Mechanisms for Wave Packet Formation and Maintenance in a Quasigeostrophic Two-Layer Model

J. G. ESLER AND P. H. HAYNES

Centre for Atmospheric Science, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, United Kingdom

(Manuscript received 30 December 1997, in final form 6 October 1998)

ABSTRACT

A quasigeostrophic, two-layer, β -plane channel model is used to investigate the dynamics of baroclinic wave packets. A series of experiments are performed in which an unstable flow is maintained by lower-level Ekman friction and radiative relaxation toward a temperature profile that corresponds to a broad parabolic upper-level jet. The final statistically steady state achieved in each experiment is found to depend on the magnitude of the hyperdiffusivity ν_0 and the supercriticality, which is controlled by β . The most important qualitative difference in such states between experiments is found to be the degree to which a waveguide in the upper level is found to develop. The mechanism for this upper-level waveguide development is the mixing effect of the eddies at the flanks of the jet, which leads to a strong potential vorticity gradient at the center of the channel, with well-mixed regions to the north and south.

Two distinct regimes with different qualitative behavior are observed and illustrated by two particular experiments. In the first regime strong hyperdiffusivity inhibits the development of the waveguide. Steady wave packets are shown to stabilize the background flow upstream by increasing the meridional shear of the jet. This upstream stabilization is argued to be a mechanism for packet maintenance in this regime. In the second regime the diffusivity is lower, and a well-developed upper-level waveguide results. The wave packets in this regime are unsteady and are shown to stabilize the background flow at, and slightly upstream of, their maxima. Wave activity diagnostics suggest that the most important mechanism in maintaining these packets is the zonal convergence of wave activity, indicating that the wave packets are undergoing a form of nonlinear self-focusing, analogous to that identified in weakly nonlinear models.

Finally, results are presented from a 10-level primitive equation model with parameter values relevant to the real atmosphere. In this experiment the nonlinear response of the background flow to the wave packets is shown to be qualitatively very similar to that observed in the low-diffusivity two-layer model experiment.

1. Introduction

Recent observational studies (Chang 1993; Lee and Held 1993; Berberry and Vera 1996) have illustrated that in the midlatitudes of both hemispheres, baroclinic eddies tend to organize spontaneously into discrete coherent wave packets. Using maps of upper-tropospheric meridional wind, they showed that these packets propagate across the Pacific and Atlantic storm tracks in the Northern Hemisphere and can often circulate around the globe in the Southern Hemisphere. More recently Chang and Yu (1999) have shown that very coherent wave packet propagation is also common across the Asian continent. They also have defined a stationary waveguide that defines a path of maximum coherence for the packets. Their results show that there is "seeding" of disturbances at the entrance to the Pacific storm track by eddies propagating from upstream. Understanding

Corresponding author address: J. G. Esler, Earth, Atmospheric, and Planetary Sciences 54-1721, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139. E-mail: gavin@rossby.mit.edu wave packet dynamics therefore seems pivotal to understanding storm track variability in both hemispheres. Not only is it an important theoretical issue, but as suggested by Lee and Held (1993) it may also may have some value in medium-range forecasting, as the wave envelope may be more predictable than individual troughs and ridges.

Modeling studies of wave packets have followed two paths. Lee and Held (1993, LH hereafter) investigated wave packets in a hierarchy of forced-dissipative models, including a quasigeostrophic two-layer channel model. The wave packets in their experiments were shown to have at the rear (or upstream) edge enhanced meridional radiation of wave activity toward the jet edges (associated with barotropic decay of the eddies). They also described a corresponding increase in the barotropic shear of the jet toward the upstream edge of the packet. They drew an analogy between this downstream to upstream packet structure and the temporal evolution of a longitudinally symmetric wave train in an initial value experiment, which is well known to undergo a cycle of baroclinic growth-barotropic decay (e.g., Simmons and Hoskins 1978; Feldstein and Held 1989).

^{© 1999} American Meteorological Society

Several other studies have focused on initial value experiments, in which an unstable flow is perturbed by a zonally localized disturbance (e.g., Chang and Orlanski 1993; Swanson and Pierrehumbert 1994, SP hereafter). Swanson and Pierrehumbert (1994) showed that in the subsequent evolution the zonal extent of such a disturbance is bounded and that the bounds are in agreement with those predicted by large-time asymptotic linear theory. The largest amplitudes were seen at the leading edge of their evolving wave packet, with the eddies upstream stabilized by lower-level potential vorticity (PV) mixing and those further upstream by an increase in the barotropic shear of the jet (see, e.g., James 1987). One hypothesis to explain the behavior seen both in the SP initial value experiments and the LH forced-dissipative experiments is that it is the upstream stabilization that is responsible for maintaining the packet structure. This upstream stabilization would occur if there were a spatial displacement between where the waves reach their maximum amplitude and where they exert their maximum influence on the background flow (e.g., this spatial displacement might be related to the time lag associated with the waves radiating outward to the jet edge and breaking).

The results discussed above may be contrasted with the wave packet behavior exhibited by the weakly nonlinear system A studied by Esler (1997, E97 hereafter). This system is based on the Phillips model: the twolayer model with a uniform flow as the basic state in each layer. If a small parameter ϵ^2 is taken as a measure of the supercriticality of this basic-state flow, then the magnitude of the background flow response is found to be constrained to be proportional to the square of the amplitude of the wave envelope given by $\epsilon A(\zeta, T')$, where $T' = \epsilon^2 t$ is a "long" timescale and $\zeta = X - c_g T$ is a function of the "intermediate" space scales and timescales $X = \epsilon x$ and $T = \epsilon t$. The wave envelope can then be shown to evolve according to the Ginsburg– Landau equation:

$$A_{T'} + \mu A_{\zeta\zeta} = \Delta \rho A + \nu A |A|^2. \tag{1}$$

Here $\Delta = \pm 1$, and ρ and μ are complex coefficients that are determined by the properties of the linear dispersion relation. The parameter ν is also a complex coefficient that is related to the correction $\delta \omega$ to the complex frequency of the fundamental wave that is induced by the background flow correction, by the relation $\delta \omega$ = $i\epsilon^2 |A|^2 \nu$. In the weakly nonlinear problem the fundamental wave is defined as that which is marginally stable, all others having negative growth rates. In the numerical experiments that follow the fundamental wave will be defined as that which is fastest growing on the time mean flow.

The constraints of this weakly nonlinear system mean the following.

1) If the amplitude of the wave envelope grows at a given location, the background flow necessarily be-

comes further stabilized at the same location. This property was shown in E97 to oppose wave packet formation. This local coupling of the background flow response to the amplitude of the wave envelope prevents upstream stabilization of the flow from having a role in packet maintenance.

2) The meridionally uniform flow also inhibits any meridional radiation of the waves, so that any part that the downstream–upstream asymmetry of the momentum fluxes plays in maintaining the wave packets in the experiments of LH can have no role in packet formation in the weakly nonlinear model.

However, the weakly nonlinear equation [(1)] allows wave packet formation under certain conditions. The nonlinear self-focusing mechanism that causes this packet formation acts as follows. For the mechanism to occur, the background flow response to the wave packet must act to reduce the frequency of the fundamental wave ($\delta \omega_r < 0$). This causes the packet to adjust to a state where the "local" wavenumber is increased toward the rear of the packet and decreased toward the front (see Figs. 7 and 9c of E97). This is because the kinematics of phase propagation imply that if there is a local minimum in frequency, then the wavenumber on the left side of the minimum, that is, where the frequency is decreasing as function of x, must increase with time and the wavenumber on the right side must decrease with time. As a consequence of $\partial^2 \omega_r / \partial k^2 > 0$ for the twolayer model with uniform flow, the group velocity of the waves then increases with time at the rear of the wave packet and decreases with time at the front. This leads to a convergence of wave activity at the packet center, leading to packet growth. An important point about wave packets described by (1) is that generally they are unsteady;1 they tend to form by nonlinear selffocusing and then decay due to dissipation (see, e.g., Balmforth 1995 and references therein).

The primary objective of this paper is to investigate wave packet dynamics in a quasigeostrophic two-layer model with an upper-level jet, the details of which are described in section 2a. In particular the viability and relevance of the mechanisms of upstream stabilization and nonlinear self-focusing are evaluated. Two methods are used to describe the dynamics of the wave packets and differentiate between the two mechanisms. The first was motivated by viewing the problem as one of wave– background flow interaction, in which the wave packets force a nonlinear response in the background flow, which feeds back upon the linear growth rate and phase speed of the fundamental wave. Complex linear phase speeds of the fundamental baroclinic wave were therefore cal-

¹ A steady wave packet solution of (1) can in fact be obtained in a periodic channel when the length of the channel is only slightly greater than the length scale associated with the sideband instability of the steady wave train.

TABLE 1. A table summarizing the parameter settings used in the experiments. Note: for the purpose of comparison with the results in E97, the scaling factors for the coefficients used in that paper do not apply here.

Expt	β	$ u_0$	r	E_2
А	0.250	5.66×10^{-2}	7.07×10^{-2}	3.54×10^{-2}
В	0.250	3.54×10^{-2}	7.07×10^{-2}	3.54×10^{-2}
С	0.250	1.41×10^{-2}	7.07×10^{-2}	3.54×10^{-2}
D	0.208	5.66×10^{-2}	7.07×10^{-2}	3.54×10^{-2}
Е	0.313	1.41×10^{-2}	7.07×10^{-2}	3.54×10^{-2}

culated with respect to the background zonal flow profiles observed across the wave packets. The complex eigenvalue method used to do this is described in section 2b. By exploiting the relationship between the constant-*k*, variable- ω linear stability problem and the dynamics of the wave envelope that was discovered for the weakly nonlinear system described by Eq. (1), we can diagnose properties of the wave envelope. For example, if we define in this case $\delta \omega = \omega - \omega_{tm}$, where ω_{tm} is the complex frequency of the fundamental with respect to the timemean flow, we can predict a tendency for the wave envelope to decay where $\delta \omega_i < 0$ and to grow where $\delta \omega_i > 0$. There should also be a tendency for wavelength shortening upstream of regions where $\delta \omega_r < 0$, as well as wavelength lengthening downstream, which can act as precursor to nonlinear self-focusing.

