The Antarctic Stratospheric Sudden Warming of 2002: A Self-Tuned Resonance?

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The extraordinary Antarctic stratospheric warming event of 2002 was characterized by a remarkable vertical structure, with the vortex observed to divide at upper levels in the stratosphere but not at lower levels: such 'partially' split vortex events are relatively rare. A simple, yet fully three-dimensional, model is constructed to investigate the dynamics of this unique event. Planetary waves are excited on the model vortex edge by a lower boundary forcing characterized by two parameters: an amplitude h_F and a frequency ω_F, measured relative to a stationary frame. For realistic forcing amplitudes, a partial vortex split resembling that observed during the 2002 event is found only within a specific, narrow band of forcing frequencies. Exploiting the relative simplicity of our model, these frequencies are shown to be those causing a 'self-tuning' resonant excitation of the gravest linear mode, during which nonlinear feedback causes an initially off-resonant forcing to approach resonance.

1. Introduction

The remarkable Antarctic stratospheric sudden warming of 2002 has attracted great interest amongst the atmospheric science community [e.g., *J. Atmos. Sci.*, 2005, special issue, no. 3], primarily because such an event is unprecedented in roughly 50 years of observations. Between September 23 and September 26, above the 600 K isentropic level ($\sim$ 26 km), the stratospheric vortex was observed to split into two parts [*Charlton et al.*, 2005], and the attendant higher polar temperatures had a dramatic impact on subsequent chemistry with substantially reduced ozone depletion [*Stolarski et al.*, 2005]. A detailed understanding of such significant events is therefore necessary in order to assess the likelihood of future occurrences, with the attendant consequences on the Antarctic ozone hole.

The short timescale associated with stratospheric warmings, typically of the order of several days, indicates that they must be essentially fluid dynamical events, with radiative and chemical processes playing a secondary role. On these time scales the dynamics are essentially adiabatic and 'balanced', and hence can be understood on the basis of the three-dimensional distribution of potential vorticity (PV). For the 2002 event, the evolution of one isosurface of scaled PV, derived from ECMWF operational analysis data, is shown in the top row of Fig. 1. The isosurface and scaling parameters have been chosen to obtain an accurate fit to the vortex edge over a large altitude range, as described further below. Approximately 20 days before the event, on September 5, the vortex is seen to be relatively cylindrical in appearance. However, at 1200UT on September 23 the vortex has become strongly elongated throughout its altitude range. The vortex is clearly split by September 26, but only at upper levels, while below $\sim$26 km it appears to have recovered its circular shape. This 'partial' split is a distinguishing feature of the 2002 event, and is distinct from the vortex behavior observed for most Northern Hemisphere events [*Manney et al.*, 2005], during which the vortex is observed to split near-simultaneously over its entire altitude range [e.g., *Manney et al.*, 1994].

Although the Antarctic 2002 event was forecast accurately [*Simmons et al.*, 2005], little insight is gained as to which specific dynamical aspects are responsible for the unusual partial split structure. Furthermore, other modeling studies [e.g. *Mukougawa et al.*, 2005] have shown that the vortex evolution is highly sensitive to initial conditions, and it remains unclear what ingredients are needed to produce a successful forecast.

The aim of this work, therefore, is to use a relatively simple model to determine the dynamical conditions necessary to generate a stratospheric sudden warming whose three-dimensional structure resembles the partial split of the 2002 event. The model's relative simplicity allows a thorough exploration of parameter space and, in particular, the delineation of the narrow region over which partial vortex splits occur. More importantly, however, the model's simplicity allows for contact to be made with analytic results [*Esler and Scott*, 2005] which yield understanding into the underlying fundamental dynamics.

Specifically, we aim to demonstrate that the Antarctic 2002 event can be understood as a 'self-tuned' resonance, in the sense of *Plumb* [1981] who showed, using a simple model of the stratosphere, that the maximum wave amplification occurs when the system is forced with a frequency that differs by a finite amount from that of a free mode. As the wave grows the system self-tunes towards resonance by nonlinear feedback. Details of the model and the numerical experiments are given in section 2, the model results are discussed in section 3, and conclusions are given in section 4 below.

