A strong stability condition on minimal submanifolds
Mu-Tao Wang (Columbia/CUHK)

It is well known that the distance function to a totally geodesic submanifold of a negatively curved ambient manifold is a convex function. We identify a strong stability condition on minimal submanifolds that generalizes the above scenario. In particular, if a closed minimal submanifold $\Sigma$ is strongly stable, then:

(1) The distance function to $\Sigma$ satisfies a partial convex property in a neighborhood of $\Sigma$, which implies that $\Sigma$ is the unique closed minimal submanifold in this neighborhood, up to a dimensional constraint.

(2) The mean curvature flow that starts with a closed submanifold in a $C^1$ neighborhood of $\Sigma$ converges smoothly to $\Sigma$.

Many examples, including several well-known calibrated submanifolds, are shown to satisfy this strong stability condition. This is based on joint work with Chung-Jun Tsai.