

SPECTRAL THEORY

HOMEWORK 4

1. Suppose X is a Banach space and let $A \in \mathcal{B}(X)$ be *nilpotent*, i.e. $A^n = 0$ for some $n \in \mathbb{N}$. Find $\sigma(A)$.

2. Show that the range of the operator $B : l^p \rightarrow l^p$, $1 \leq p < \infty$,

$$(Bx)_n := \frac{1}{1+n^2} x_n, \quad n \in \mathbb{N}, \quad x = (x_1, x_2, \dots),$$

is not closed.

3. Let $P \in \mathcal{B}(X)$ be a projection, i.e. $P^2 = P$. Construct $R(P; \lambda)$.

4. Let $A_l^{-1} \in \mathcal{B}(Y, X)$ be a left inverse of $A \in \mathcal{B}(X, Y)$, i.e. $A_l^{-1}A = I_X$. Find $\sigma(AA_l^{-1})$.

Let $B_r^{-1} \in \mathcal{B}(Y, X)$ be a right inverse of $B \in \mathcal{B}(X, Y)$, i.e. $BB_r^{-1} = I_Y$. Find $\sigma(B_r^{-1}B)$.