Multiresolution Markov random field wavelet shrinkage for ripple suppression in sonar imagery

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Detect mines in synthetic aperture sonar image

Want very high true positive rate (near 100%) and a relatively low false positive rate.
(Non-linear) matched filter (cf. Dobeck et al., 1997)

The mine template is constructed with a superposition of 3 shifted raised cosines. The shadow length is adaptive with respect to range.
False positives occur mainly in ripple field. We propose to suppress the ripples (adaptively) using ideas from fractal analysis and wavelet shrinkage.

Thanks to Dave Bull and Paul Hill (Bristol University) for data ground truth.
Dual-tree complex wavelets (Kingsbury ACHA2001)

Dual-tree wavelets provide 6 (complex) directionally selective filters with good shift invariance. (Other fast, ‘real’, wavelet transforms have poor shift invariance and provide 3 filters, only two of which have a dominant direction.)
Wavelet energy of (non-)rippled seabed patches

The extent to which the spectrum satisfies this power law can be described by scale invariant, fractal dimension based, measure (Nelson & Kingsbury ICIP2010 and IOA2010).
Definition 1 (weak statistical self-similarity)

Let \( \lambda \in \mathbb{R} \), \( H \in (0, 1] \), \( t \in T \subseteq \mathbb{R}^2 \). Stochastic field \( g : \Omega \times T \mapsto \mathbb{R} \) has weak statistical self-similarity (WSS) if

\[
\mathbb{E}g(\lambda \cdot) = \lambda^H \mathbb{E}g
\]
\[
\mathbb{E}g(\lambda t)g(\lambda \cdot) = \lambda^{2H} \mathbb{E}g(t)g(\cdot)
\]

Furthermore, \( H \) is called the Hurst parameter.

Example 2 (fractional Brownian surface \( \in \text{SS} \subset \text{WSS} \))

\[
B_H(\cdot + \delta \cdot) - B_H(\cdot) \sim \mathcal{N}\left(0, \sigma_H^2 \|\delta \cdot\|_{2H}^{2H}\right), \quad B_H(0) = 0
\]

Definition 3 (wavelet transform)

\[
(\mathcal{W}g)(\cdot ; k, m) := 2^{-k} \int_{\mathbb{R}^2} g(t) \overline{\psi}(\cdot - 2^{-k} t) \, dt
\]

Proposition 4 (e.g. Nelson & Kingsbury, IOA2010)

If \( g \) is WSS, then \( \mathbb{E}|(\mathcal{W}g)(\cdot ; k, m)|^2 \propto 2^{2k(H+1)} \)
In practice \( \mathbb{E} | \cdot |^2 \) approx. by localised sample 2nd moment in \( \delta_t \):

\[
E_{km}(t) := |\delta_t|^{-1/2} \| (\mathcal{W}g)(\cdot; k, m) \|_{\ell_2(\delta_t)} \propto 2^k(H_m(t)+1)
\]

Since \( H_m(t) \in (0, 1] \), we have \( g \) WSS on \( \delta_t \) if

\[
\frac{1}{4} \leq \frac{E_{k,m}(t)}{E_{k+1,m}(t)} = 2^{-(H_m(t)+1)} \leq \frac{1}{2}, \quad \forall k, m
\]

**ripple condition**

At scale \( k \), orientation \( m \), location \( t \), the wavelet coefficient \( (\mathcal{W}g)(t; k, m) \) contributes to a ripple region if

\[
y_{km}^-(t) := \frac{E_{k-1,m}(t)}{E_{k,m}(t)} < \theta_1 \quad \text{or} \quad y_{km}^+(t) := \frac{E_{k,m}(t)}{E_{k+1,m}(t)} > \theta_2
\]
wavelet shrinkage

- wavelet transform: \( c_{km}(\cdot) = (\mathcal{W}g)(\cdot; k, m) \)
- shrinkage: \( \tilde{c}_{km}(\cdot) = S_{km}(\cdot)c_{km}(\cdot), \ S : (\cdot; k, m) \rightarrow [0, 1] \)
- inverse wavelet transform: \( g^\sim(\cdot) = (\mathcal{W}^{-1}\tilde{c}^\sim)(\cdot; k, m) \)

Nelson & Kingsbury (IOA2010, IET2012) proposed \( S = S^{(1)}S^{(2)} \) with

\[
S^{(1)}_{km}(\cdot) := \min \left( 1, \max \left( 0, \frac{y_{k,m}(\cdot) - \theta_0}{\theta_1 - \theta_0} \right) \right)
\]

\[
S^{(2)}_{km}(\cdot) := \min \left( 1, \max \left( 0, \frac{\theta_3 - y_{k,m}^+(\cdot)}{\theta_3 - \theta_2} \right) \right)
\]
Shrinkage functions

Rippled data

DTCWT FIR
Typical rippled image
Ripple suppressed image
Unsuppressed correlation surface
Suppressed correlation surface
aggressive choice of shrinkage parameters $\theta$ over-suppresses
conservative choice of shrinkage parameters $\theta$ under-suppresses
Solution: be aggressive but spatially regularise

For each scale $k$, direction $m$...

