UCL

Measurement as a constructive act - a statistician's view

Christian Hennig

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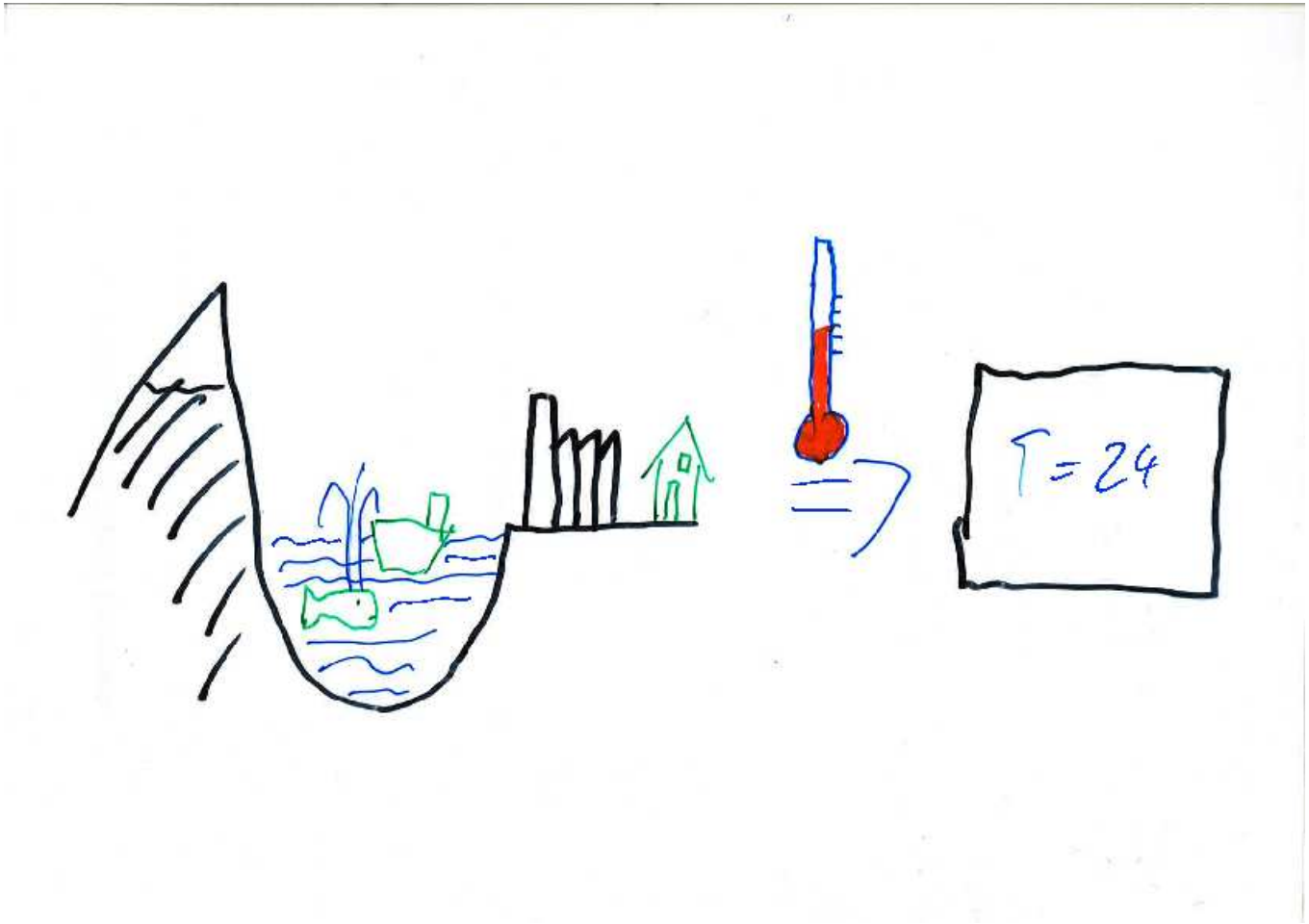
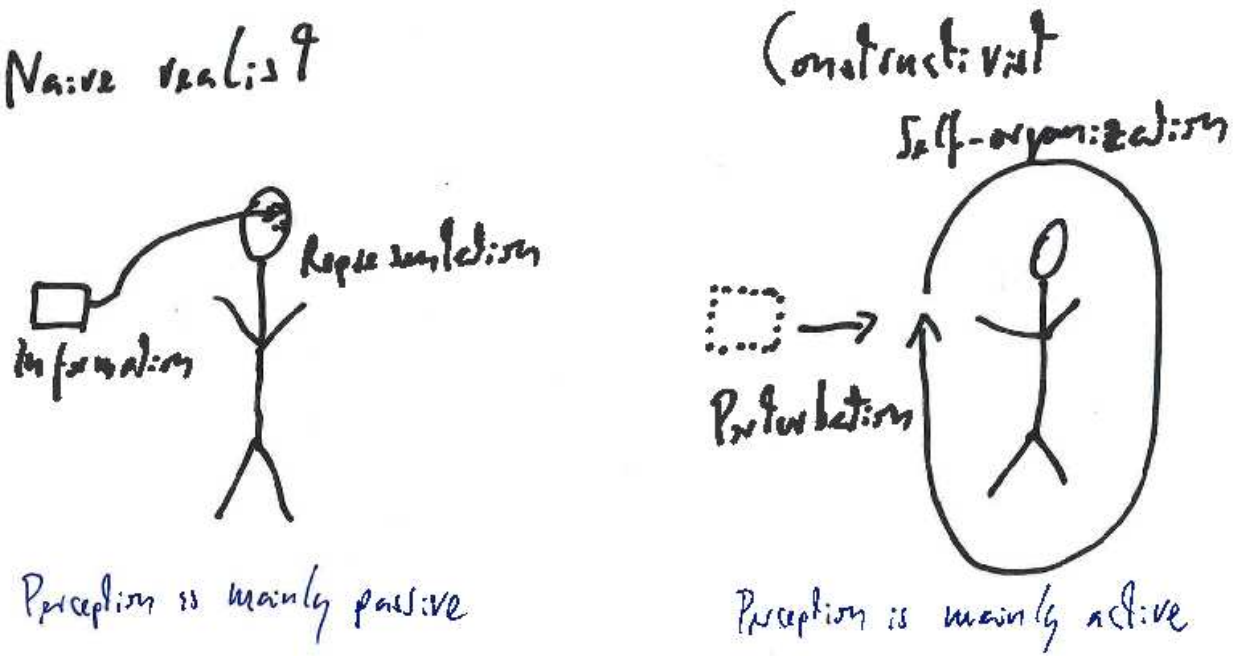
1. Constructivism, measurement, mathematics

