Topology and Groups

Week 10, Monday

1 Preparation

- 8.05 (Galois correspondence, 1),
- 8.06 (Galois correspondence, 2).

2 Classwork

1. Draw the covering spaces of $S^1 \vee S^1$ associated to the following subgroups:
   - the subgroup generated by a^2 and b^2;
   - the normal subgroup generated by a^2 and b^2.
2. What are all the subgroups of $\pi_1(T^2)$? What do the corresponding covering spaces look like? What are the deck groups? Given two such covering spaces Y_1, Y_2, what do the covering transformations $Y_1 \to Y_2$ look like? Is there a covering space of T^2 which is not a product of covering spaces of the two factors?
3. Show that the torus is a double cover of the Klein bottle.
4. What is the universal cover of the Möbius strip?
5. Suppose that X is a graph which is homotopy equivalent to a wedge of n circles. What is the Euler characteristic of X? Suppose that $p\colon Y \to X$ is a finite-to-one covering space of X. Show that $\pi_1(Y, y)$ is a free group $\mathbf{Z} \star \cdots \star \mathbf{Z}$ for some number N of factors. What is N?
6. Show that any subgroup of a free group is free (Nielsen-Schreier theorem). If G is a free group of rank n and H is a subgroup of index d, H is free by the Nielsen-Schreier theorem. What is the rank of H?