2. Formulation of Model and Experiments

Arguably the simplest model to capture the fundamental fluid dynamics of stratospheric sudden warmings is that of a three-dimensional quasi-geostrophic flow in a compressible

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atmosphere on an f-plane [*Dritschel and Saravanan*, 1994]. In this model the columnar polar vortex is represented, at each log-pressure height z, by a patch $\mathcal{S}(z)$ of uniformly high PV, which is initially circular with radius $R(z)$. Outside the vortex, the PV q is constant; inside q is a function of z. The flow is adiabatic and frictionless, and hence conserves PV; its dynamics thus obey:

$$(\partial_t - \psi_y \partial_x + \psi_x \partial_y)\, q = 0, \tag{1}$$

where q is defined by

$$\begin{aligned} q(\mathbf{x}, z) &= f + \nabla_H^2 \psi + \frac{1}{\rho}\left(\rho \frac{f^2}{N^2}\psi_z\right)_z \\ &= \begin{cases} f + \Delta(z) + \Omega & \mathbf{x} \in \mathcal{S}(z), \\ f + \Omega & \text{otherwise.} \end{cases} \end{aligned} \tag{2}$$

Here ψ is a streamfunction for the horizontal velocity, $\mathbf{u} = -\nabla_H \times \psi\mathbf{k}$, ∇_H^2 is the horizontal Laplacian operator, f is the Coriolis parameter, N is the buoyancy frequency, Ω is a constant PV value, and $\rho = \exp(-z/H)$ is the density, with H a constant scale height. The function $\Delta(z)$ denotes the potential vorticity jump at the vortex edge. The lower boundary condition of the model, is

$$f\psi_z + N^2 h = 0, \quad \text{on} \quad z = 0. \tag{3}$$

where h is a 'topographic' forcing which excites planetary-scale waves that propagate on the vortex edge.

The values of the model parameters are chosen as in *Waugh and Dritschel* [1999]: $H = 6.14$km, $f = 4\pi\text{days}^{-1}$ and $N = 2.13 \times 10^{-2}\text{s}^{-1}$, which yields a Rossby radius $L_R = NH/f = 900$km. The functional forms for $\Delta(z)$ and $R(z)$, are obtained by making a crude fit to the observed PV on September 11, 2002, as given by ECMWF operational analysis datasets and plotted in Fig. 2. The position of the observed vortex edge, defined as being the location of the maximum gradient in PV with respect to equivalent latitude on each isentropic surface, is marked with a set of crosses on Fig. 2. In order to fit this surface, we choose $\Delta(z)$ and $R(z)$ as follows

$$\Delta(z) = \begin{cases} 0 \\ 0.6f \\ 0.4f \\ 0.4f \end{cases} \quad R(z) = \begin{cases} \text{n/a} & z < H \\ 7L_R & H < z < 2H \\ r(z)\, L_R & 2H < z < 8H \\ 2L_R & z > 8H \end{cases} \tag{4}$$

As Fig. 2 shows, the observed PV is roughly uniform inside the vortex, which has a weak poleward slant with height; we capture this by letting $r(z) = 4.1 - 0.27z/H$. A PV gradient is also added in the upper troposphere ($H < z < 2H$), to represent the subtropical tropospheric jet. A choice of $\Omega = -0.07f$ was found to give the best fit of the model winds to the observed ones. The resulting model tropospheric and stratospheric jets (not shown), have strengths 48.3 and 59.2 ms^{-1} respectively, and are then co-located with the observed jets (whose maxima are 43.4 and 63.4 ms^{-1}, respectively, as shown in Fig. 2).

The forcing function is chosen to be,

$$h(r, \phi, t) = h_F\, J_2\left(\frac{r}{r_F}\right)\cos(2\phi - \omega_F t), \tag{5}$$

with $r_F = 2.58L_R$ and J_2 a Bessel function. With this choice, $h_F = 0.1H$ corresponds to a peak to trough difference in height of 597m. It was found that the results are insensitive to the specific choice of h. A set of calculations identical to those reported below, but with h given by two Gaussian mountains, centered an equal distance on opposite sides of the origin, gave results with no qualitative differences to report.

