M111 (Spectral Theory)

Year:	2015–2016
Code:	MATHM111
Level:	Masters
Value:	Half unit (= 7.5 ECTS credits)
Term:	1
Structure:	3 hour lectures per week
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH3103, MATH3109, (MATH3101 recommended)
Lecturer:	Prof L Parnovski

Course Description and Objectives

Spectral theory came to prominence when quantum mechanics was introduced in modern physics. In quantum mechanics classical quantities (position, momentum etc) are represented by operators (bounded, unbounded, self-adjoint etc). The eigenvalues of these operators are the only precise measurements of the quantity. Without requiring knowledge of physics, this course introduces the fundamentals of such operators and the space of their eigenvalues, which is called the spectrum.

Recommended Texts

Recommended books are (i) E. Brian Davies, *Linear Operators and Their Spectra* (Cambridge Studies in Advanced Mathematics), (ii) W. Arveson, *A short course on spectral theory* (Springer Graduate Texts in Mathematics).

Detailed Syllabus

Banach and Hilbert Spaces. Orthogonal projections. Orthonormal bases. Fourier expansion. Riesz representation theorem. Linear operators (bounded and unbounded). Adjoint operator. Symmetric, normal, self-adjoint and compact operators. Resolvent. Spectra of linear operators: classification and properties. Spectral theorem for compact self-adjoint operators. Applications to differential and integral equations. Further topics are chosen from: spectral theorem for bounded self-adjoint operators; unbounded operators and applications; Fredholm operators; extensions of symmetric operators; quadratic forms.