The second method introduced in section 2c was based on the use of a wave activity conservation relation to diagnose the wave packets. An upper-level diagnostic equation for the wave packets is derived from this conservation relation in section 2d. This diagnostic equation allows the maintenance of the wave packet to be interpreted as a competition between a baroclinic source from the level below, a barotropic and dissipative sink, and a source or sink of wave activity associated with the zonal convergence of the wave activity flux. The occurence of nonlinear self-focusing is associated with zonal convergence of this flux near the wave packet center.

A range of numerical experiments were performed. In section 3 a subset of the experiments is chosen to illustrate the different behaviors found. The dynamics are found to be characterized by the extent to which an upper-level waveguide develops. This is controlled by the extent to which PV mixing occurs in the criticallayer regions located at the edges of the upper-level jet, as well as the center of the channel in the lower level. The degree to which this PV mixing takes place is de-



FIG. 1. Illustrating the difference between the time-mean state and the (initial) radiative states in expts A and C ("a" and "c"). (left) The zonal mean winds; (right) the zonal mean potential vorticity gradients. In each panel the upper-level time-mean profiles are illustrated by solid lines, and the lower-level profiles by dashed lines. The dotted lines show the radiative profiles in each level.

termined by both the mean wave amplitude in the upper layer, which is controlled by the supercriticality, and the strength of the small-scale dissipation.

The two experiments chosen to represent the different regimes of behavior are shown to differ in several fundamental ways. The more dissipative experiment (in which the upper-level waveguide is not well developed) is characterized by steady wave packets that force a barotropic response in the background flow. As in LH the latitudinal shear of the jet is found to increase toward the rear of the packet. It is shown that this additional shear has the effect of stabilizing the flow upstream of the packet. Wave activity diagnostics show that nonlinear self-focusing has no role in maintaining these steady packets. However, when the waveguide is well developed (e.g., in a low-dissipation experiment) the maximum background flow response occurs near the packet maximum, stabilizing the flow there, as in the weakly nonlinear system A. In this regime wave activity diagnostics indicate that nonlinear self-focusing is the more important mechanism for packet maintenance.

The results from integrations of a 10-level primitive equation model are briefly analyzed in section 4. These show that the response of the background flow to the wave packets in the low-dissipation experiment is not simply an artifact of the two-layer channel model.

2. The numerical model and methodology

a. The quasigeostrophic two-layer model

The coupled PV equations that govern the evolution of the quasigeostrophic two-layer model are of the form

$$\frac{D_i Q_i}{Dt} = d_i + \nu_0 \nabla^4 \phi_i \qquad i = 1, 2, \qquad (2)$$

where the subscripts denote the layer number (i = 1 is the upper layer and i = 2 the lower layer). In this equation Q_i is the PV, d_i the forcing or dissipation, and D_i/Dt is the quasigeostrophic advective derivative given by

$$\frac{D_i}{Dt} = \frac{\partial}{\partial t} - \Phi_{iy} \frac{\partial}{\partial x} + \Phi_{ix} \frac{\partial}{\partial y}.$$
(3)

The quasigeostrophic streamfunction Φ_i is related to the PV through the relation

$$Q_i = \beta y + \nabla^2 \Phi_i + (-1)^{i+1} F(\Phi_2 - \Phi_1), \quad (4)$$

where F is the nondimensional internal Froude number given by

$$F = \frac{f_0^2 L^2}{g' H},\tag{5}$$

and β is the nondimensionalized gradient in the Coriolis parameter *f*. Here *H* and *L* are the vertical and horizontal length scales, respectively, and *g'* is the reduced gravity due to the density difference between the layers. The reader is referred to Pedlosky (1987, pp. 416–430) for details.

The domain is periodic in the zonal direction, with sidewalls at y = 0 and $y = L_y$. The flow is maintained by the dissipation d_i , which is given the form

$$d_{i} = -E_{i} \nabla^{2} \Phi_{i} + (-1)^{i+1} r F[(\Phi_{1} - \Phi_{2}) - (\Phi_{1}^{s} - \Phi_{2}^{s})].$$
(6)

The streamfunction of the radiative equilibrium state Φ_i^s corresponds to a shear flow that has a broad parabolic jet structure in the upper level (unlike in E97). This jet structure is given by

$$U_{1}^{s} = -\Phi_{1y}^{s} = 4U_{0} \left(1 - \frac{y}{L_{y}}\right) \frac{y}{L_{y}} \text{ and}$$
$$U_{2}^{s} = -\Phi_{2y}^{s} = 0.$$
(7)

Only the lower-layer Ekman friction, denoted by E_2 , is nonzero to provide a crude parameterization of surface friction.

The numerical model is spectral in the zonal direction, with 64 waves, and grid point in the meridional direction, with 100 grid points. The maximum shear $U_0 =$ 1.0, F = 0.5, and β is the parameter that controls the criticality. The channel length $L_x = 20\sqrt{2}\pi$ and channel width $L_y = 5\sqrt{2}\pi$, respectively.

The hyperdiffusion acts only on the perturbation streamfunction ϕ_i (defined as a perturbation to the radiative equilibrium state, so that $\Phi_i = \Phi_i^s + \phi_i$). The hyperdiffusivity ν_0 is treated as a parameter that acts as a control on the extent of the wave breaking and PV mixing allowed at the edge of the jet.

The values of β and the hyperdiffusivity ν_0 are varied between the experiments. In each case the model was given sufficient time to "spin up" into a state of statistical equilibrium before results were analyzed. The different experiments are summarized in Table 1.²

b. The method of linear stability analysis of zonal flows

In section 3 composite diagrams showing various quantities calculated for an "average" wave packet are presented, including the response of the "background" zonal flow. This is defined as the zonal mean flow plus the contributions from the first seven zonal wavenumbers. This effectively filters out the baroclinic waves that have maximum enstrophy at wave 10 and wave 11. All the composites shown are calculated by first identifying the maxima of the wave packets at each model day. This is achieved by generating an envelope function, by Fourier decomposition, of the field of latitu-

 $^{^{2}}$ It is worth noting that when the value of radiative relaxation *r* was decreased below the values shown in Table 1, oscillations of the zonal jet such as those described in Lee and Feldstein (1996) became so pronounced that it became difficult to apply some of the statistical methods used in this paper.

dinally averaged upper-level perturbation meridional wind squared³ $v_1^{e^2}$, and identifying its maximum at each time. The longitude of the packet maximum is taken as the zero longitude in the composite diagram. The composite quantity at a given longitudinal lag, denoted by x_{lag} , from the packet maximum can then be calculated at each latitude, effectively averaging the quantity in question about each packet maximum.

The complex phase speeds for the fundamental baroclinic mode at each longitudinal lag can then be calculated. The background zonal flow is assumed to be slowly varying in longitude, in order that the effect of the background meridional flow on the waves can be neglected. This is consistent with a weakly nonlinear approach to the wave packet behavior. To calculate the complex phase speeds for the two-layer model with a general zonal flow in each layer, it is necessary to solve a complex eigenvalue problem. The basic method used to do this is reviewed in SP. In this paper the method is adapted in order that accurate linear growth rates could be calculated for zonal flows that were close to the time-mean zonal flow for each individual experiment. This involved including the dissipation for each experiment in the growth rate calculation.

The eigenvalue problem in matrix form is

$$\mathbf{L}\boldsymbol{\phi} = c\mathbf{M}\boldsymbol{\phi},\tag{8}$$

where, in the absence of dissipation,

$$\mathbf{L} = \begin{pmatrix} Q_{1y} + U_1(d^2/dy^2 - k^2 - F) & U_1F \\ U_2F & Q_{2y} + U_2(d^2/dy^2 - k^2 - F) \end{pmatrix},$$
(9)

$$\mathbf{M} = \begin{pmatrix} d^2/dy^2 - k^2 - F & F \\ F & d^2/dy^2 - k^2 - F \end{pmatrix},$$
(10)

and

$$\boldsymbol{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \tag{11}$$

In order to add dissipation, it is necessary to add extra terms to the matrix L, which generalizes the problem by making L complex:

$$\mathbf{L}^{p} = \mathbf{L} + i \begin{pmatrix} E_{1}(d^{2}/dy^{2} - k^{2}) - rF - \nu_{0}(d^{4}/dy^{4} - 2k^{2}d^{2}/dy^{2} + k^{4}) & rF \\ rF & E_{2}(d^{2}/dy^{2} - k^{2}) - rF - \nu_{0}(d^{4}/dy^{4} - 2k^{2}d^{2}/dy^{2} + k^{4}) \end{pmatrix}.$$
(12)

The eigenvalue problem is then solved by inverting **M**, and using a standard complex eigenvalue routine for the complex matrix $\mathbf{M}^{-1}\mathbf{L}^{D}$. This method was tested for the case where $U_i = -\Phi_{iy}$ is uniform in each layer. Phase speeds were compared with those calculated from the linear dispersion relation [Eq. (6) in E97]. Typical errors were 0.2% for a grid of 26 points in each layer. For the calculations presented in this paper a grid of 51 points was used.

The complex phase speeds that are calculated in this way give information about the wave–background flow interaction that is taking place along a typical wave packet. The linear growth rate ($\omega_i = k_0 c_i$) of the fundamental indicates where the nonlinear response to the wave packet has caused the background zonal flow to be stabilized (where $\omega_i < 0$) or destabilized (where $\omega_i > 0$). Equally important, however, is the real frequency of the fundamental ($\omega_r = k_0 c_r$), and how it is adjusted by nonlinear response to the wave packet. As noted in the introduction, changes in the sign and magnitude of the correction to the real frequency ($\delta \omega_r$) are, in systems such as "system A" described by E97, associated with nonlinear self-focusing.

³ The superscript e is used to denote a perturbation quantity with respect to the time-mean field, as opposed to the radiative equilibrium field.

c. A three-dimensional wave activity relation

In order to provide a framework in which to understand the evolution of the wave packets that are observed in the experiments that follow, it is helpful to construct a suitable wave activity relation. For the twolayer system the appropriate form for the relation is

$$\frac{\partial A_i}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{F}_i + S_i = \mathcal{D}_i, \qquad i = 1, 2, \qquad (13)$$

where A_i is a measure of the wave activity in layer *i*, \mathbf{F}_i the horizontal flux or transport of wave activity in that layer, \mathcal{D}_i the nonconservative sources or sinks of A_i , and S_i an exchange term between the two layers (with $S_1 = -S_2$). The exchange term is the analogue in the two-layer system of the term involving the vertical derivative of the vertical flux that appears in conservation relations for continuously stratified systems.