The model is implemented numerically using the CASL (Contour Advective Semi-Lagrangian) algorithm to advect the boundaries of the vortex patches on each model level [*Macaskill et al.*, 2003]. For each calculation, the vortex has an initially circular cross-section, and is allowed to evolve for 30 days under the influence of the above forcing. The parameters h_F and ω_F are varied between experiments, with the aim of creating a regime diagram illustrating the regions of parameter space that correspond to different behaviors.

3. Results

The bottom row of Fig. 1 shows the evolution of the model vortex during an experiment with parameters $h_F = 0.1H$ and $\omega_F = -0.01305f$. Clearly there is a strong resemblance between the evolution of the model vortex and that of the the observed vortex during the 2002 event (top row). On day 0, the model vortex is axisymmetric, but is rapidly distorted under the influence of the forcing. Immediately prior to day 24 the wave-2 disturbance on the model vortex is seen to propagate eastwards. By day 24 the vortex has become both elongated and 'pinched' at all levels, and resembles the 2002 vortex as observed on 1200UT September 23, just before the sudden warming. During the next 60 hours, the model vortex splits at upper levels, but recovers at lower levels to form a single vortex, as does the observed vortex over a similar timescale. The model vortex undergoes very little rotation during this time, and there is no significant phase tilt with height. Similarly, the observed vortex remains oriented approximately along the 35°E-155°W great circle at all levels between 1200UT September 23 and 0000UT September 26. Over the following two days, up to model day 28.25, the 'arms' of the partially split vortex begin to wrap around each other anticyclonically, developing strong phase tilts with height. A similar process occurs in the observed event up to 1800UT on September 27 [see e.g., *Manney et al.,* 2005]. Some details of the observed evolution begin to differ from the model at this stage, in part due to the imposed wave-2 symmetry in the model. An anticyclonic vortex, advected from the subtropics at upper levels, now has a significant role in the observed dynamics. Nevertheless, we submit that the model experiment in Fig. 1 captures both the main qualitative aspects of the nonlinear dynamical evolution and, equally importantly, the dynamical timescale of the 2002 event.

A very large number of model runs were necessary to obtain the qualitative and quantitative agreement with observations shown in Fig. 1. In particular, the model exhibits very strong sensitivity to the parameter ω_F. Due to the rotational invariance of the model, two alternative interpretations of ω_F exist: it is either the frequency of a transient lower boundary forcing as in (5) above, or it is a measure of the strength of an anticyclonic solid body rotation added to the initial flow ($\Omega \to \Omega - \omega_F$) for the case of a stationary lower boundary forcing. What is important, therefore, is the angular velocity at the vortex edge *relative* to the forcing. Considering the latter interpretation, it is easily shown that an increase in ω_F of $0.01f$ is equivalent to reducing the initial stratospheric jet strength by a mere 1.7ms^{-1}: this, however, can have dramatic consequences on the evolution of the flow, as shown by the regime diagram in Fig. 3. In Fig. 3 the outcome of the model experiments, as a function of (ω_F, h_F), are summarized. Vortex splits, denoted by red diamonds, occur first for $h_F \sim 0.09H$, and then only within a vanishing small range of forcing frequencies around $\omega_F = -0.01305f$. As h_F increases, the range of forcing frequencies over which the vortex splits broadens considerably: however, in most cases, the vortex splits over its entire height. Partial vortex splits (blue squares), such as the one shown in the bottom row of Fig. 1, occur over a much narrow range, and only for $0.09H \leq h_F \leq 0.13H$.

