preparing a 'mini-report' in time for the first interview (week 8 of the first term).
- To get a top mark you are expected to show initiative and demonstrate your ability to work independently.
- Make sure you understand the rules on plagiarism.

Important dates

- End of Michaelmas term: Summary of progress to date and plans for the future.
- Epiphany, week 17: Poster + presentation.
- End of Epiphany term: Draft report.
- Start of Easter term: Submission of final report.

Reading list

Most books on probability will have something on random walk and gambler's ruin. The best treatment is in Feller's classic book. More modern books might be easier to read in places, but most do not cover the material as well as Feller. The connection with electrical networks is explored thoroughly in Doyle and Snell's notes. Several of these books say something about applications (to finance etc.) but you might want to search for other references.

- Random walks and electric networks, P.G. Doyle and J.L. Snell, 2000.
  Available from `arxiv.org/abs/math/0001057`.
- Introduction to Probability Theory and Its Applications, Volume I, W. Feller, 3rd ed., 1968.
  Chapters 3 and 14 for random walks and gambler's ruin, also Chapters 15 and 16 are relevant.
- Probability and Random Processes, G. Grimmett and D. Stirzaker, 3rd ed., 2001.
- Lectures on Contemporary Probability, G.F. Lawler and L.N. Coyle, 1999.
  Chapters 1 and 2 give a streamlined discussion of simple random walk.
- Problems and Snapshots from the World of Probability, G. Blom, L. Holst, and D. Sandell, 1994.
  Chapter 10.
- Random Walks and Random Environments, Volume 1: Random Walks, B.D. Hughes, 1995.
  Covers applications and various other models of random walks.