

# G 2 Days 2014

## 14 – 16 July

The timetable for the workshop will be as follows.

Monday: Gauge theory	Tuesday: Topology	Wednesday: Open Session
09:00 <i>Refreshments/registration</i>	09:00 <i>Refreshments</i>	09:00 <i>Refreshments</i>
09:30 Bryant	10:00 Salamon	10:00 TBA
11:00 <i>Break</i>	11:00 <i>Break</i>	11:00 <i>Break</i>
11:30 Oliveira	11:30 Nordström	11:30 TBA
12:30 <i>Lunch</i>	12:30 <i>Lunch</i>	12:30 <i>Close</i>
14:00 Haydys	14:00 Crowley	
15:00 <i>Break</i>	15:00 <i>Break</i>	
15:30 Becker	15:30 Goette	

Talks will all take place in the Harrie Massey Lecture Theatre at UCL (25 Gordon Street).

Morning refreshments and breaks will all be in the 5th Floor Common Room in the Mathematics Department at UCL (25 Gordon Street).

### Titles and abstracts

**Katrin Becker** (Texas A&M) – The  $\alpha'$  expansion on a compact manifold of exceptional holonomy

In the approximation corresponding to the classical Einstein equations, which is valid at large radius, string theory compactification on a compact manifold  $M$  of  $G_2$  or  $\text{Spin}(7)$  holonomy gives a supersymmetric vacuum in three or two dimensions. Do  $\alpha'$  corrections to the Einstein equations disturb this statement? Explicitly analyzing the leading correction, we show that the metric of  $M$  can be adjusted to maintain supersymmetry. Beyond leading order, a general argument based on low energy effective field theory in spacetime implies that this is true exactly (not just to all finite orders in  $\alpha'$ ).

**Robert Bryant** (Duke) – On  $G_2$  and complex and almost-complex structures on 6-manifolds

**Diarmuid Crowley** (MPI Bonn/Aberdeen) – New invariants in  $G_2$  topology

I will define the  $\nu$ -invariant, a  $\mathbb{Z}/48$ -valued homotopy invariant of  $G_2$ -structures on 7-manifolds, and also a generalisation of the Eells–Kuiper invariant which is valid for all spin 7-manifolds. These invariants play an important role in joint work with Johannes Nordström and Sebastian Goette.

The  $\nu$ -invariant can be used to distinguish between connected components of the moduli space of  $G_2$  metrics and it is the topological fore runner to the analytic invariant discussed in the following talk by Sebastian Goette. The generalised Eells–Kuiper invariant can be used to distinguish the smooth type of homeomorphic but not diffeomorphic  $G_2$ -manifolds.

The examples where these invariants are computed will be described in the preceding talk by Johannes Nordström. In this talk I will also describe the 8-dimensional co-boundaries used for some computations of these invariants.

**Sebastian Goette** (Freiburg) – Analytic invariants of  $G_2$  manifolds

Diarmuid Crowley has described a  $\mathbb{Z}/48$ -valued invariant  $\nu$  for topological  $G_2$  structures on compact manifolds in his talk. Using an intrinsic description in terms of  $\eta$ -invariants and Mathai–Quillen currents, we lift  $\nu$  to an integer-valued invariant for  $G_2$  holonomy metrics. We compute this invariant for some of the examples from Johannes Nordström’s talk and describe how it can distinguish different connected components of the

space of  $G_2$  holonomy metrics, even if the underlying topological  $G_2$  structures are homotopic.

**Andriy Haydys** (Bielefeld) – A compactness theorem for the Seiberg–Witten equations with multiple spinors

This is a joint project with Th. Walpuski. Motivated by higher dimensional gauge theory, we consider the compactness problem for the Seiberg–Witten equations with multiple spinors in dimension three. We show that a sequence of solutions of the Seiberg–Witten equations has a subsequence converging to a Fueter section, which is a non-linear version of a harmonic spinor.

**Goncalo Oliveira** (Imperial/Duke) – Monopoles in  $G_2$  manifolds

The Bogomolnyi equation is a PDE for a connection and a Higgs field on a bundle over a 3-dimensional Riemannian manifold. In the talk I will explain an extension of this PDE to 7 dimensional  $G_2$  manifolds. Its solutions are known as  $G_2$ -monopoles and Donaldson and Segal proposed that “counting” them may give an invariant of certain noncompact  $G_2$  manifolds. Moreover, this may be related to the conjectural invariants of Joyce obtained by counting coassociative submanifolds. I plan to motivate all this by exploring examples of monopoles on the Bryant–Salamon  $G_2$  manifolds.

**Johannes Nordström** (Bath) – Disconnecting the  $G_2$  moduli space

**Simon Salamon** (King’s) – Index theory and special geometries in dimension 8

After reviewing basic facts concerning characteristic classes, Betti numbers, the Dirac operator, and triality, I shall contrast the geometries arising from the structure groups  $\text{Spin}(7)$  and  $\text{Sp}(2)\text{Sp}(1)$  and their representation theory, and touch on topics such as formality and Witten rigidity.