Is it possible to understand such complex nonlinear behavior from the predictions of linear theory? In order to answer this question, the frequencies of the linear normal modes of the initial vortex were calculated, using the eigenvalue method described by *Waugh and Dritschel* [1999]. The three gravest vertical modes were found to have frequencies corresponding to $\omega_F = \{0.0114, 0.149, 0.160\}f$ respectively. Those model experiments leading to vortex splits, as described above, occur for forcing frequencies closest to that of the gravest mode (or barotropic mode), highlighted by the vertical dotted line on Fig. 3 ($0.0114f$). Can the vortex splits be associated with the excitation to finite amplitude of the gravest linear mode? A series of model experiments with comparatively low h_F ($0.01H \rightarrow 0.08H$) was performed to address this question. For each h_F, the value of ω_F which caused the largest disturbance to the model vortex, as measured by the maximum vortex angular impulse recorded up to the end of each experiment [see e.g. eqn. (12) of *Esler and Scott*, 2005], was determined; these values are shown by the black crosses in Fig. 3. It is clear that as $h_F \rightarrow 0$, the maximum response occurs at the linear frequency of the gravest mode of the vortex. For finite h_F, a 'frequency offset' between the frequency yielding the maximum response and the linear frequency becomes apparent, and this frequency offset increases as h_F increases. It is clear that $(\omega_F, h_F) = (-0.01305f, 0.09H)$, where the vortex split first occurs, lies on a continuation of this same maximum response curve. It may therefore be concluded that the vortex split illustrated in Fig. 1 is caused by the excitation to finite amplitude of the gravest vertical mode of the vortex.

4. Discussion and Conclusions

In this letter, it has been demonstrated that a simple quasi-geostrophic model is able to capture the unusual 'partial split' evolution of the Antarctic vortex observed during the September 2002 sudden warming. The simplicity of the model allows both a thorough exploration of parameter space and a connection to be made with linear theory. In particular it is possible to test the idea that there is a connection between vortex split sudden warmings and the resonant excitation of a linear mode of the flow [*Tung and Lindzen*, 1979]. Previous theory [*Plumb*, 1981] predicts that the maximum response should occur when the forcing is initially off-resonant and the flow 'self-tunes' towards resonance as the disturbance grows to finite amplitude. The model experiments provide strong evidence that a self-tuning resonant excitation of the gravest linear mode of the vortex is precisely the cause of both partial and complete vortex splits. Model experiments designed to excite other vertical modes of the vortex were also attempted and were not found to cause either partial or complete vortex splits. Hence we conclude that only the gravest linear mode is important for wave-2 sudden warmings.

Different initial vortex structures have also been investigated. The initial inward tilt of the vortex was found to be important in allowing partial, rather than complete, vortex splits. For example if the vortex is initially cylindrical only complete splits occur. The model tropopause at $H < z < 2H$ also appears to have a role in transmitting disturbances to the vortex, although vortex splits also occur in its absence. Each of the vortex structures investigated could be made to split by exciting the gravest linear mode at the correct 'off-resonant' frequency.

It is intended that variations of the methods used in this study might be used in practice to assess the likelihood that a specific observed vortex will subsequently undergo a self-tuning resonant excitation, leading to a sudden warming event. However, since the observed vortex is never in a truly undisturbed state it is unclear how one might calculate *a priori* the frequencies associated with linear resonances. Nevertheless, some progress may be possible. In addition, however, two clear and important qualitative results have emerged from this study, and it is worth emphasizing them. First, it was found that partial vortex splits occur only over a very small range in parameter space: this might explain why such events are relatively rare. Second, it was shown that the vortex evolution in its nonlinear stage is extremely sensitive to characteristics of the forcing: this provides one reason why such events are in practice, rather difficult to forecast.

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References

Charlton, A. J., O'Neill, A., Lahoz, W. A., and Berrisford, P., The splitting of the stratospheric polar vortex in the southern hemisphere, September 2002: Dynamical evolution. *J. Atmos. Sci.*, *62*, 590–602, 2005

Dritschel, D. G. and Saravanan, R., Three-dimensional quasi-geostrophic contour dynamics, with an application to stratospheric vortex dynamics, *Q. J. R. Meteorol. Soc.*, *120*, 1267–1297, 1994.

Esler, J. G., and Scott, R. K., The excitation of transient Rossby waves on the stratospheric polar vortex and the barotropic sudden warming, *J. Atmos. Sci.*, *62*, 3661–3682, 2005.

