Handout 10: Laplace Transforms

Rules

We assume that we already know that Laplace transform of a given function $f(t)$. i.e. we know what

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t) dt$$

is. We can then find the Laplace transform of a number of functions related to $f(t)$ by applying some rules.

Rule 1: Multiplying by a constant a is a constant:

$$\mathcal{L}\{\ af(t)\ \} = aF(s)$$

Rule 2: Adding functions together

If we know the Laplace transform of two functions $\mathcal{L}\{f_1(t)\} = F_1(s)$ and $\mathcal{L}\{f_2(t)\} = F_2(s)$, then

$$\mathcal{L}(f_1(t) + f_2(t)) = F_1(s) + F_2(s).$$

Also, if a and b are constants

$$\mathcal{L}(af_1(t) + bf_2(t)) = aF_1(s) + bF_2(s).$$

Rule 3: Multiplying by e^{at}

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a).$$

Rule 4: Differentiation

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0).$$

Rule 5: Second Derivative

$$\mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0).$$

Rule 6: Multiply by $-t$

$$\mathcal{L}\{-tf(t)\} = \frac{\mathrm{d}F(s)}{\mathrm{d}s}.$$

Rule 7: Multiply be t^2

$$\mathcal{L}\{t^2 f(t)\} = \frac{\mathrm{d}^2 F(s)}{\mathrm{d}s^2}.$$

Handout 11: Laplace Transforms of Common Functions

Result 1:

$$\mathcal{L}\{1\} = \frac{1}{s}.$$

Result 2:

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

Result 3:

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

Result 4: For a constant a,

$$\mathcal{L}\{e^{at}\} = \frac{1}{(s-a)}$$

Results 5 and 6: $\sin(at)$ and $\cos(at)$ for a constant a.

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2}, \qquad \mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2}$$

Results 7 and 8: $\sinh(at)$ and $\cosh(at)$ for a constant a.

$$\mathcal{L}\{\sinh(at)\} = \frac{a}{s^2-a^2}, \qquad \mathcal{L}\{\cosh(at)\} = \frac{s}{s^2-a^2}$$