If all models are wrong, are the robust ones better?

The role of robust statistics in a constructivist philosophy of statistics

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1. A constructivist philosophy of mathematical models

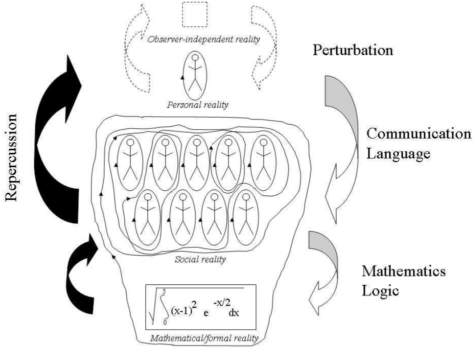
Most people know models are “wrong but sometimes useful”, but most philosophy of statistics ignores this (and many researchers do, too, when using them).

Missing: philosophy of mathematical modelling.

Mathematical modelling (broad and naive):
mapping (perceived) reality to mathematical objects.

Note: in this sense, not only “$\mathcal{N}(\mu, \sigma^2)$”
is a mathematical model,
but also a breakdown point is a mathematical model of
what we think of as the “quality of robustness”.

Mathematics and reality
(H., 2010, Foundations of Science)



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“Absolute agreement” within mathematics makes people believe that mathematical modelling can achieve agreement about reality.

But this relies on connection between formal models and informal reality, which is inaccessible to formal analysis.

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  Models live in "world of mathematics"
  and cannot be "true" in informal reality.
- Pragmatist attitude: what do we get out of it?

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Relevant questions when discussing models

- What are the implications for modelling of not really believing in the “truth” of models?
- How is perceived reality mapped on models? Do people agree about this reality?
- What aims can modelling help to achieve, and how are different modelling methods/strategies related to these aims?
- What are the implications for modelling of models influencing communication, thoughts, and reality?

2. Some thoughts on frequentism

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“We think of the situation as . . . ”

- potentially infinite repetition (of experimental conditions),
- no systematic dependence between repetitions,
- $P(A)$: relative frequency limit of occurrence of A

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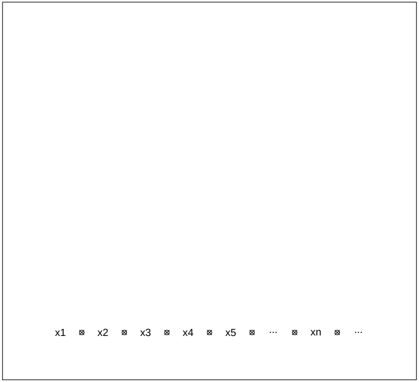
"Infinite repetition" is usually modelled as i.i.d.,
but i.i.d. is defined *in terms of* probability models,
so cannot explain what such models mean.

In order to define “i.i.d.” sequences, i.i.d. repetitions and defining repetitions are required on different levels.



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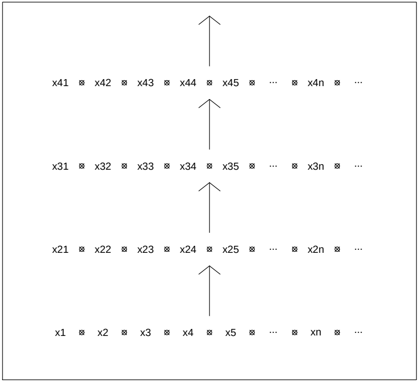

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Repetition is constructed by selective ignorance.

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(Similarly, Bayesians need exchangeability.)

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“Approximately true”-arguments are *within* mathematical models;
not about models vs. informal reality.

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However, with every data set,
huge classes of models are compatible.

So this doesn’t give a useful notion of
“approximately true models”.

3. Frequentism-as-model

Exploit frequentist modelling to

- communicate researcher's perception of situation,
- inspire methodology,
- understand implications of methods,
- approximate data,
- check quality of methods
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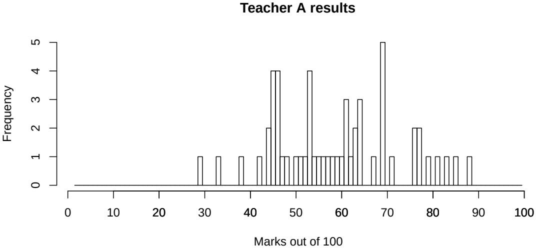
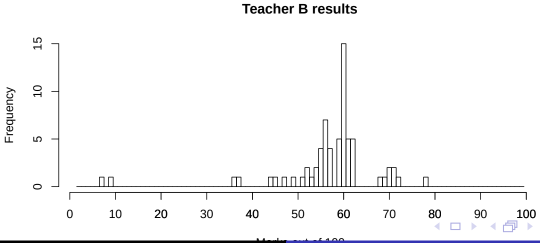
Keep in mind that
models are neither true nor approximately true;

consider other aspects of "usefulness".