Lait, L. R., 1994, An alternative form for potential vorticity. *J. Atmos. Sci.*, *51*, 1754–1759.

Macaskill, C., Padden, W. E. P., and Dritschel, D. G., The CASL algorithm for quasi-geostrophic flow in a cylinder, *J. Comput. Phys.*, *188*, 232–251, 2003.

Manney, G. L., Farrara, J. D., and Mechoso, C. R., Simulations of the February 1979 stratospheric sudden warming: Model comparisons and three-dimensional evolution. *Mon. Wea. Rev.*, *122*, 1115–1140, 1994.

Manney, G. L. and Coauthors, Simulations of dynamics and transport during the September 2002 Antarctic major warming. *J. Atmos. Sci.*, *62*, 690–707, 2005.

Mukougawa, H., Sakai, H., and Hirooka, T., High sensitivity to the initial condition for the prediction of stratospheric sudden warming, *Geophys. Res. Lett.*, *32*, L17806, doi:10.1029/2005GL022909, 2005.

Plumb, R. A., Instability of the distorted polar night vortex: a theory of stratospheric warmings. *J. Atmos. Sci.*, *38*, 2514–2531, 1981.

Simmons, A. Hortal, M., Kelly, G., McNally, A., Untch A. and Uppala, S., ECMWF Analyses and Forecasts of Stratospheric Winter Polar Vortex Breakup: September 2002 in the Southern Hemisphere and Related Events. *J. Atmos. Sci.*, *62*, 668–689, 2005.

Stolarski, R. S., McPeters, R. D., and Newman, P. A., The Ozone Hole of 2002 as Measured by TOMS, *J. Atmos. Sci.*, *62*, 716-720, 2005.

Tung, K. K., and Lindzen, R. S., A theory of stationary long waves. Part I: A simple theory of blocking, *Mon. Wea. Rev.*, *107*, 714–734, 1979.

Waugh, D. W. and Dritschel, D. G., The dependence of Rossby wave breaking on the vertical structure of the polar vortex, *J. Atmos. Sci.*, *56*, 2359–2375, 1999.

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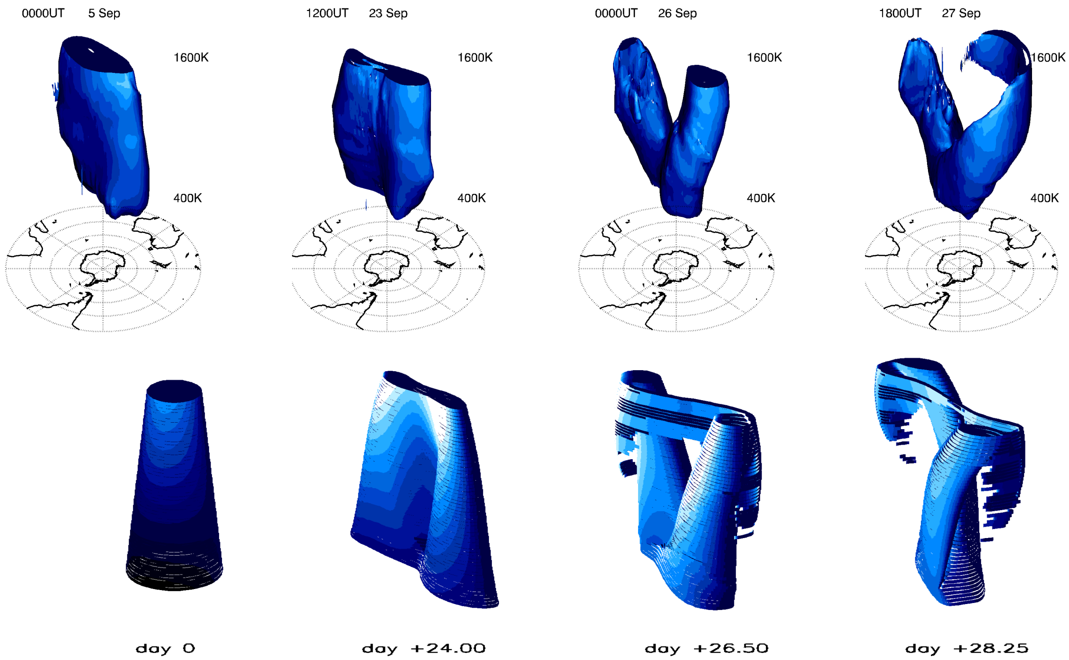

