

Topology and Groups

Week 1, Thursday

1 Preparation

- 1.02 (Paths, loops, homotopies),
- 1.03 (Concatenation, π_1).

2 Discussion

1. (PCQ) Is the homotopy γ_R from our sketch-proof of the fundamental theorem of algebra a *based* homotopy or a *free* homotopy?
2. (PCQ) Let x be a basepoint. Suppose that α_t is a based homotopy between loops α_0 and α_1 and β_t is a based homotopy between β_0 and β_1 . Can you write down a based homotopy between $\beta_0 \cdot \alpha_0$ and $\beta_1 \cdot \alpha_1$? This verifies one of the claims from the lemma in Section 1.03: which claim?

3 Classwork

In your learning groups, tackle the following questions:

1. Prove that based homotopy of loops is an equivalence relation.
2. Prove that the fundamental group is a group (concatenation is associative on homotopy classes and admits inverses).

In the final 20 minutes of the lecture, we will need volunteers to present their solutions.