It will be shown, following Plumb (1985), that such a relation, valid for small-amplitude waves, may be constructed in a form that each term is independent of the phase of the wave in the limit of a slowly varying basic state. Furthermore, Plumb's approach will be generalized to allow for the possibility of nonstationary waves.

A starting point is the two-layer pseudomomentum conservation relation of Shepherd (1988), without phase averaging and valid to order a^2 in wave amplitude *a*, which has components in (13) given by

$$A_i = \frac{q_i^2}{2Q_{iy}^0},\tag{14}$$

$$\mathbf{F}_{i} = \frac{1}{2} \begin{pmatrix} 2U_{i}A_{i} + (\phi_{ix})^{2} - (\phi_{iy})^{2} + (-1)^{i}F\phi_{i}(\phi_{1} - \phi_{2}) \\ 2\phi_{ix}\phi_{iy} \end{pmatrix},$$

$$S_{i} = \frac{(-1)^{i+1}}{2} F(\phi_{2x}\phi_{1} - \phi_{2}\phi_{1x}), \qquad (16)$$

and

$$\mathcal{D}_i = \frac{d_i q_i}{Q_{iv}^0},\tag{17}$$

where U_i and Q_i^0 are basic-state quantities as before, and the other quantities represent perturbations from those. Equations (14)–(15) are generalized to give the required phase-invariance properties as follows. First following Plumb (1985), a vector \mathbf{G}_i with the property $\nabla \cdot \mathbf{G}_i =$ 0 (in the nondissipative limit) is added to \mathbf{F}_i . The vector chosen is

$$\mathbf{G}_{i} = \frac{1}{4} \begin{pmatrix} (\phi_{i}^{2})_{yy} \\ -(\phi_{i}^{2})_{xy} \end{pmatrix}.$$
 (18)

The nonuniqueness of the conservation relation is exploited yet further in order to obtain a more general relation that will have the required properties for traveling as well as stationary waves. This is achieved by making the transformation

$$A_i \to A_i - \frac{(\eta_i q_i)_x}{4Q_{iy}^0}, \qquad F_i^{(x)} \to F_i^{(x)} + \frac{(\eta_i q_i)_t}{4Q_{iy}^0},$$
 (19)

with no net effect on (13). Here η_i is a perturbation quantity defined by $\eta_{ix} = q_i - \overline{q_i}$, where the overbar denotes a time-mean zonal mean. The components of (13) can now be shown to be [correct to $O(a^2)$ in wave amplitude]

$$A_{i} = \frac{q_{i}^{2} - q_{ix}\eta_{i}}{4Q_{iy}^{0}},$$
(20)

$$\mathbf{F}_{i} = \frac{1}{2} \begin{pmatrix} (\phi_{ix})^{2} - \phi_{i}\phi_{ixx} + 2U_{i}A_{i} + \frac{1}{2}(\phi_{i}q_{i} - \phi_{ix}\eta_{i}) \\ \phi_{ix}\phi_{iy} - \phi_{i}\phi_{ixy} \end{pmatrix}, \quad (21)$$

and

(15)

$$\mathcal{D}_i = \frac{2d_iq_i - r_iq_{ix} - d_{ix}\eta_i}{4Q_{iy}^0},\tag{22}$$

with S_i unchanged. Here r_i is another perturbation quantity defined by $r_{ix} = d_i - \overline{d_i}$. All the terms in (13) can now be shown to be phase independent.

This conservation relation has the following properties, which are analogous to those given by Plumb.

- 1) For steady $(\partial A_i/\partial t = 0)$, conservative $(\mathcal{D}_i = 0)$ flow $\nabla \cdot \mathbf{F}_i + S_i = 0$ or \mathbf{F}_i is nondivergent where there is no exchange of wave activity between the layers.
- 2) Where $A_i > 0$, regions of convergence, where $\nabla \cdot \mathbf{F}_i + S_i < 0$, indicate the import of wave activity, whereas regions of divergence indicate its export.
- In the appropriate WKBJ limit, the group velocity property

$$F_1^{(x)} + F_2^{(x)} = c_g(A_1 + A_2) \tag{23}$$

holds, where $c_g = \partial \omega_r / \partial k$ is the linear group velocity. (This property is verified for the case of nondissipative, nongrowing waves in the appendix).

4) In the zonal average, the flux reduces to the twolayer analogue of the Eliassen–Palm flux, with the exception of the addition of a zonal component, which is of no consequence.

The fact that the relation defined above does not rely upon phase averaging gives two advantages over Shepherd's flux (which does rely on some kind of averaging in order that it is easily interpreted). First, phase averaging will tend to substantially smooth quantities that vary on the packet length scale, as typically a wave packet may measure only two to three wavelengths. Second, taking a phase average over a fixed zonal wavelength may cause difficulties when flux quantities are sensitive to variations in the actual zonal wavelength within the wave packet or between different wave packets. The main disadvantage, as discussed at some length in section 3d, is that advection of wave activity by the eddy wind fields is neglected.

d. A diagnostic equation for the wave packet maintenance mechanisms

The upper-level wave activity equation [(13) with i = 1] may be rewritten in the form

$$\begin{pmatrix} \frac{\partial}{\partial t} + c_{g_1} \frac{\partial}{\partial x} \end{pmatrix} A_1 = -\frac{\partial}{\partial x} (F_1^{(x)} - c_{g_1} A_1) - \frac{\partial F_1^{(y)}}{\partial y} \\ \xrightarrow{\text{Zonal convergence}}_{\text{of wave activity}} Barotropic \\ \text{sink} \\ - \underbrace{S_1}_{\text{Baroclinic}} + \underbrace{\mathcal{D}_1}_{\text{Dissipation}}, \\ \xrightarrow{\text{Baroclinic}}_{\text{source}} Dissipation$$
(24)

where $c_{g1} = c_{g1}(y)$ can be chosen to be representative of the zonal mean upper-level group velocity of the waves. In what follows, c_{g1} is defined as the time-mean zonal mean of $F_1^{(x)}$ divided by the time-mean zonal mean of A_1 . This definition of c_{g1} was found to give a better first approximation to the wave packet group velocity in the experiments to be described, compared with either the local WKBJ group velocity relevant to both layers $c_g = (F_1^{(x)} + F_2^{(x)})/(A_1 + A_2)$ (see the appendix) or the group velocity c_g^{\lim} calculated from linear theory with respect to the time-mean flow (as in section 2b). This was perhaps because low-level mixing changed the nature of the coupling between the eddies in the two levels from that in linear theory.

When the wave activity equation is in this form, the terms on the right-hand side may be regarded as representing effective sources and sinks of wave activity for a wave packet moving with the group velocity c_{g1} . These sources and sinks can be related to the dynamical processes that are responsible for maintaining the wave packet and have been labeled appropriately in (24). In section 3 composites of these quantities are presented. The interpretation of these composites is facilitated by dividing each source or sink term into three separate contributions.

- 1) The contribution of the term to the time-mean zonal mean budget of upper-level wave activity.
- 2) The correction to the group velocity c_{g1} of the wave packet due to the dipolar component of the source or sink term. For example, if a wave activity source is disproportionately strong to the front of the wave packet, and disproportionately weak to its rear, the main effect of the term may be to cause the wave packet to move faster than c_{g1} .
- 3) A residual term that may "focus" or "defocus" the wave packet.

The residual terms are defined for a generic source term $\langle S \rangle$ as follows:

$$\langle S \rangle^{\text{res}} = \langle S \rangle - \frac{\overline{S}}{\overline{A_1}} \langle A_1 \rangle - c_{g_1}^{\langle S \rangle}(y) \langle A_1 \rangle_x.$$
 (25)

The angular brackets denote composites, as defined in section 2b, and the overbars time and zonal averaging. Here $c_{g1}^{(S)}(y)$ is a correction to the group velocity chosen to effectively remove the dipolar component of $\langle S \rangle$. A suitable choice was found to be

$$c_{g1}^{\langle S \rangle}(y) = -\frac{\langle S \rangle_x|_{x_{\text{lag}}=0}}{\langle A_1 \rangle_{xx}|_{x_{\text{lag}}=0}}.$$
 (26)

Subtracting out the time-mean zonal mean contribution [the second term on the right-hand side of (25)] from each source/sink term is equivalent to subtracting $\langle A_1 \rangle$ × the time-mean zonal mean wave activity equation

$$\overline{S_1 + F_{1y}^{(y)} + \mathcal{D}_1} = 0 \tag{27}$$

from (24). By then subtracting the group velocity corrections (i.e., the third term on right-hand side of (25)] we are in effect rewriting (24) as

$$\left(\frac{\partial}{\partial t} + (c_{g1} + c_{g1}^{zc} + c_{g1}^{bt} + c_{g1}^{bc} + c_{g1}^{ds})\frac{\partial}{\partial x}\right)\langle A_{1}\rangle$$

= ... residual sources/sinks, ..., (28)

where c_{g1}^{cc} , c_{g1}^{bt} , ... are the corrections associated with each term on the right-hand side of (24), respectively.

Each term on the right-hand side of (24) is left with a residual part that may now be considered to be competing tendencies to focus or defocus the wave packet. The first term, the zonal convergence of wave activity, can be associated with nonlinear self-focusing mechanism for wave packet formation described in E97. This is distinct from the second term, the barotropic sink due to the meridional radiation and divergence of wave activity. This second term, so called as a zonally symmetric wave train undergoing barotropic decay [in the sense of, e.g., Feldstein and Held (1989)], would lose wave activity from the center of the channel through this term. The remaining terms are the baroclinic source of wave activity from the lower level and the sink of wave activity due to dissipation of the waves.

3. Results from the two-layer model experiments

Over the course of many experiments, of which those reported below and listed in Table 1 form only an illustrative subset, it was observed that the dynamics of the wave packets in this model are sensitive primarily to the extent to which wave breaking and PV mixing are allowed at the edge of the jet. Experiments A and C have been chosen for detailed analysis in this section, as they illustrate the two different regimes of behavior. In section 3a the stabilization of the time-mean flow in these experiments is discussed. In section 3b results from the linear stability analysis and wave activity diagnostics are presented for experiments A and C, and the transition between the two regimes, illustrated by experiment B, is also briefly discussed. In section 3c the results from experiments with higher and lower values of β are presented, which are intended to illustrate that similar qualitative behavior can be identified at different criticalities. Section 3d contains a discussion of the results obtained when the full nonlinear wave activity flux of Shepherd (1988) is used in place of the corrected flux derived in section 2c [(20)–(21)]. The results from other experiments (not presented) in which the values of E_2 and r were varied showed that although the structure of the time-mean jet was sensitive to these parameters, the wave packet behavior was qualitatively similar.

a. Stabilization of the time-mean zonal mean winds

In each experiment the time-mean zonal mean flow was investigated in order to compare its stability properties to that of the radiative equilibrium flow profile given by (7). Figure 1 shows the time-mean zonal winds, as well as the time mean PV gradients in experiments A and C. The zonal winds in both experiments differ from the radiative profile by the addition of a barotropic component, which is stronger in the case of experiment C. In both experiments the PV gradient in the lower level is reduced, and in the upper level a region of strong gradient is created near the center of the channel. The degree of this PV mixing is greater in experiment C (less dissipative) than in experiment A (more dissipative).