Mathematical modelling (broad and naive):
mapping reality to mathematical objects.

Measurement: “anchor” of modelling:
methods for assigning values to aspects of reality.



Constructivism



(Constructivism is not typical for statisticians!)

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A constructivist view of mathematical modelling

- Science is about establishing agreement in open exchange.
  Mathematics is about creating a system that makes absolute agreement possible.
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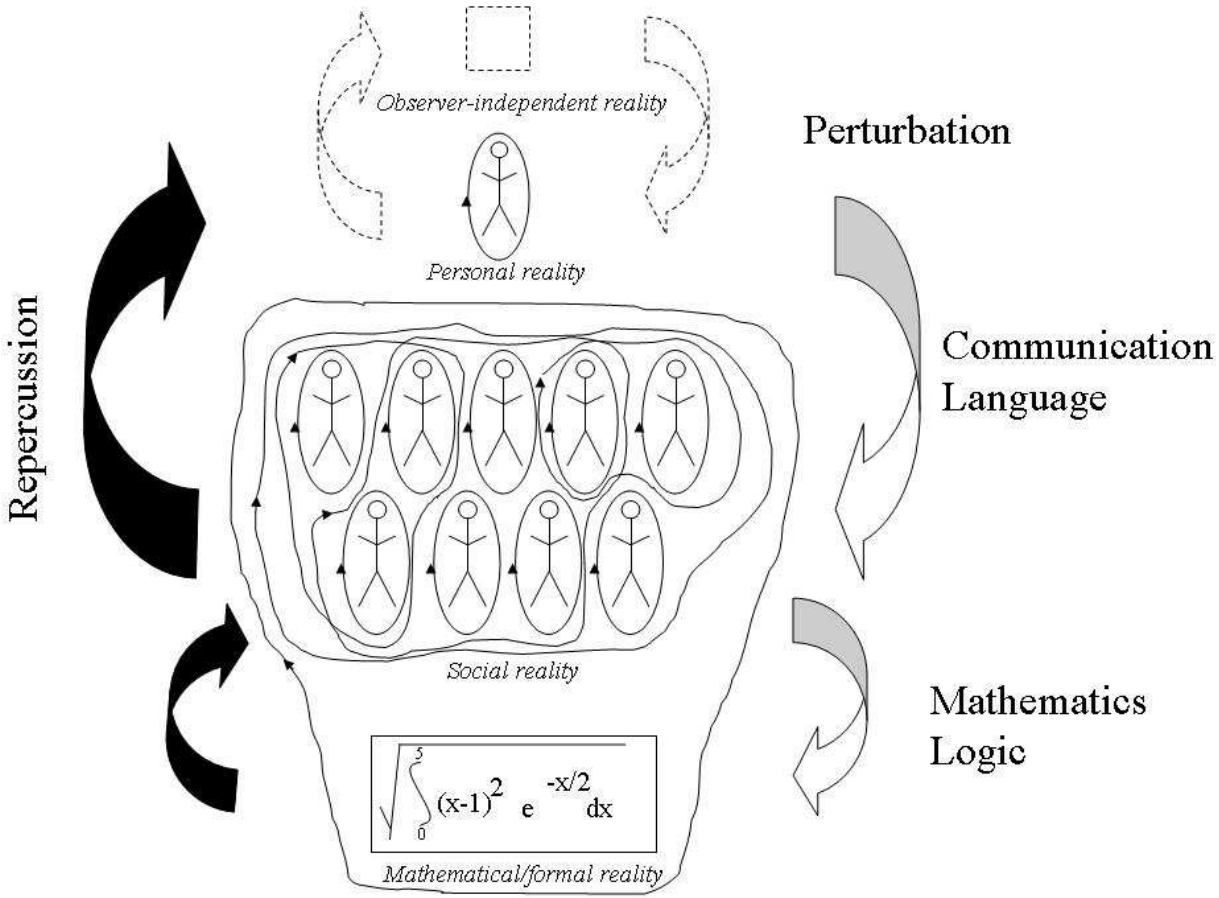
  (But that's *within mathematics*.)
- Mathematical modelling is not about how things are, but about how we perceive them
  and how we think and communicate about them.
- Mathematics is human construction,
  but not "as opposed to real"; it's a *perspective*.