Figure 1. The evolution of the Antarctic polar vortex during September 2002 from ECMWF operational analyses (top row) and of the model vortex during an idealized calculation (bottom row). The top row shows three-dimensional iso-surfaces $q = q_0$ of Lait's modified potential vorticity, $q = (\theta/\theta_0)^{\alpha}(\nabla\theta).(f + \nabla \times \mathbf{u})/\rho$, between the potential temperature (isentropic) surfaces $\theta = 400K$ and $\theta = 1600K$ on September 5 (0000UT), 23 (1200UT), 26 (0000UT) and 27 (1800UT) 2002. The parameters $q_0 = 39.5$ PVU (1 PVU=10^{-6} K kg^{-1} m^2 s^{-1}) $\alpha = -4.25$, and $\theta_0 = 475$K are chosen in order to best identify the vortex edge over the altitude range being investigated (see Fig. 2 and discussion). In the lower panels the model parameters are $h_F = 0.1H$, $\omega_F = -0.01305f$, and the surfaces are generated from the model contours between layers $z = 2H$ (12.25 km) and $z = 8H$ (49 km), at model times $t = 0$, 24.0, 26.50 and 28.25 days.

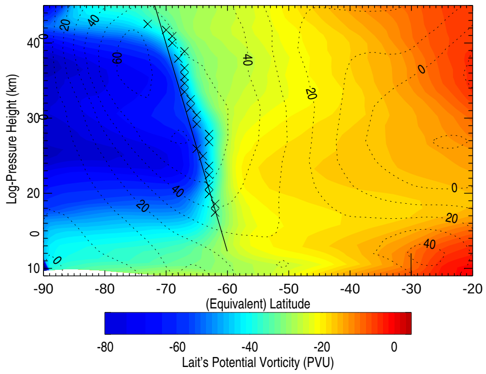

Figure 2. Lait's modified potential vorticity (see Fig. 1 caption) at 0000UT on September 11, 2002 (1 PVU = 1 $\times 10^{-6}$ K kg^{-1} m^{2} s^{-1}). Each cross denotes a local maximum in the gradient of potential vorticity with respect to equivalent latitude on an isentropic surface. Dotted contours denote zonal mean wind (c.i. 10ms^{-1}.) The solid black lines show the locations of the model vortex edge and tropopause PV jumps.

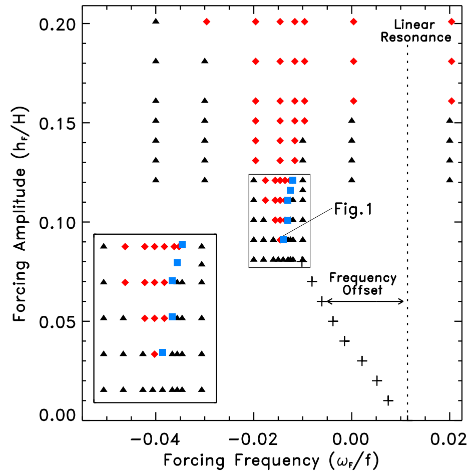

Figure 3. Regime diagram of model behavior as a function of forcing amplitude h_F and forcing frequency ω_F. Red diamonds denote model experiments which result in a complete vortex split, and black triangles denote those experiments in which no split occurs. Blue squares denote experiments where a partial split, such as that illustrated in Fig. 1, occurs. The inset is an enlargement of the boxed region. The value of ω_F causing resonant excitation of the gravest linear mode of the vortex is given by the dotted line. For each forcing amplitude $h_F = 0.01H$, $0.02H$,...,$0.08H$, a black cross marks the particular forcing frequency ω_F that causes the greatest disturbance to the vortex.