A review of

Partial Differential Equations in General Relativity
Alan D. Rendall
2008 New York: Oxford University Press
296pp. UK £29.95, US $60.00
ISBN: 978-0-19-921541-6.

Although many books on general relativity contain an overview of the relevant background material from differential geometry, very little attention is usually paid to background material from the theory of differential equations. This is understandable in a first course on relativity but it often limits the kinds of problems that can be studied rigorously.

Einstein's field equations lie at the heart of general relativity. They are a system of partial differential equations (PDEs) relating the curvature of spacetime to properties of matter. A central part of most problems in general relativity is to extract information about solutions of these equations. Most standard texts achieve this by studying exact solutions or numerical and analytical approximations. In the book under review, Alan Rendall emphasises the role of rigorous qualitative methods in general relativity. There has long been a need for such a book, giving a broad overview of the relevant background from the theory of partial differential equations, and not just from differential geometry. It should be noted that the book also covers the basic theory of ordinary differential equations.

Although there are many good books on the rigorous theory of PDEs, methods related to the Einstein equations deserve special attention, not only because of the complexity and importance of these equations, but because these equations do not fit into any of the standard classes of equations (elliptic, parabolic, hyperbolic) that one typically encounters in a course on PDEs. Even specifying exactly what ones means by a Cauchy problem in general relativity requires considerable care. The main problem here is that the manifold on which the solution is defined is determined by the solution itself. This means that one does not simply define data on a submanifold.

Rendall's book gives a good overview of applications and results from the qualitative theory of PDEs to general relativity. It would be impossible to give detailed proofs of the main results in a self-contained book of reasonable length. Instead, the author concentrates on providing key definitions together

with their motivations and explaining the main results, tools and difficulties for each topic. There is a section at the end of each chapter which points the reader to appropriate literature for further details. In this way, Rendall manages to describe the central issues concerning many subjects.

Each of the twelve chapters (except for one on functional analysis) contains an important application to general relativity. For example, the chapter on ODEs discusses Bianchi spacetimes and the Einstein constraint equations are discussed in the chapter on elliptic equations. In the chapter on hyperbolic equations, the Einstein dust system is considered in the context of Leray hyperbolicity and Gowdy spacetimes are analysed in the section on Fuchsian methods. The book concludes with four chapters purely on applications to general relativity, namely *The Cauchy problem for the Einstein equations*, *Global results*, *The Einstein-Vlasov system* and *The Einstein-scalar field systems*.

On reading this book, someone with a basic understanding of relativity could rapidly develop a picture, painted in broad brush strokes, of the main problems and tools in the area. It would be particularly useful for someone, such as a graduate student, just entering the field, or for someone who wants a general idea of the main issues. For those who want to go further, a lot more reading will be necessary but the author has sign-posted appropriate entry points to the literature throughout the book. Ultimately, this is a very technical subject and this book can only provide an overview. I believe that Alan Rendall's book is a valuable contribution to the field of mathematical relativity.

<div align="right">

— R. G. Halburd, University College London.

</div>