When the linear stability analysis described in section 2b is applied to the time-mean flows illustrated in Fig. 1, the flow is found to be stabilized in each case. Table 2 shows a comparison of the complex phase speeds of the fundamental for the radiative and time-mean states in each experiment. For experiments A and C the meridional and vertical structure of the fundamental wave calculated for the time-mean flow was broadly similar to the structure of the waves observed in the full non-linear experiments (Figs. 2a and 10a).

This spinup of a stabilizing barotropic jet by the eddies is well documented in zonally symmetric initial value experiments in the two-layer model by, for example, Feldstein and Held (1989), and in similar experiments by LH. Importantly, it is analogous to the spinup of a barotropic jet upstream of a developing wave packet in an initial value problem (e.g., SP). Increased meridional shear was also observed at the upstream edge of the wave packets in the experiments of LH. However, the following experiments will show that the development of wave packets *need not* be associated with a large upstream barotropic correction to the background zonal flow.

b. The effect of varying the small-scale diffusivity 1) EXPERIMENT A: HIGH DIFFUSIVITY

Figure 2 shows a snapshot of various fields from experiment A, in which the higher-order diffusion ν_0 is

set at a relatively high value. In this experiment the waves in both layers behave almost as linear waves, even in the upper-level critical-layer regions. As the diffusion is strong, the waves are dissipated in these regions before the PV contours overturn. The wave activity snapshot (Fig. 2c) shows the two-layer model wave activity flux, which is described in section 2c, corrected for the time-mean zonal mean group velocity at each latitude, $c_{g1}(y)$. Relative to this group velocity, in this experiment the wave activity flux is entirely directed in the meridional direction. The critical-layer regions can be described as largely absorptive, in the sense of Killworth and McIntyre (1985), as the flux can be seen to be strongly convergent there.⁴

There is also evidence in this picture to complement the results of LH that the wave packet structure is asymmetric from its downstream to upstream side. If one considers the wave packet in the stationary frame, at the east or downstream end of the packet the waves grow from radiation of wave activity from the larger upstream disturbances. Evidence of this downstream development occuring for baroclinic waves in observations was presented by Chang (1993). The waves at this end of the packet have less latitudinal phase tilt toward the jet edges in Fig. 2a than the waves at the upstream end of the packet. This is consistent with weaker radiation of wave activity toward the jet edges. As the relative rate of radiation of wave activity to the jet edges increases toward the rear of the packet, one would expect wave decay through absorption at the critical layer to become more important there. It could be argued that the packet adjusts to this higher rate of wave activity absorption at its upstream end by decaying away sharply there, compared with the gradual increase in wave amplitude at its downstream end.

The cross-sectional EOFs method described in E97 (appendix 3) was used here in order to compare the evolution in longitude and time of the baroclinic waves to that of the leading-order background flow response. This method separates the variability of a field into "optimal" structures in the meridional-height plane that describe the maximum possible variance. The principal components of each structure are defined by its projection onto the flow field at a given longitude and time. The evolution of these principal components in longitude and time can give a greatly simplified picture of the important coherent variability of the field.

Figure 3 shows the leading symmetric and antisymmetric cross-sectional EOFs for the perturbation zonal wind field u^e from experiment A. The principal anti-

⁴ Absorptivity in Killworth and McIntyre (1985) is defined as the jump in the value of (minus) the zonally averaged momentum flux across the critical layer or, equivalently, the convergence of wave activity flux into the critical layer. The meridional component of the flux \mathbf{F}_1 is a three-dimensional analogue of the zonally averaged momentum flux.

TABLE 2. Comparing the complex phase speeds of the fundamental wave calculated for the radiative equilibrium flow and the time-mean flow.

Expt	Phase speed <i>c</i> (radiative equilibrium flow)	Phase speed <i>c</i> (time-mean flow)
(wave 10, $k = 0.7071$)	0.1734 + 0.1067i	0.3757 + 0.0136 <i>i</i>
(wave 11, $k = 0.7778$)	0.2133 + 0.1127i	0.7295 + 0.0002i

symmetric EOF describes the baroclinic waves, which consist of the two coherent, rapidly propagating wave averaged momentum flux packets seen in Fig. 2. These wave packets can be seen in Fig. 3 to remain steady throughout the course of the integration. The first symmetric EOF (top right) shows the nonlinear wave-2 response to the wave forcing due to these packets. It has structure that is very close to being barotropic. The positive phase of this mode is located at the downstream (east) side of each packet, showing that the local zonal jet is broader and weaker there. The negative phase is located on the westward side of the packets, which indicates that the local jet is stronger and narrower there. This is reasonable as the extra breaking and dissipation of the waves at the rear of the packet leads to more effective mixing and weakening of the PV gradient at the jet edges. This is consistent with the stronger narrower jet that is observed. The phase speed for the waves calculated from this picture agrees well with that calculated from linear theory, shown in Table 2.

Figures 4a and 4b show the composites of background zonal wind correction around a packet maximum (at zero longitude), the calculation of which is described in section 2b. A barotropic correction is observed, which is nearly identical to the leading symmetric EOF in Fig. 3. The barotropic correction is similar in structure to the correction to the time-mean zonal flow forced by the eddies, shown in Fig. 1. The linear stability of this composite background zonal flow can be calculated along the length of the wave packets. Figures 4c and 4d show how the real and imaginary linear phase speeds c_r and c_i of the fundamental baroclinic wave vary in longitude around the packet maximum. The real part of the phase speed c_r varies smoothly along the packets, with minima just downstream of the packet maxima. The imaginary part of the phase speed c_i shows that the background flow is unstable to the downstream side of the wave packets, and slightly unstable at the packet maximum. However, it becomes strongly stabilized by the barotropic correction to the background flow to its upstream side, apparently due to the barotropic governor effect (e.g., James 1987). As the fundamental wave would decay in the stabilized region and grow in the unstable downstream region, this background flow structure should maintain the twin packet structure, provided that nonlinear self-focusing-defocusing is not important. For this experiment this turns out to be the case as the analysis of the wave activity equation [(24)], which shows that for this experiment the nonlinear selffocusing term is negligible compared with the other terms (Figs. 5b, 7c). It therefore seems that there is a viable mechanism, which we might call the "upstream stabilization mechanism" for maintenance of the wave packets.

Overall, the background flow correction causes nonlinear enhancement of the linear group velocity of the waves, simply by causing wave growth at the downstream edge of the packet and wave decay at the upstream edge. The magnitude of this effect is proportional to the degree of instability of the flow downstream of the packet. It can be thought of as analogous to the (linear) enhancement of packet group velocity described in the experiments of SP, which similarly is a function of the degree of instability of the undisturbed flow downstream of the packet. In our experiment it causes the observed group velocity ($c_g^{obs} = 1.484$) to be much greater than that predicted by the linear calculations ($c_g^{in} = 1.076$ with respect to the time-mean flow).

An important property of the proposed upstream stabilization mechanism is that it feeds back positively upon itself. If one considers a perturbed wave train that induces a tiny barotropic correction to the flow, with structure similar to that in Fig. 4, it will cause wave decay at the upstream edge (due to the barotropic governor effect), as well as growth at the downstream edge and packet maximum, therefore steepening the wave packet. This leads to an increased barotropic correction, reinforcing the process. It is therefore a viable secondary instability of a symmetric wave train in the sense of E97.

Further insight into the nature of this mechanism can be obtained by considering the upper-level wave activity equation [(24)]. As discussed in section 2d, it is the zonal variation of the wave activity statistics along the wave packets that is of most interest. Estimates of these are obtained by taking composites about the packet maximum, as for the background zonal wind (see section 2b).

Figure 5 shows longitude-latitude contour plots of composite quantities of the terms in (24) before the residuals have been calculated. Note that different contour intervals have been used in order to illustrate the spatial structure of each field. Figure 5a also shows the composite wave activity $\langle A_1 \rangle$ itself. Note that the wave activity maxima occur away from the center of the channel in this experiment. Figure 5b confirms that little zonal convergence of wave activity is taking place. Figure 5c shows the remaining terms on the right-hand side of the equation, $\langle \mathcal{D}_1 - S_1 - F_{1\nu}^{(y)} \rangle$. This picture shows a net increase of wave activity due to these terms downstream of the wave activity maxima, and a net decrease upstream of the maxima. Figures 5d and 5e show the barotropic sink term $\langle -F_{1y}^{(y)} \rangle$ and the baroclinic source term $\langle -S_1 \rangle$, which largely cancel with the dissipation $\langle \mathcal{D}_1 \rangle$ (note the different contour intervals), leaving a small residual.

Figure 6 shows how the dipolar part of these tendencies influence the wave packet group velocity as described in section 2d. Figure 6a shows $c_{v1}(y)$ (dashed line), the corrected group velocity (solid line), as well as the upper-level mean flow (dotted line). It shows that the net effect of the dipolar structure of the total tendency (shown in Fig. 5c) is to accelerate the fringes of the wave packet so that it all moves at a near-constant velocity (close to the observed group velocity $c_{g}^{obs} =$ 1.484). [Note that this method of obtaining the total wave packet group velocity neglects advection by the eddy wind field, which is not included in (20)–(21).] Figure 6b shows the how the individual terms influence the group velocity. Because the baroclinic term is disproportionately strong toward the rear of the packet (in the channel center), as can be seen in Fig. 5e, it therefore has a decelerating effect. The barotropic sink is also disproportionately strong toward the rear of the packet, and therefore has an accelerating effect. Figure 6c shows $\langle F_1^{(x)} \rangle / \langle A_1 \rangle$, averaged across the channel center, showing the variations in the local, uncorrected zonal group velocity along the length of the wave packet. It is nearly constant everywhere except where the wave amplitude is very small, again indicating that zonal convergence of wave activity is not important here.