(H 2010, Foundations of Science)



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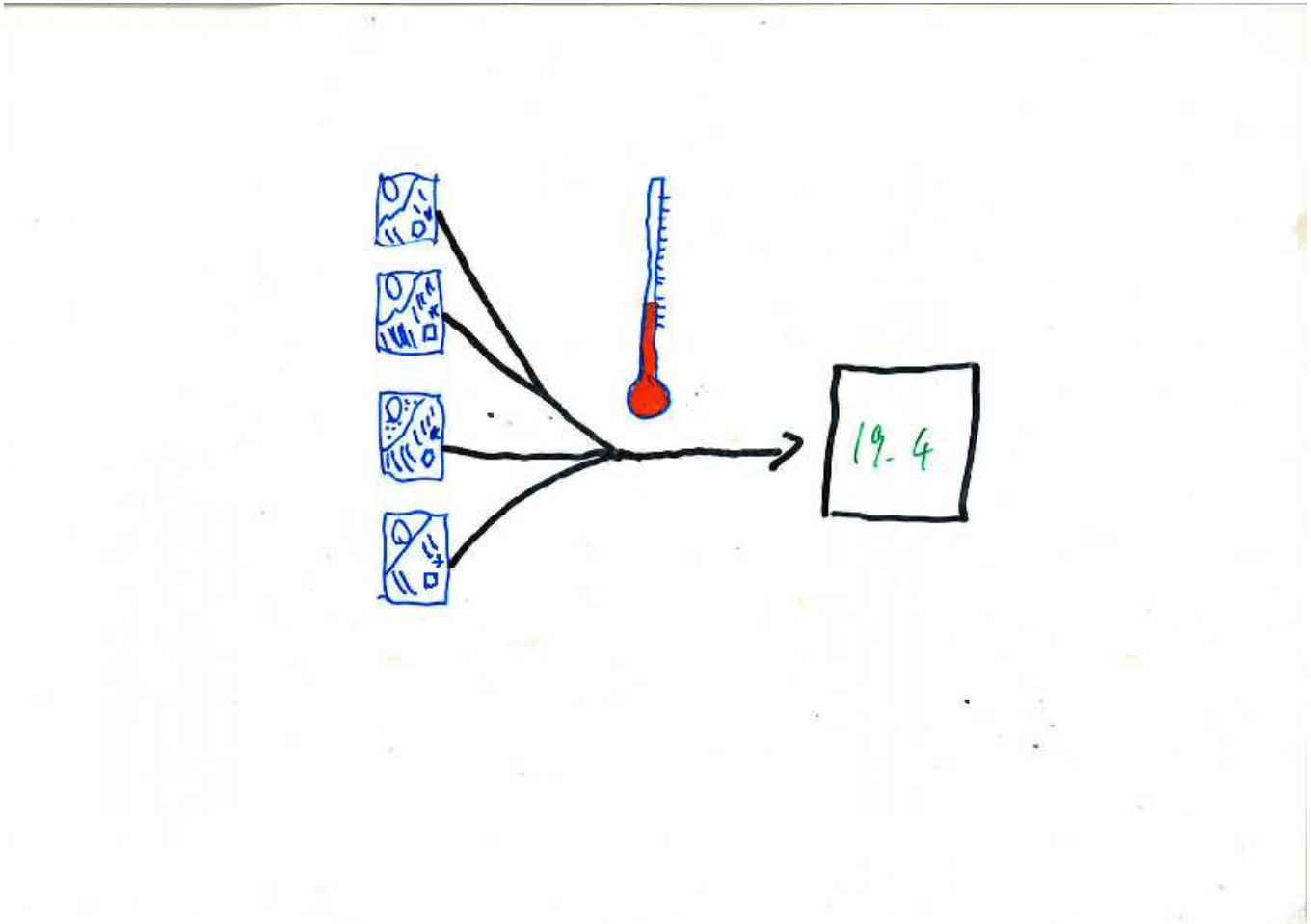
