SPECTRAL THEORY

HOMEWORK 2

1. Let $B \in \mathcal{B}(C([0,1]))$ be defined by the formula

$$Bf(t) = tf(t), \quad t \in [0,1].$$

Find $\sigma(B)$ and the set of all eigenvalues of B.

2. Let $g \in C([0,1])$ be a given function and let $A \in \mathcal{B}(C([0,1]))$ be defined by the formula

$$Af(t) = g(t)f(t), \quad t \in [0,1].$$

Find $\sigma(A)$ and construct effectively the resolvent $R(A;\lambda)$. Find the eigenvalues and eigenvectors of A.

3. Let $K \subset \mathbb{C}$ be an arbitrary nonempty compact set. Construct an operator $B \in \mathcal{B}(l^p)$, $1 \leq p \leq \infty$, such that $\sigma(B) = K$.

4. Let $k \in C([0,1])$ be a given function. Consider the operator $B \in \mathcal{B}(C([0,1]))$ defined by the formula

$$(Bu)(s) = \int_0^s k(t)u(t)dt.$$

Construct effectively (not as a power series!) the resolvent of A. How does this resolvent $R(B;\lambda)$ behave when $\lambda \to 0$?

5. Let $A, B \in \mathcal{B}(X)$. Show that for any $\lambda \in \rho(A) \bigcap \rho(B)$,

$$R(B;\lambda) - R(A;\lambda) = R(B;\lambda)(A-B)R(A;\lambda).$$