Figure 7a shows a snapshot of upper-level meridional velocity v_1 shifted in longitude so that a packet maximum is at the zero point, to act as a reference for the curves below. Figure 7b shows the composite quantities plotted in Fig. 5, now averaged across the channel center, from $y = 0.42L_y$ to $y = 0.58L_y$, to act as a reference for the residual tendencies shown in Fig. 7c, also averaged across the channel center. It is clear from Fig. 7c that the only residual term that causes the wave packet to be concentrated, by causing growth at its maximum and decay at its upstream and downstream edges, is the barotropic sink term. This can be explained by noting that the barotropic sink at the packet maximum is disproportionately weak, as there the jet is somewhat broader and weaker than the time-mean jet (see Fig. 4). This focusing is balanced by defocusing due to the baroclinic term (which is disproportionately weak at the packet maximum as the jet is weaker) and the dissipation term. Note that there is a net tendency to defocus the packet, which is most likely balanced by nonlinear advection (see the discussion in section 3d).

2) EXPERIMENT B: MEDIUM DIFFUSIVITY

Experiment B, in which the value of the hyperdiffusion ν_0 has been decreased from that in A, is a tran-

sitional case. Figure 8 shows a snapshot of the same dynamical fields shown for experiment A. Note that wave breaking takes place to a greater degree in the upper-level critical layers at the edge of the jet. This results in a region of PV that is relatively well mixed, stretching behind the wave packet at the edges of the jet. As in experiment A, the PV gradient is slowly restored in the critical-layer regions behind the wave packet by the effects of radiative relaxation. There is also clear evidence of wave breaking in the lower-level PV picture. This mixes the PV gradient right across the center of the channel. Viewed relative to the group velocity $c_{p1}(y)$, the zonal fluxes are eastward (and hence disproportionately strong) toward the rear of the wave packet, where the local wavelength is shorter. The meridional fluxes also increase to the rear of the packet. Of the two wave packets visible, it is clear that one has much greater wave amplitude than the other. Over time, the two wave packets visible in the channel can be observed to grow and decay with a timescale of around 100-150 model days.

Figure 9 shows the mean background flow correction along the wave packet for experiment B. The barotropic correction extends less far upstream of the wave packet than that for experiment A, and there is now a baroclinic component to the flow correction at the packet maximum. Linear stability analysis shows that the region at and upstream of the packet maximum is strongly stabilized by this baroclinic component. This is seen to a much greater extent in the following experiment.

3) EXPERIMENT C: LOW DIFFUSIVITY

Figure 10 shows the equivalent snapshots for experiment C. These show that the upper-level waveguide is well developed in this experiment in the sense that in the upper-level there is a region of tight PV gradient near the center of the channel, surrounded by strongly mixed regions to each side where the PV gradient is very weak. This is true at all longitudes, and although it is noticeable that there is more mixing taking place at the upstream edge of the wave packets, the criticallayer regions remain well mixed everywhere. The lowerlevel PV is well mixed toward the center of the channel, with stronger PV gradients located at the latitudes of the upper-level critical layers.

The relatively strongly mixed upper-level critical layers appear to act as partial reflectors (or rather nonabsorbers) of wave activity (see, e.g., Killworth and McIntyre 1985). This is consistent with the wave activity vectors in Fig. 10c pointing more in the zonal di-

 $[\]rightarrow$

FIG. 2. A snapshot of various dynamical fields from expt A. From the top they are (a) upper-layer meridional velocity ϕ_{1x} , (b) upperlayer potential vorticity Q_1 , (c) wave activity exchange S_1 between the lower and upper level, and (d) lower-layer potential vorticity Q_2 . (c) Arrows show the upper-level wave activity flux, vector $(F_1^{(c)} - c_{g1}A_1, F_1^{(o)})$, defined in section 2d. Contour intervals are (a) 0.16, (b) 1.4, (c) 0.035, and (d) 0.27. Arrow scaling is 1 length unit to 0.1.





FIG. 3. (top) The latitudinal profile of the most important antisymmetric and symmetric cross-sectional EOFs of u, for expt A. The solid line shows the upper-level structure and the dotted line the lower-level structure. The modes describe 52.8% and 6.1% of the variance in u, respectively. (bottom) Longitude–time plots showing the evolution of the corresponding principal components in longitude and time. Contour intervals are (left) 0.2 and (right) 0.1.

rection than those in experiments A and B. (The more zonal the arrows, the lower the jump in the value of the wave activity flux across the critical-layer regions.) As in experiment B, stronger zonal fluxes (eastward-pointing arrows) are associated with regions where the local wavelength is shorter. This is usually the rear of the wave packets.

Figure 11 shows the structure of the leading antisymmetric and symmetric EOFs for experiment C, and the time evolution of their principal components. The antisymmetric EOF, which describes the baroclinic waves, shows that individual wave packets tend to remain well defined for a shorter period of time in this experiment (around 50 days). Unlike in experiment A, the symmetric EOF has a strong baroclinic component. It also differs in that only its negative phase can be strongly correlated with wave packet position in the evolution pictures of Fig. 11. Figures 12a and 12b, which show the composite background zonal winds around the wave packet maximum for this experiment, illustrate why this is the case. Near the composite packet maximum there is a strong baroclinic component to the correction to the background flow, with a relatively weak barotropic correction slightly upstream. It is important to emphasize that in this experiment, the correction to the radiative flow due to the presence of eddies (see Figs. 1c and 1d) is quite different in structure to the correction to the background flow due to the wave packets. The more baroclinic structure of the background flow correction suggests that as the upper-level critical layers are well mixed, the background flow is in some sense partially saturated to further barotropic correction.

Figures 12c and 12d show the real and imaginary parts of the linear phase speed of the fundamental wave, calculated with respect to the composite background flow around the packet maximum. The correction to the real phase speed of the waves is in phase with the correction to the zonal flow at the center of the channel. The strong reduction in the phase speed of the fundamental suggests that, as in the case of the weakly nonlinear model described in the introduction, the packet will tend to adjust to a state in which the wavelength of the waves at the rear of the packet is reduced relative to those at the front. This adjustment is a precursor to nonlinear self-focusing taking place.

The plot of imaginary phase speed shows that the flow has also been strongly stabilized at and just upstream of the packet maximum. Both the barotropic and baroclinic components of the correction appear to have contributed to stabilizing the flow. The negative values in the region of the packet maximum suggest that the growth rate of the waves may be more strongly influenced by the baroclinic component of the flow correction rather than the barotropic component. To confirm this the linear stability of a composite zonal flow with a barotropic correction (bt) only, and a similar flow with a baroclinic correction only (bc) were investigated. These flows are given by

$$u_i^{\rm bt} = \overline{u_i} + \frac{u_1^c + u_2^c}{2} \tag{29}$$

$$u_i^{\rm bc} = \overline{u_i} + (-1)^{i+1} \frac{u_1^c - u_2^c}{2},\tag{30}$$

where $\overline{u_i}$ is the time-mean zonal flow, and u_i^c is the composite background zonal wind correction (shown in Figs. 14a,b for i = 1, 2). The barotropic correction flow u_i^{bt} was found to be unstable in the region of the packet maximum, with maximum growth rate equal to 0.0570. The baroclinic correction flow u_i^{bc} was by contrast strongly stabilized near the packet maximum with fastest decay rate equal to -0.0907. The stabilizing effect of the baroclinic component of the flow correction is therefore dominant, as seen in Fig. 12d.

Figure 13 shows longitude–latitude composites of the quantities in Eq. (24) for this experiment. Figure 13a shows the upper-level wave activity $\langle A_1 \rangle$, which has a maximum slightly upstream of the packet maxima defined in section 2b. Figures 13b and 13c illustrate the zonal convergence term and the sum of the remaining terms in Eq. (24), respectively, for this experiment. Note that the zonal convergence term has the effect of concentrating wave activity toward the packet maximum. Figure 13d shows the barotropic sink term, which at the packet maximum has a magnitude less than half that of the baroclinic source term shown in Fig. 13e.

Figure 14 illustrates the effect of these tendencies on the wave packet group velocity. Figure 14a shows that the corrected group velocity (solid line) is substantially greater than c_{g1} (dashed line). Figure 14b shows that it is the zonal convergence term that is chiefly responsible for this increase. However, for this experiment in particular, we cannot expect to accurately predict the observed wave packet group velocity $(c_g^{obs} = 1.5465)^5$ without including the effect of advection by the eddy wind fields (see section 3d). Figure 14c shows the local uncorrected group velocity at the channel center, given by $\langle F_1^{(x)} \rangle / \langle A_1 \rangle$. It has a minimum at the front of the packet, with a maximum just behind the packet maximum, and another minimum to the rear of the packet. This suggests that there will be convergence of wave activity at the packet maximum, and that zonal convergence is an important mechanism in maintaining the wave packet.

Figure 15a shows a composite meridional wind field from this experiment, which has been derived as follows. The reference point is taken to be the longitude at which the maximum of the square of the eddy meridional wind field v_1^2 occurs. If $v_1 < 0$ at this point for the wave packet in question, then the transformation v_1 $\rightarrow -v_1$ is made before adding the wave packet to the composite. The composite was taken over 661 wave

⁵ For comparison the group speed from the linear calculations $c_g^{\text{lin}} = 1.2387$.



packets and is intended to illustrate the characteristic length scale and structure of the wave packets recorded in the experiment. A shortening of the wavelength of the waves toward the center and rear of this composite packet may be seen. The composite process tends to smooth out the wavelength shortening, which is much more visible in the snapshot Fig. 10a.

Figure 15b shows averages across the channel center, again from $y = 0.42L_y$ to $y = 0.58L_y$ of the composite quantities shown in Fig. 13, for the purpose of comparison with their residuals (calculated according to the method described in section 2d) shown in Fig. 15c. The residual terms are dominated by a relatively strong focusing tendency due to the zonal convergence of wave activity. There is also a weak tendency toward focusing due to the barotropic term (as in expt A) and a weak tendency toward defocusing due to the baroclinic and dissipative terms.