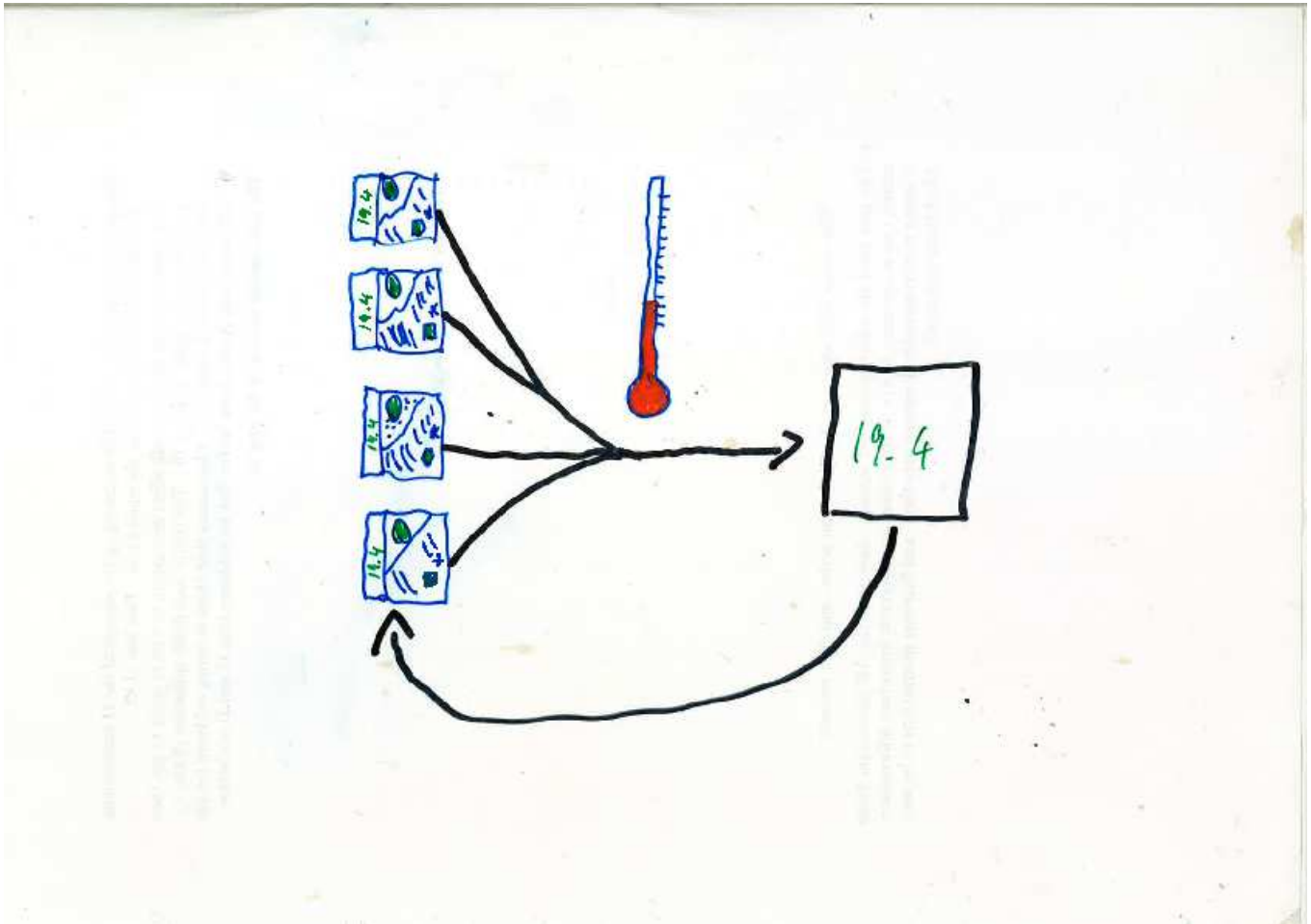
This requires a *change* of perception,
and constructive negotiation.