Example


Teacher A results
Frequency
0 1 2 3 4 5
0 10 20 30 40 50 60 70 80 90 100
Marks out of 100


Teacher B results
Frequency
0 5 10 15
0 10 20 30 40 50 60 70 80 90 100

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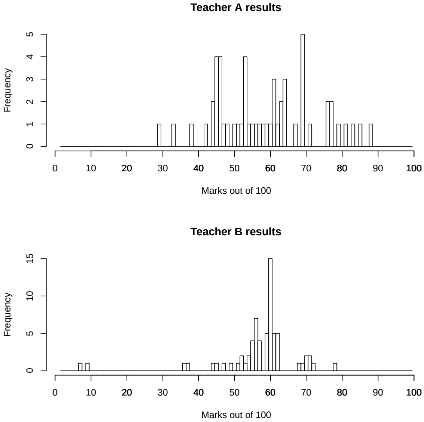
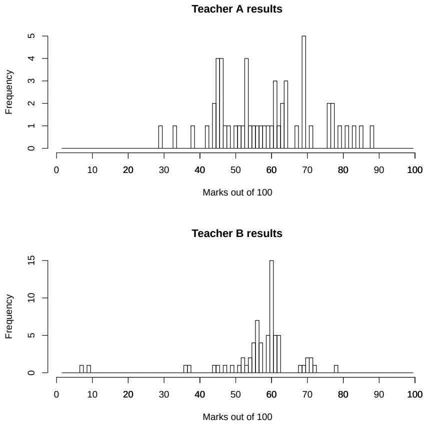
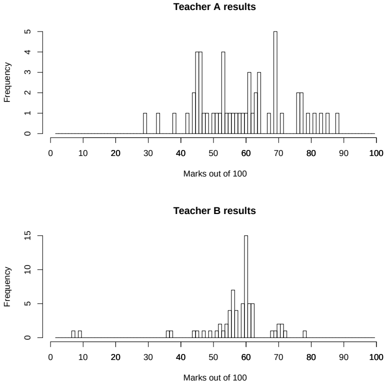
Mean vs. median vs. . . . by optimality in model?
Max breakdown?
(No ultimate criteria, but aspects of understanding.)



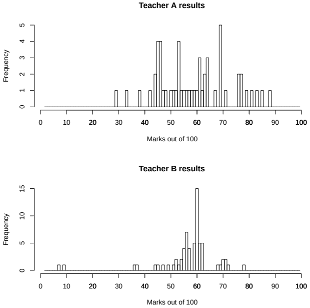
Means: 58.6 (A), 56.9 (B)
Medians: 58 (A), 59 (B).



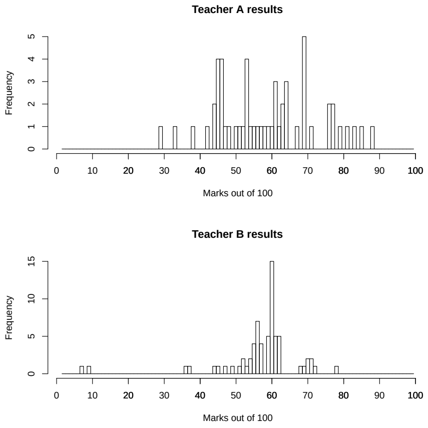
For median: lower outliers in B.



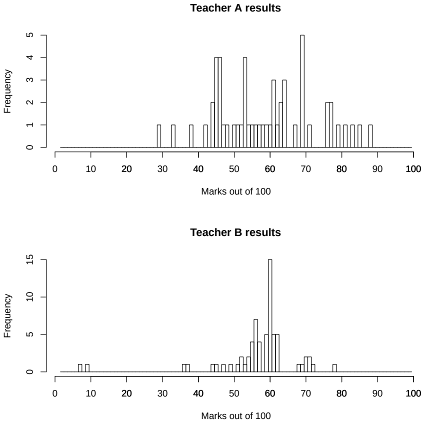
For mean: observations are not erroneous,
so *all* observations should contribute!?
High breakdown may imply ignoring relevant information.



Against mean: it's really not very relevant
how bad the clearly failed students exactly are.
(Would be different for upper outliers.)



Could look at pass ($\geq$ 50) rate (B clearly better)
and other meaningful statistics instead.



"Which aspect is of interest?"/meaning of data dominates "Which model is close to the truth."

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(top students; insurance payouts,…);
although theory then warns us about instability.

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Though problem with deviation extends to *this* model.

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- Influences researcher's perception (as all models): e.g., "good observations" vs. "outliers",
- Do standard robustness measures really measure what's desirable? Sometimes...
  May not want to throw as much information away as high breakdown does.
- May tempt researchers into believing that the "violation of assumptions"-problem will go away.

Conclusion

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vs.
decisions made by “the data alone, not the researcher”
or mathematical optimisation
(popular in science in general)

Robust statistics models
what happens when models are wrong.

That's a big step forward,
but also a paradox.