These results show that the maintainence of the wave packets in this experiment is due almost entirely to the zonal convergence of wave activity. The qualitative similarity of the background flow stability characteristics with those of the weakly nonlinear system A of E97 suggest that the same nonlinear self-focusing mechanism is taking place. The net focusing of the wave packet shown by the heavy solid curve is likely to be at least partially balanced by the residual effect of advection by the eddy wind fields (see section 3d).

c. The effect of varying the supercriticality

1) EXPERIMENT D: HIGH SUPERCRITICALITY

In order to illustrate that the wave packet dynamics are controlled by the effective amount of PV mixing allowed in the critical layers, in a further two experiments, D and E, the parameter β was varied. Reducing β effectively increases the supercriticality of the system. Figure 16 shows a snapshot of fields from experiment D, in which the criticality was increased, with $\beta =$ 0.208. The dissipation v_0 is set to a relatively high value, equal to that in experiment A. The high criticality has the effect of increasing the eddy amplitude in the timemean state. The larger waves are observed to mix the PV in the upper-level critical layers to a greater extent than those in experiment A, and, in fact, to a similar extent as in experiment B. As a result the wave packets remain well defined for a similar length of time as in experiment B, around 100-150 days. Other diagnostics, such as the wave activity flux snapshots are also qualitatively similar to experiment B. A composite of the

 \leftarrow

background zonal wind (not shown) was also found to be very similar to that for experiment B (Fig. 9), although its amplitude was found to be slightly larger, most likely due to the wave amplitude being larger.

2) EXPERIMENT E: LOW SUPERCRITICALITY

Figure 17 shows the snapshots from experiment E. This experiment is characterized by a low criticality, β = 0.313. The small-scale dissipation ν_0 is in this case set to a low value (equal to that for expt C). These parameters result in wave behavior that is almost linear everywhere, except in the upper-level critical layers. As a result this experiment gives a very good example of asymmetric downstream-upstream packet structure, indicating upstream stabilization by a barotropic correction. There is a trail of breaking waves in the critical layers upstream of the packet maximum, and the wave activity flux vectors are in the meridional direction relative to the group velocity $c_{g1}(y)$. Although the time and length scales on which the wave packet evolves are much longer, as the radiative jet is only weakly supercritical, the wave packet shares these features in common with those seen in experiment a (Fig. 2). This substantiates the claim that it is the degree to which the waves are allowed to break and in particular the reflectivity or absorptivity of the critical layers that determines the fundamental dynamics of these experiments. If a higher value of dissipation was used when the β parameter was at this value, a steady wave train was formed. (This in fact was found to be the case for experiments with lower β also.)

d. On the use of a full nonlinear wave activity flux

In this section we discuss the results obtained using the full nonlinear wave activity relation of Shepherd (1988). This relation, which also has the form (13), has wave activity given by

$$A_{i}(Q_{i}, q_{i}^{e}) = \int_{0}^{q_{i}^{e}} Y(Q_{i} + \tilde{q}) - Y(Q_{i}) d\tilde{q}, \quad (31)$$

where $Y(Q_i)$ is the inverse function of the basic-state PV, $Q_i(y)$ in layer *i*. The flux is given adding an eddy advection term to \mathbf{F}_i as given in Eq. (15):

$$\mathbf{F}_i \to \mathbf{F}_i + \begin{pmatrix} u_i^e A_i \\ v_i^e A_i \end{pmatrix}. \tag{32}$$

Here S_i remains unchanged.

FIG. 4. (a) A composite showing the mean upper-level background zonal flow correction around a packet maxima in expt A. The contour interval is 0.02, positive contours are solid, and negative contours are dashed. (b) As in (a) but for the lower layer. (c) The real phase speed c_r of the fundamental baroclinic wave ($k = 2^{-1/2}$) calculated with respect to the time-mean flow plus the background flow correction illustrated in (a) and (b). The dotted line shows c_r calculated with respect to the time-mean flow alone. (d) As in (c) but for the imaginary phase speed c_i .



Group Velocities





FIG. 6. (a) The latitudinal structure of the upper-level group velocity $c_{g1}(y)$ (dashed line), the corrected group velocity $c_{g1} + c_{g1}^{zc} + c_{g1}^{bt} + c_{g1}^{bc} + c_{g1}^{dt}$ (solid line), and the upper-level mean wind in expt A (dotted line). (b) The individual corrections to the wave packet group velocity due to each term in Eq. (24), c_{g1}^{xc} (dot-dash line), c_{g1}^{bt} (dashed line), c_{g1}^{bc} (solid line), and c_{g1}^{ds} (dotted line). (c) Longitudinal structure of $\langle F_1^{(x)} \rangle / \langle A_1 \rangle$ averaged across the center of the channel from $y = 0.42L_y$ to $y = 0.58L_y$, showing the variations in the notional local group velocity along the wave packets. The dotted line shows $\overline{F_1^{(x)}}/\overline{A_1}$.

This fully nonlinear flux is then used in place of the corrected flux derived in section 2c [Eqs. (20) and (21)], which is formally valid only up to $O(a^2)$ in wave amplitude *a*. The main advantage of the nonlinear relation is that it includes the effect of the advection of wave activity by the eddy wind field. Its main disadvantage is that unlike (20)-(21) it relies on some form of phase averaging across individual eddy wavelengths. When a fixed wavelength is chosen for this phase averaging, we have discovered that it is difficult to evaluate terms in (24) that are sensitive to the local variation in the zonal wavenumber, such as $F_{1}^{(x)}$, in a consistent method-in-

1.20

dependent manner.6 The other composites calculated (e.g., $\langle A_1 \rangle$, $\langle S_1 \rangle$, $\langle F_{1\nu}^{(y)} \rangle$) gave nearly identical results between (14)-(15) and (20)-(21).

+L_/2

By analogy with tracer transport problems, mere advection of wave activity by an incompressible eddy

⁶ The most consistent results between Eqs. (15) and (21) were obtained for $\langle F_x^{(1)} \rangle$ when the compositing alone was the method of phase averaging, although this led to very noisy results. This suggests that the problem may have been due to the smoothing effect of phase averaging prior to compositing.

[←]

FIG. 5. Composites of various fields taken around the packet maxima in expt A. (a) The-upper level wave activity A_1 ; (b) $-\langle F_{12}^{(i)} - c_{el}A_1 \rangle_{x}$, the zonal convergence of wave activity term in Eq. (24); (c) $\langle \mathcal{D}_1 - S_1 - F_{12}^{(i)} \rangle$, the remaining terms on the rhs of Eq. (24); (d) the barotropic sink term $\langle -F_{12}^{(i)} \rangle$; (e) the baroclinic source term $\langle -S_1 \rangle$. In each panel solid contours denote positive values and dotted contours negative values. The zero contour is omitted. Contour intervals are (a) 0.5, (b) 0.002, (c) 0.005, (d) 0.01, and (e) 0.01.



FIG. 7. From expt A. (a) A snapshot of upper-level meridional velocity ϕ_{1x} , with a packet maxima shifted in longitude to the "zero" reference point. The contour interval is 0.1. (b) A comparison of the relative magnitude of the composite terms in Eq. (24), averaged across the channel center from $y = 0.42L_y$ to $y = 0.58L_y$. The dot-dash curve is the zonal convergence term $\langle F_1^{(2)} - c_{s1}A_1 \rangle_x$, the thin solid curve is the baroclinic term $\langle -S_1 \rangle$, the dashed curve is the barotropic term $\langle -F_{1y}^{(2)} \rangle_x$ and the dotted curve is the dissipation $\langle \mathcal{D}_1 \rangle$. The thick solid curve is the total of all these terms. (c) A comparison of the corresponding *residual* terms, when contributions to the time-mean zonal mean wave activity budget, and effective changes to the wave packet group velocity, have been subtracted out, as described in section 2d.



FIG. 8. As in Fig. 2 but for expt B. Contour intervals are (a) 0.44, (b) 1.4, (c) 0.165, and (d) 0.28. Arrow scaling is 1 length unit to 0.125.



FIG. 9. As in Figs. 4a and 4b but for expt B. The contour interval is 0.05.

wind field cannot lead to concentration of a maximum in wave activity, although stirring of the wave activity field by the eddies might be expected to contribute to its eventual dissipation through diffusion. One might therefore expect the major effect of including the extra advective tendencies in Shepherd's flux to be a correction to the wave packet group velocity. By this argument, a tendency to defocus the wave packet would be apparent when the residual tendency defined in section 2d is calculated for the eddy advection term.

Figure 18 shows some results when the composite method is applied with the full nonlinear flux to experiment C. Figure 18a shows the composite wave activity calculated using Eq. (31). There is a slight rearrangement compared with Fig. 13a, with the wave activity more concentrated in the center of the channel. Figure 18b shows the composite tendency due to the (phase averaged) advective term: $\langle (u_1^e A_1)_x + (v_1^e A_1)_y \rangle$ minus its zonal mean component (see section 2d). As it is largely dipolar about the packet center, one would expect the main effect of this term to be a correction to the group velocity. In Fig. 18c the residual of this term is shown, averaged across the center of the channel as before. From this curve we see that the advective term has a tendency to defocus the packet, as speculated above. Although we are using a different wave activity relation and it is not possible to compare directly, it is probably not a coincidence that the residual eddy advection tendency is close to being equal and opposite to the total net tendency for packet focusing due to the waves, illustrated by the heavy curve in Fig. 15c. Figure 18d shows the correction to the group velocity due to the advective term. Unsurprisingly, this is similar in structure and magnitude to the background flow correction at zero lag shown in Fig. 12a.

For experiment A, the eddy advection terms were found to have the effect of advecting the wave packet with a velocity comparable to the zero lag background flow velocity, as in experiment C. Also, in experiment A the residual eddy advection tendency in the *center* of the channel was to focus the wave packet, with an intensity close to being equal and opposite of the total net tendency illustrated by the heavy curve in Fig. 7c. This focusing is due to the background flow field advecting wave activity into the center of the channel from the outer maxima shown in Fig. 5a. [Here $\langle A_1 \rangle$ is not greatly changed by using the nonlinear wave activity density (31) in place of the linear density (20).]

In conclusion, it seems that the main effect identified by including the eddy advection terms in the nonlinear flux is the resultant correction to the wave packet group velocity, which is close in magnitude to the background flow correction defined in section 2b. The residual effect is the defocusing of wave activity away from the wave maximum (or maxima in the case of experiment A) by eddy stirring of the wave activity. The magnitude of the eddy stirring effect is close to being equal and opposite to the sum of the eddy flux residual tendencies identified in section 3a, suggesting that the total net focusing balances net defocusing in the composite mean.