19.4

. . . and measurement influences perception.



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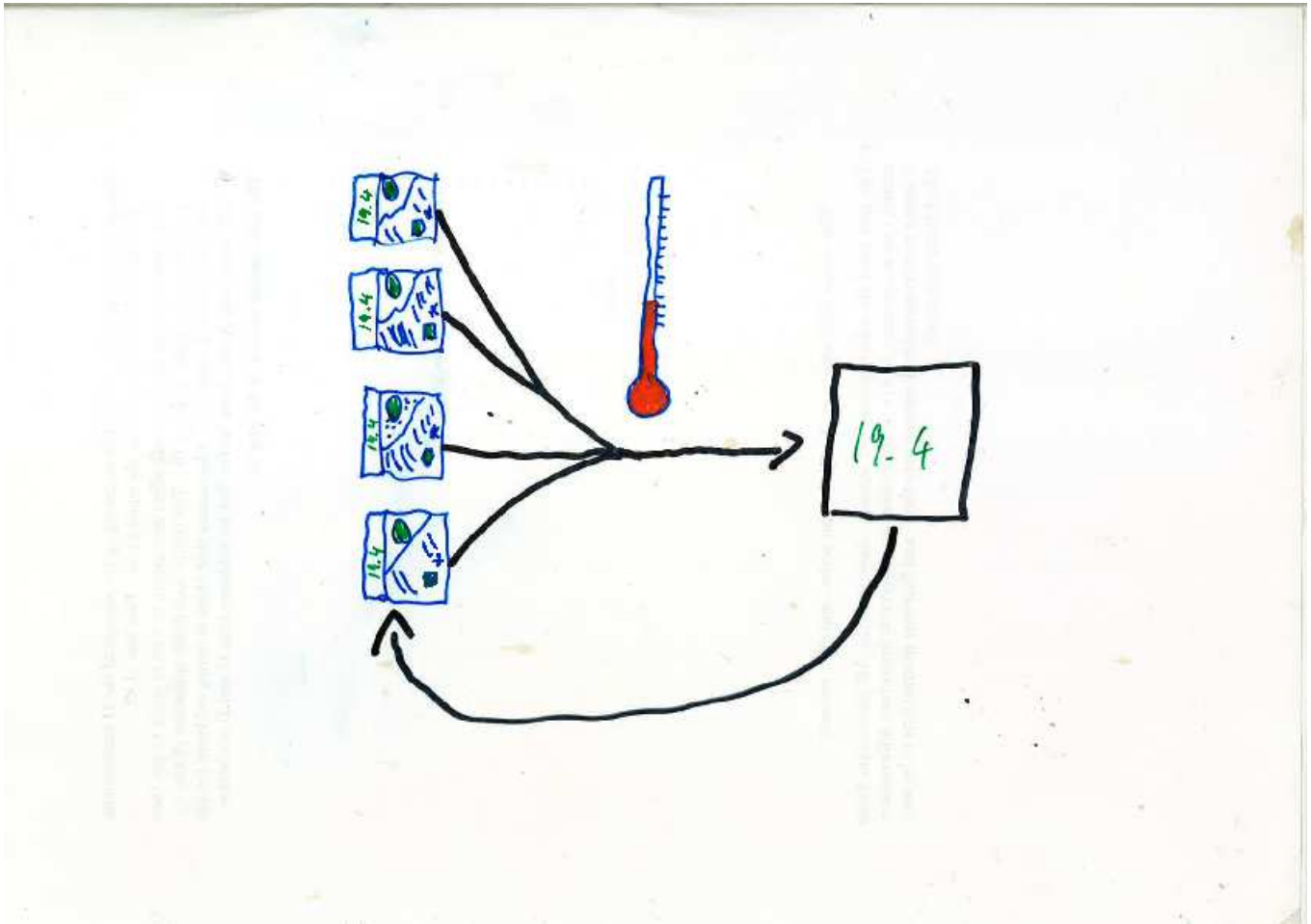

Measurement does more than just measuring.