- $x_i$ denotes state (ripple $= -1$ or non-ripple $= +1$) of the wavelet coefficient at location $t_i$
- $y_i$ denotes observation at location $t_i$
- $y = \{y_i\}, x = \{x_i\};$ joint posterior $p(x|y) \propto p(y|x)p(x)$

Aim: estimate marginal posteriors $p(x_i = 1|y_i)$ and then do:

**MRF-based wavelet shrinkage**

- wavelet transform: $c_{km}(t_i) = (\mathcal{W}g)(t_i; k, m)$
- shrinkage: $c_{\sim km}(t_i) = p(x_i = 1|y_i; k, m)c_{km}(t_i)$
- inverse wavelet transform: $g^{\sim}(t_i) = (\mathcal{W}^{-1}c^{\sim})(t_i; k, m)$
Assume $x$ Markov: $p(x_i|x_{-i}) = p(x_i|x_{\partial i})$. Choose Ising model as prior, then $p(x_i) \propto \exp \beta \sum_{j \in \partial i} x_i x_j$. Define

$$p(y_i|x_i = 1) \propto S^*_{km}(t_i|x_i = 1) \eta := (S^+_{km}(t_i)S^-_{km}(t_i)) \eta$$

$$S^-_{km}(\cdot) := \min \left(1, \frac{1}{2} \frac{y^-_{k,m}(\cdot)}{\theta_-}\right), \quad S^+_{km}(\cdot) := \min \left(1, \frac{1}{2} \frac{\theta_+}{y^+_{k,m}(\cdot)}\right)$$

Assuming $y_i \sim \text{uniform}(\theta^-_{\infty}, \theta^+_{\infty})$, we have

$$p(y_i|x_i = -1) \propto S^*_{km}(t_i|x_i = -1) \eta$$

$$:= 1 - S^*_{km}(t_i|x_i = 1) \eta$$
Multiresolution, multidirectional, Metropolis sampler

for scale $k$ and subband direction $m$ do
  $\forall i, x_i^{(1)} \leftarrow 1$, if $S_{k,m}^*(t_i|x_i = 1) > \frac{1}{2}$, and $-1$ otherwise
  $\nu_i \leftarrow 0$
  for $n = 1$ : number_iterations do
    $\ell \leftarrow$ random permutation of $1 : T_k$
    for $i = \ell(1 : T_k)$ do
      compute $\alpha \leftarrow p(-x_i^{(n)}|y_i) / p(x_i^{(n)}|y_i)$
      if $\alpha > \alpha' \sim$ uniform([0, 1]) then
        set $x_i^{(n+1)} \leftarrow -x_i^{(n)}$
      end if
      if $n >$ burn_in_period then
        $\nu_i \leftarrow \nu_i + \delta(x_i^{(n+1)}, 1)$,  $(\delta(\cdot, \cdot) =$ Kronecker's delta)  
      end if
    end for
  end for
  $p^\sim(x_i = 1|y_i; k, m) \leftarrow \nu_i / (\text{number_iterations} - \text{burn_in_period})$
end for

The acceptance ratio $p(-x_i^{(n)}|y_i)/p(x_i^{(n)}|y_i)$ is computed via the unnormalised marginal priors and likelihoods:

$$\frac{p^*(−x_i|y_i)}{p^*(x_i|y_i)} = \frac{1 - S_{k,m}^*(t_i|x_i)\eta}{S_{k,m}^*(t_i|x_i)\eta} \exp \left(−2\beta \sum_{j\in\partial i} x_i x_j \right)$$
deterministic conservative suppression

correlation surface

S_{6,1}  S_{6,2}  S_{6,3}  S_{6,4}  S_{6,5}  S_{6,6}
motivation

wavelets and self-similarity

MRF framework

detector results

examples

deterministic aggressive suppression

correlation surface

$S_{6,1}$ $S_{6,2}$ $S_{6,3}$ $S_{6,4}$ $S_{6,5}$ $S_{6,6}$
MRF suppression

correlation surface

$$S_{6,1}^*(\cdot|x_i = 1)$$  $$S_{6,2}^*(\cdot|x_i = 1)$$  $$S_{6,3}^*(\cdot|x_i = 1)$$  $$S_{6,4}^*(\cdot|x_i = 1)$$  $$S_{6,5}^*(\cdot|x_i = 1)$$  $$S_{6,6}^*(\cdot|x_i = 1)$$
Number of false positives incurred to recover 90%, 95%, 98%, 99%, and 100% of the total number of true positives using: no suppression; conservative and aggressive deterministic method; and MRF-based method.

<table>
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<tr>
<th>Seabed type</th>
<th>Number of true positives</th>
<th>Number of false positives</th>
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The end. (PTO for notes)
Context: standard detector/classification scheme

Detector finds (true and false) positives. **Training phase**: a binary classifier is trained on features extracted from both true and false positives. **Testing phase**: the detector is applied, features are extracted from positives, which are then classified by learned decision function.
Proposed detector/classification scheme

Pre-processing suppresses ripples. Detector finds (true and false) positives. **Training phase**: a unary classifier is trained on features extracted from false positives only. **Testing phase**: Pre-processing suppresses ripples. Detector is applied, features are extracted from positives, which are then classified by learned decision function (true positives are anomalies).
More examples (using aggressive deterministic suppression without spatial prior)...
Shrinkage functions

Rippled data

DTCWT FIR

$S_{5,1}$

$S_{5,2}$

$S_{5,3}$

$S_{5,4}$

$S_{5,5}$

$S_{5,6}$

$S_{6,1}$

$S_{6,2}$

$S_{6,3}$

$S_{6,4}$

$S_{6,5}$

$S_{6,6}$
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Ripple suppressed image
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$S_{6,1}$  $S_{6,2}$  $S_{6,3}$  $S_{6,4}$  $S_{6,5}$  $S_{6,6}$
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