FIG. 10. As in Fig. 2 but for expt C. Contour intervals are (a) 0.56, (b) 1.4, (c) 0.23, and (d) 0.28. Arrow scaling is 1 length unit to 0.25.



FIG. 11. (top) The latitudinal profile of the most important antisymmetric and symmetric cross-sectional EOFs of u, for expt C. The solid line shows the upper-level structure and the dotted line the lower-level structure. The modes describe 48.2% and 15.0% of the variance in u, respectively. (bottom) Longitude–time plots showing the evolution of the corresponding principal components in longitude and time. Contour intervals are (left) 0.8 and (right) 0.5.

catitud**e** ۲/۲

0 -L./2

0

(a)

Upper Layer

 \cap

 \bigcirc

0







FIG. 12. As in Fig. 4 but for expt C. (a), (b) The contour interval is 0.08.

0 Longitudinal Distance from Reference Point



FIG. 13. As in Fig. 5 but for expt C. Contour intervals are (a) 1.0, and (b)–(e) 0.025.



4. Results from a multilevel primitive equation model

a. The model and equations

The model integrates the primitive equations in spherical geometry and pressure coordinates. These are given by

$$\frac{d\mathbf{u}}{dt} = -f\mathbf{k} \times \mathbf{u} - \nabla_{\!_{H}} \Phi^{_{G}} - \kappa \mathbf{u}, \quad (33)$$

$$\frac{d\Theta}{dt} = -r(\Theta - \Theta_0), \qquad (34)$$

$$\frac{\partial \omega_P}{\partial p} + \boldsymbol{\nabla}_H \cdot \boldsymbol{\mathbf{u}} = 0, \qquad (35)$$

and

$$\frac{\partial \Phi^G}{\partial \zeta} = -c_p \Theta, \qquad (36)$$

where Φ^G is geopotential, and ∇_H denotes the horizontal component of the gradient operator. Here $\zeta = (p/p_s)^{\kappa}$ denotes an auxiliary vertical coordinate, ω_P is pressure velocity, Θ denotes potential temperature, and Θ_0 is the equilibrium potential temperature field, to which Θ is being relaxed on a timescale r^{-1} , which is set to 25 days. The model has mechanical damping on a timescale of κ^{-1} , nonzero and equal to 0.5 days only in the lowest level. This represents surface friction effects. Components of motion are **u** and ω_p , and c_p is specific heat of constant pressure. The model used has 10 equally spaced pressure levels, with spectral discretization in the horizontal of resolution T42. Here ∇^8 hyperdiffusion is included in (33) and (34) to prevent the cascade of enstrophy to grid scales. The timescale of this diffusion acting upon the largest wavenumber was 6 h. The boundary conditions on the upper and lower surfaces are taken to be $\omega_p = 0$, which is equivalent to constraining the vertically integrated horizontal divergence to be zero.

Figure 19a shows the radiative equilibrium zonal wind field, and Fig. 19b shows the time-mean zonal mean wind that results when the model has spun up to statistical equilibrium. Wave packets were found to be ubiquitous throughout the model experiment. The EOFs method identified the propagating wave packets but was not able to isolate the background flow response as in the two-layer model experiments above. This was due to the excitation of long waves by the internal variability of the system. These experiments will be discussed in greater detail in a future paper.



FIG. 15. (a) A composite wave packet calculated for expt C. Positive contours are solid, the zero contour is dashed, and the negative contours are dotted. The contour interval is 0.3. (b),(c) As in Figs. 7b,c but for expt C.



FIG. 16. As in Fig. 2 but for expt D. Contour intervals are (a) 0.48, (b) 1.3, (c) 0.36, and (d) 0.36. Arrow scaling is 1 length unit to 0.25.



FIG. 17. As in Fig. 2 but for expt E. Contour intervals are (a) 0.04, (b) 1.5, (c) 0.005, and (d) 0.18. Arrow scaling is 1 length unit to 0.02.

b. The composite method applied to the background flow

An important question raised by the above two-layer model results is what form the background zonal flow correction forced by a wave packet takes in a more realistic three-dimensional model. Does the flow appear to be stabilized by a barotropic correction to the jet upstream of the wave packet, or does the wave packet tend to force a semibaroclinic correction near its maximum as in the low-dissipation two-layer model experiment C? In order to answer this question the composite method was applied to several 100-day periods of model results. No attempt was made to address the more subtle and complex question of whether the system can be viewed in the same weakly nonlinear framework as the two-layer model. Future work may include calculating linear growth rates using an analogous eigenvalue method, but such an analysis seems likely to be complicated by the presence of unstable shallow modes.

Figure 19c shows a cross section of the composite background zonal wind at the the packet maxima. For comparison, Fig. 19d shows the same picture for the far-field flow.⁷ Unsurprisingly, the far-field flow is close in structure to the time-mean flow (see Fig. 19b). The composite flow at the packet maxima, however, shows that the jet is considerably weaker at the location of the packet maximum.

Figure 20 shows a plot of the composite background zonal wind correction on the 250-mb level and the 750mb level. These levels can be thought to be equivalent to the upper and lower levels in the two-layer model. The contour plots show that the jet becomes weaker and broader near the packet maximum, and slightly sharper and stronger just upstream. The meridional and vertical structure is strikingly similar to the semibaroclinic correction to the background flow in the low-dissipation two-layer model experiment C (see Figs. 12a and 12b). This suggests that the wave packets maintain themselves in this model by a similar semibaroclinic mechanism. Independent results by J. F. Scinocca (1996, personal communication) showed that if the small-scale dissipation in a similar model was much stronger, then steady wave packets would result, as in experiment A for the two-layer model. This suggests that the dynamics of the wave packets in this type of experiment are controlled, as in the two-layer model, by the degree of wave breaking and PV mixing that takes place.

5. Summary and conclusions

The main focus of this paper has been to investigate the wave packet dynamics observed in a series of experiments in a quasigeostrophic two-layer model. When the model was relaxed radiatively toward a baroclincally unstable state with a jet profile in the upper level, two different regimes of wave packet behavior were observed. In the first regime the wave packets were argued to be maintained by upstream stabilization, and in the second by nonlinear self-focusing. What determined the type of behavior was found to be the extent of PV mixing that took place in the upper-level critical layers at the sides of the jet, and at the center of the channel in the lower level. The degree to which the PV was well mixed controlled how well developed the upper-level waveguide became in each experiment.

When this waveguide was not allowed to develop, for example, when ν_0 was large (expt A) or when β was large (expt E), the resultant wave packets were steady in time. The wave packets were characterized by a local balance between baroclinic growth and barotropic decay, and nonlinear self-focusing was found not to be important. Upstream stabilization by a barotropic correction to the flow was suggested as a mechanism for maintaining these packets. Linear calculations revealed that the jet with stronger meridional shear to the rear of the wave packets in experiment A was stabilized relative to the broader weaker jet at the packet maximum and to its downstream side. As the flow remains unstable in the region of the packet maximum, packet growth is encouraged rather than inhibited by the background flow response. This barotropic correction appears to be excited because there is a time delay τ between waves reaching their maximum amplitude at the center of the wave packet and their eventual dissipation at the jet edges. The waves effectively transport eastward zonal momentum from the center of the channel to the jet edges upstream of the packet where it is deposited. One estimate for τ might therefore be $L_d/c_{g1}^{(y)}$, where L_d is the meridional distance from the channel center to the region of maximum dissipation of the waves, which depends upon various factors such as wave amplitude, and $c_{\ell 1}^{(y)}$ is the upper-level meridional group velocity given by $F_{\perp}^{(y)}/A_{\perp}$. This would give a length scale for the packets of $c_{e1}\tau = c_{e1}L_d/c_{e1}^{(y)}$. While a more detailed study would be necessary to test the validity of this scaling, it does give appropriate length scales for the packets observed in experiments A and E.

The increase in the meridional shear of the jet toward the rear of the wave packets was also observed in the experiments of LH. They also noted that it is related to increased barotropic decay of the eddies toward the rear of the wave packet, which was evident in the wave activity composites for experiment A. These experiments also appear to have much in common with the results of the initial value experiments of SP. They too describe stabilization upstream of the packet maximum by low-level mixing and the spinup of a barotropic jet. The enhanced group velocity caused by growth at the downstream edge of the packet in experiment A also seems analogous to the enhanced group speed of the leading edge modes in the experiments of SP. In initial

⁷ This composite is an average over those points with spacial lag greater than $+160^{\circ}$ or less than -160° from the packet maxima.





FIG. 19. (a) The zonal wind field in balance with the radiative temperature profile Θ_s ; (b) the time-mean zonal mean zonal wind field; (c) a composite of the background zonal wind at the packet maxima; (d) a composite far-field background zonal wind. Contour intervals are 4 m s⁻¹ and values greater than 20 m s⁻¹ are shaded. The zero contour is dashed and the dotted contours denote negative values.

value problems of this type there is necessarily a strong gradient of baroclinicity along the wave packet from the undisturbed fluid downstream to the stabilized flow upstream. This gradient, which does not necessarily exist in the forced-dissipative experiments reported here, appears to constrain upstream stabilization to be important in initial value experiments.

One of the main results of this paper is that upstream stabilization alone cannot account for wave packet formation when the waveguide is well developed (e.g., in expt C). This is because the wave packets act to stabilize the background flow at the packet maximum as well as just upstream. Nonlinear self-focusing, which was diagnosed in this paper as the zonal convergence of wave activity in the upper level, was shown to have an important role. In the simple weakly nonlinear model (system A) of E97, in which the wave packets form through nonlinear self-focusing alone, the dynamics of the amplitude equation allow the wave packets to exist for a relatively short period before they are dissipated. This may account for the short lifetimes of the wave packets in experiment C (\approx 50 model days). This regime is almost certainly of more relevance to the dynamics of the midlatitudes, as the extratropical tropopause is well known to be characterized by local regions of strong PV gradients on isentropic surfaces, surrounded by relatively well-mixed regions (e.g., Hoskins 1991).

To help understand why the background flow re-

 \leftarrow

FIG. 18. (a) Composite upper-level wave activity $\langle A_1 \rangle$ as in Fig. 13a, but from Shepherd's relation. (b) A composite of the nonlinear advection term $\langle (u_1^cA_1)_x + (v_1^cA_1)_y \rangle$ in Shepherd's equation (minus its time-mean zonal mean component). The contour interval is 0.025. (c) A curve such as those in Fig. 15c showing the residual effect of this term at the channel center. (d) A curve such as those in Fig. 14b showing the group velocity correction due to this term.