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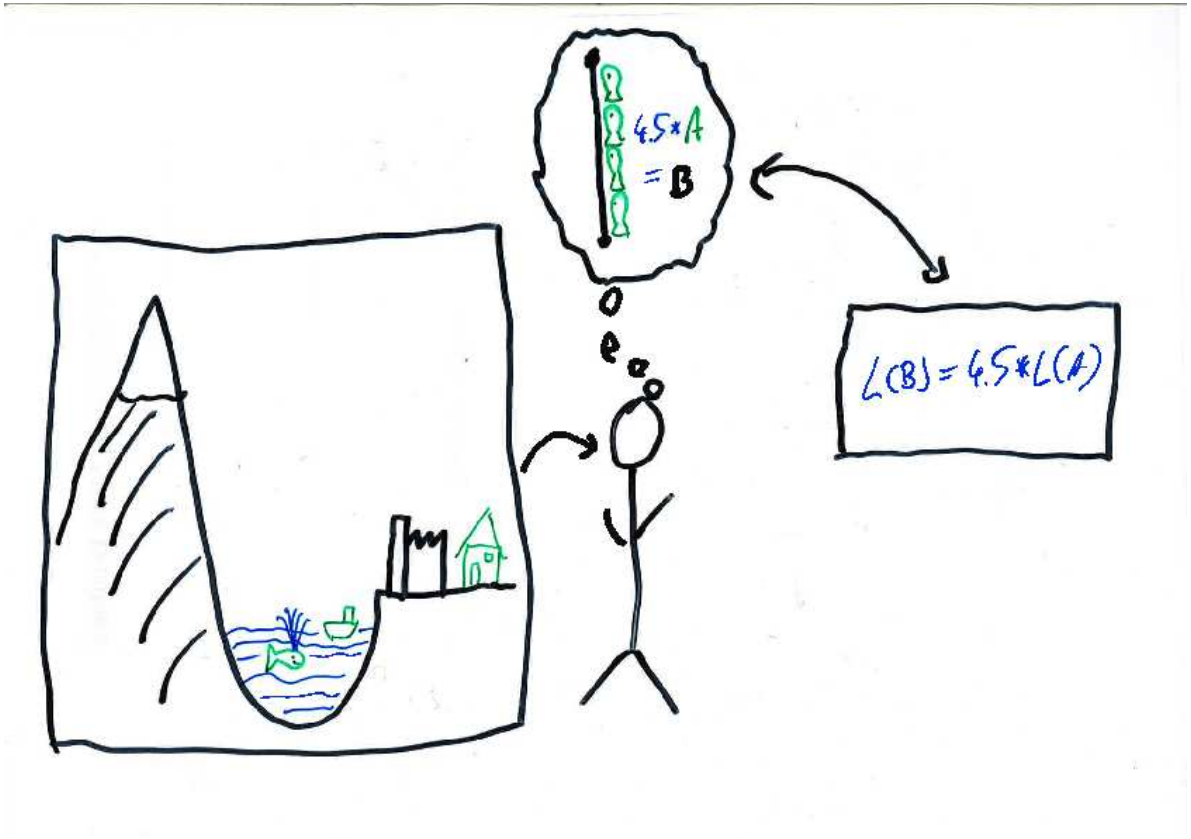
The operational concept: (Bridgman 1927)

What is measured is defined
by the measurement procedure.

2.1 A constructivist discussion of measurement concepts

Classical and representational concept require measurement to be reflection of formal aspects of reality.



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Whether measurement captures quantitative reality,
or exists homomorphism between reality and measurement,
can only be found out if real quantities or structures
are measured/modelled first.

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ERS is *created* before representation.

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Cuts out the “negotiation”;
how does measurement relate to what is intended?

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Iteration to (temporary) stability/agreement
(as in Chang 2004).

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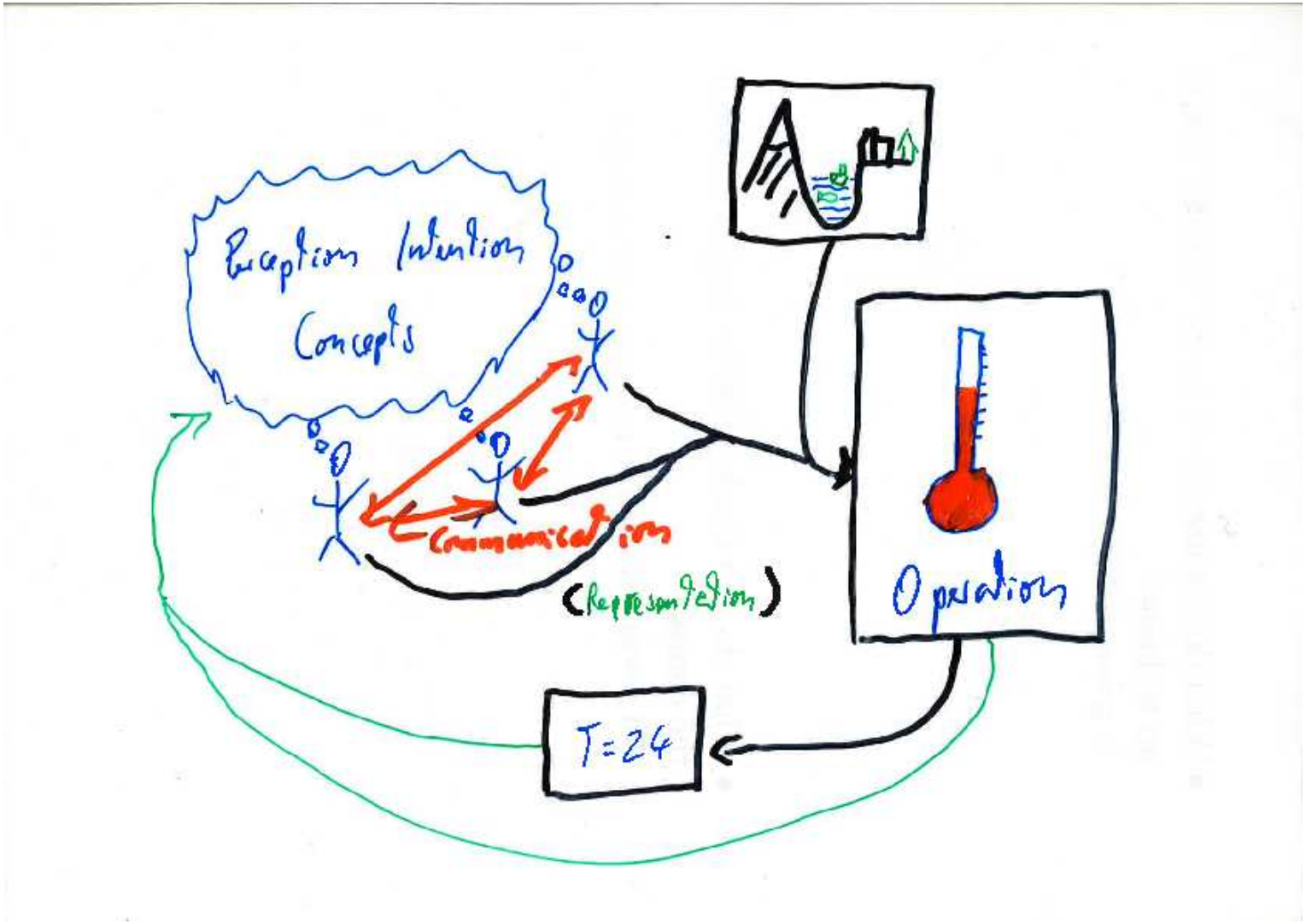

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Basic measurement *generating*
mathematical objects and structures is different
from using existing mathematical objects
to measure something new.

Primary measurement: measurement connected to basic sources of mathematical thinking.
Counting, lengths, weights, partitioning.

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Classical/representational views seem “natural”, because mathematical structures were originally *identified* with such tasks.

Secondary measurement: existing mathematical structures are used to measure something “new”.

More difficult job to justify representation.
Meaningful in classical concept?

Tertiary measurement (index construction): measurement defined by mathematical aggregation of primary and/or secondary measurements (explicit or “latent variable”).

3. Measurement and statistics
3.1 Measurement error

Typical statistical model for measurement T (Gauss, Laplace...):

$$T = X + \epsilon, \ \epsilon \text{ i.i.d. random (normal)}, \ E\epsilon = 0.$$

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Identifiability condition *defines* truth as ET!
Truth is not primary; it's derived from measurement.

Why believe in “errors”?

- Imperfect match of operation and theory/intention,
- known measurement instrument malfunction,
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“Random errors” convenient conceptual tool
to deal with this and to avoid useless complexity.