FIG. 20. A longitude–latitude plot of the composite background zonal wind correction along the wave packet (a) at the 250-mb level and (b) at the 750-mb level. The contour interval is 1 m s^{-1} , positive contours are solid, and negative contours dashed.

sponse due to the wave packets is so different in experiment C compared with experiment A, it is useful to consider the dynamical similarities between experiment C and the weakly nonlinear model. In the weakly nonlinear model the eddies are constrained by the sidewalls, and the baroclinic background flow response is determined by the structure of the in situ wave forcing and the rate of restoration of the background flow in each layer. In experiment C, the eddies are confined to the waveguide by (partially) reflecting critical-layer regions. This means that the weak in situ dissipation of the waves becomes a more important component of the wave forcing of the flow than the barotropic decay (see Fig. 15c), which may account for the large baroclinic component of the response. The reason for the inclusion of the results from the primitive equation experiment (section 4) was to illustrate that the background flow response observed in experiment C could be observed in a more realistic model.

The nonlinear self-focusing mechanism is closely related to the zonal convergence of wave activity, which can take place on zonally varying basic flows (see, e.g., Swanson et al. 1997). The simplest case shown in that paper describes the dynamics of a single contour that separates regions of constant vorticity. This simple model can be thought to crudely represent the upper-level waveguide of experiment C. The conservation of wave activity W for that system was derived to be

$$\frac{\partial \mathcal{W}}{\partial T} + \frac{\partial}{\partial S}(c_s \mathcal{W}) = 0, \qquad (37)$$

where $c_g = U(S)$ is the group velocity of the waves that is equal to the basic flow speed (S is distance along the contour). Equation (37) can be considered a greatly simplified version of (24) for zonally varying flows. From (37) it is clear that nonlinear self-focusing of wave packets is likely to be greatly enhanced by convergence of wave activity in regions where U'(S) < 0 and diminished where U'(S) > 0.

Another important issue that these experiments raise is related to the modeling of internal variability of the troposphere. Scinocca and Haynes (1997) discussed the importance to the stratospheric circulation of the long waves generated by nonlinearity in the troposphere. The experiments reported here show that such internal variability will be grossly distorted if the numerical model is overdamped (or equivalently underresolved), since the wave packets will then be steady, as in experiment A, rather than variable, as in experiment C. This issue will be touched on again in a future paper.

Acknowledgments. This work was done primarily at the University of Cambridge under the financial support of the Natural Environment Research Council (United Kingdom), as well as by the U.K. Meteorological Office. The work was completed and written up at the Massachusetts Institute of Technology under NSF Grant 9528471-ATM. Helpful discussions with Edmund Chang, Alan Plumb, and Isaac Held are gratefully acknowledged, as well as the contribution of the anonymous referees. Mark Taylor also deserves thanks for providing the complex eigenvalue routine, and Tieh Yong Koh for comments on the manuscript. The Centre for Atmospheric Science is a joint initiative of the Department of Applied Mathematics and Theoretical Physics and the Department of Chemistry. The work of the Centre is supported by the U.K. Natural Environment Research Council through the U.K. Universities Global Atmospheric Modelling Programme and by the Isaac Newton Trust.

APPENDIX

Derivation of the Group Velocity Property for the Wave Activity Relation

In this appendix, the group velocity property [(23)] is verified for the wave activity relation defined in section 2c for the case of nongrowing waves in the absence of dissipation. A new discretized two-layer model group velocity is also introduced.

In the appropriate WKBJ limit, of a slowly varying

background flow and infinitesimal sinusoidal waves, we can write

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \mathcal{A} \sin(kx + ly - \omega t) \\ \gamma \mathcal{A} \sin(kx + ly - \omega t) \end{pmatrix}.$$
 (A1)

As the waves are nongrowing and nondissipative, $\omega = ck$ and γ are real and are given by (e.g., Pedlosky 1987, pp. 416–430)

$$\frac{\omega}{k} = c = \frac{U_1 + U_2}{2} - \frac{\beta(a^2 + F)}{a^2(a^2 + 2F)}$$
$$\pm \frac{1}{a^2(a^2 + 2F)} [4\beta^2 F^2 - U_s^2 a^4 (4F^2 - a^4)]^{1/2}$$
(A2)

and

$$\gamma = \frac{(c - U_s)(a^2 + F) + \beta + FU_s}{(c - U_s)F}.$$
 (A3)

Here $a^2 = k^2 + l^2$ and $U_s = U_1 - U_2$, and U_2 can be set equal to zero without loss of generality. Then, if the relation $\sin^2 \theta + \cos^2 \theta = 1$ is exploited, it can be derived from (20) that

$$A_{1} + A_{2} = \frac{\Re^{2}}{4} \left\{ \frac{[\gamma F - (a^{2} + F)]^{2}}{\beta + FU_{s}} + \frac{[F - \gamma(a^{2} + F)]^{2}}{\beta - FU_{s}} \right\}$$
$$= \frac{\Re^{2} F}{4\mathcal{E}\mathcal{F}} \{ \mathcal{F}[\gamma F - a^{2} - F] + \mathcal{E}[F - \gamma(a^{2} + F)] \},$$
(A4)

since $F(\beta + FU_s) = \mathcal{E}(\gamma F - a^2 - F)$, and $F(\beta - FU_s) = \mathcal{F}(F - \gamma(a^2 + F))$. The constants \mathcal{E} and \mathcal{F} are given by

$$\mathcal{E} = (c - U_s)F$$
 and $\mathcal{F} = c(a^2 + F) + \beta - FU_s$.
(A5)

Similarly, from (21),

$$F_{1}^{(x)} + F_{2}^{(x)} = \frac{\mathcal{A}^{2}}{4} \left\{ \frac{U_{s}(\gamma F - a^{2} - F)^{2}}{\beta + FU_{s}} + 2k^{2}(1 + \gamma^{2}) + (\gamma F - a^{2} - F) + \gamma[F - \gamma(a^{2} + F)] \right\}$$
$$= \frac{\mathcal{A}^{2}F}{4\mathcal{E}\mathcal{F}} \{\mathcal{F}U_{s}(\gamma F - a^{2} - F) + 2k^{2}[(c - U_{s})\mathcal{F} + c\gamma\mathcal{E}] + \mathcal{F}(\beta + FU_{s}) + \gamma\mathcal{E}(\beta - FU_{s})\}.$$
(A6)

From these it is a small step to Eq. (23),

$$F_1^{(x)} + F_2^{(x)} = c_e(A_1 + A_2),$$

as the relation

$$c_g = \frac{\partial \omega}{\partial k} = \frac{\mathcal{F}[U_s(\gamma F - a^2) + 2k^2(c - U_s) + \beta] + \mathcal{E}\gamma[2ck^2 + \beta - FU_s]}{\mathcal{F}(\gamma F - a^2 - F) + \mathcal{E}(F - \gamma(a^2 + F))}$$
(A7)

can be obtained by implicit differentation of (A2). This wave activity relation is not only consistent with the idea of the overall global group velocity c_g for the two-layer model, but interestingly it also defines a separate group velocity vector in each layer, which is exploited in section 3. This is given by $c_{gi} = F_i^{(x)}/A_i$. This vector gives the group velocity of waves in one layer in the absence of coupling with the other layer. In the experiments of section 3, c_{g1} was found to be much closer to the observed group velocity of the wave packets, perhaps because lower-level mixing had the effect of reducing the coupling between the layers.

REFERENCES

- Balmforth, N. J., 1995: Solitary waves and homoclinic orbits. Annu. Rev. Fluid Mech., 27, 335–373.
- Berbery, E. H., and C. S. Vera, 1996: Characteristics of the Southern Hemisphere winter storm track with filtered and unfiltered data. *J. Atmos. Sci.*, 53, 468–481.
- Chang, E. M., 1993: Downstream development of baroclinic waves as inferred from regression analysis. J. Atmos. Sci., 50, 2038– 2053.
- —, and I. Orlanski, 1993: On the dynamics of a storm track. J. Atmos. Sci., 50, 999–1015.
- —, and D. B. Yu, 1999: Characteristics of wave packets in the upper troposphere. Part I: Northern Hemisphere winter. J. Atmos. Sci., 56, 1708–1728.

- Esler, J. G., 1997: Wave packets in simple equilibrated baroclinic systems. J. Atmos. Sci., 54, 2820–2849.
- Feldstein, S., and I. M. Held, 1989: Barotropic decay of baroclinic waves in a two-layer beta-plane model. J. Atmos. Sci., 46, 3416– 3430.
- Hoskins, B. J., 1991: Towards a PV- θ view of the general circulation. *Tellus*, **43AB**, 27–35.
- James, I. N., 1987: Suppression of baroclinic instability in horizontally sheared flows. J. Atmos. Sci., 44, 3710–3720.
- Killworth, P. D., and M. E. McIntyre, 1985: Do Rossby-wave critical layers absorb, reflect or over-reflect? J. Fluid Mech., 161, 449– 492.
- Lee, S., and I. M. Held, 1993: Baroclinic wave packets in models and observations. *J. Atmos. Sci.*, **50**, 1413–1428.
- —, and S. Feldstein, 1996: Mechanism of zonal index evolution in a two-layer model. J. Atmos. Sci., 53, 2232–2246.

Pedlosky, J., 1987: Geophysical Fluid Dynamics. Springer-Verlag.

- Plumb, R. A., 1985: On three-dimensional propagation of stationary waves. J. Atmos. Sci., 42, 217–229.
- Scinocca, J. F., and P. H. Haynes, 1998: Dynamical forcing of stratospheric planetary waves by tropospheric baroclinic eddies. J. Atmos. Sci., 55, 2361–2392.
- Shepherd, T. G., 1988: Nonlinear saturation of baroclinic instability. Part I: The two-layer model. J. Atmos. Sci., 45, 2014–2025.
- Simmons, A. J., and B. J. Hoskins, 1978: The life cycles of nonlinear baroclinic waves. J. Atmos. Sci., 35, 1364–1373.
- Swanson, K. L., and R. T. Pierrehumbert, 1994: Nonlinear wave packet evolution on a baroclinically unstable jet. J. Atmos. Sci., 51, 384–396.
- —, P. J. Kushner, and I. M. Held, 1997: Dynamics of barotropic storm tracks. J. Atmos. Sci., 54, 791–810.