But “error” is a loaded term.
Meaningful variation, dependence,
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Model confounds “error” with “instable reality”.
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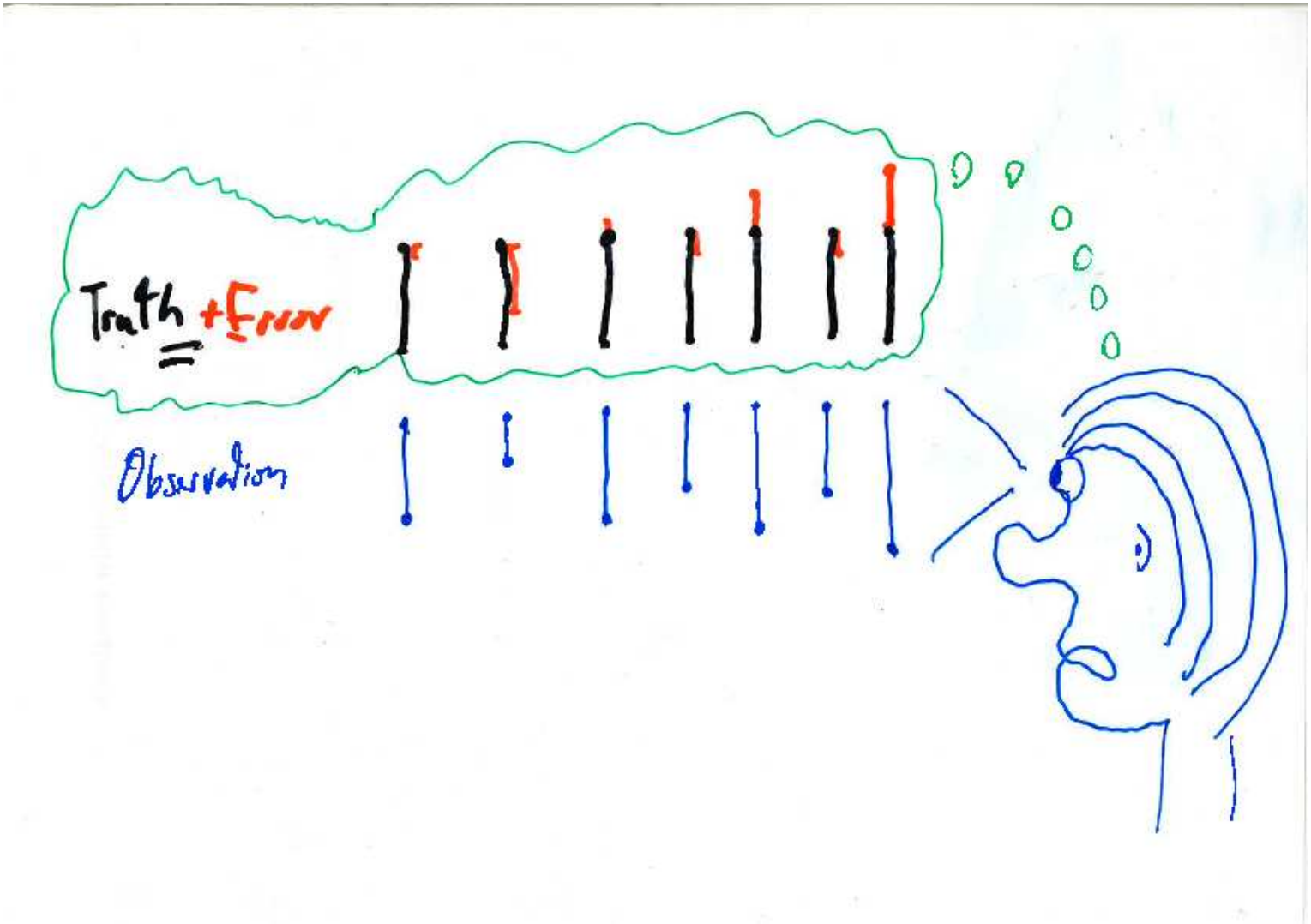
Truth is estimated by minimising observed error.
Misinterpretation: “we know truth, error is small.”


Truth
+Error
=
Observation

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Validity: the procedure measures what it's supposed to measure.
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Can estimate validity and reliability from data
under assumptions of measurement error model.
But crucial assumptions cannot be checked.

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- any two observable situations are different,
  reliability assumes them to be the same.

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Focus on *both* measurement instrument and researcher's constructs.

3.3 Scale types, index construction

Statisticians/mathematicians tend to be happy
about formal prescriptions for measurements.
Many of these are inspired by representationalism.

Scale types: Stevens (1946) distinguishes
nominal, ordinal, interval, ratio, absolute scales.

Scale types are claimed (by mathematical arguments) to
license only specific statistical methods
(e.g., arithmetic mean requires interval scale).

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Such requirements tend to ignore more subtle perceptions
such as variable importance for researcher's aim,
symmetric meaning of scales such as
"strongly disagree/disagree/neutral/agree/strongly agree"

Mathematical properties are popular
because they look “objective”.
“Let the data make the decisions!”

Task of “translation”
of researcher’s concepts and aims
into mathematical formula too often ignored
(e.g., implications of variable weighting).

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Statistics tends to start from naive “truth and error” idea.
Should not take as confirmation what is only a tool.