Math 7502

Example from last lecture

Consider the transportation problem (T) with cost matrix (c_{ij}), supply vector (p_i) and demand vector (q_j) displayed in the following array:

8	6	7	5	12
4	3	5	4	8
9	8	6	7	11
7	6	10	8	

We use the north-west rule to find the feasible solution

7	5	0	0
0	1	7	0
0	0	3	8

Since the number of nonzero entries is $6 = 3+4-1$ this is a basic solution (the seven equations for the transport problem are not independent, one follows from supply=demand). We find λ_i, $i = 1, 2, 3$ and μ_j, $j = 1, 2, 3, 4$ satisfying complementary slackness, even if they do not all satisfy the constraints $\lambda_i + \mu_j \leq c_{ij}$. There is freedom in the choice of one of them, so we choose $\lambda_1 = 0$.

$\lambda_i \setminus \mu_j$	8	6	8	9
0	$^{7}8$	$^{5}6$	7	5
-3	4	$^{1}3$	$^{7}5$	4
-2	9	8	$^{3}6$	$^{8}7$

We check the constraints $\lambda_i + \mu_j \leq c_{ij}$. If a constraint is not satisfied we put a cross and write how much we are off. This gives the table

$\surd$	$\surd$	$\times^{-1}$	$\times^{-4}$
$\times^{-1}$	$\surd$	$\surd$	$\times^{-1}$
$\surd$	$\surd$	$\surd$	$\surd$

We choose the largest number -4 and decide to include x_{14} in our basic variables. We decide to increase it to ϵ. This gives as new transport solution

7	$5-\epsilon$	0	ϵ
0	$1+\epsilon$	$7-\epsilon$	0
0	0	$3+\epsilon$	$8-\epsilon$

This is feasible as long as $\epsilon \leq 5$ and we choose this value to exit x_{12} from our basic variables. This gives as new basic feasible solution

7	0	0	5
0	6	2	0
0	0	8	3

To check whether it is optimal we find dual variables λ_i and μ_j satisfying complementary slackness. We choose $\lambda_1 = 0$. This gives

$\lambda_i \setminus \mu_j$	8	2	4	5
0	$^{7}8$	6	7	$^{5}5$
1	4	$^{6}3$	$^{2}5$	4
2	9	8	$^{8}6$	$^{3}7$

We check the constraints $\lambda_i + \mu_j \le c_{ij}$. If a constraint is not satisfied we put a cross and write how much we are off. This gives the table

$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$\times^{-5}$	$\checkmark$	$\checkmark$	$\times^{-2}$
$\times^{-1}$	$\checkmark$	$\checkmark$	$\checkmark$

We choose the largest number -5 and decide to include x_{21} in our basic variables. We decide to increase it to ϵ. This gives as new transport solution

$7-\epsilon$	0	0	$5+\epsilon$
ϵ	6	$2-\epsilon$	0
0	0	$8+\epsilon$	$3-\epsilon$

This is feasible as long as $\epsilon \le 2$ and we choose this value to exit x_{23} from our basic variables. This gives as new basic feasible solution

5	0	0	7
2	6	0	0
0	0	10	1

To check whether it is optimal we find dual variables λ_i and μ_j satisfying complementary slackness. We choose $\lambda_1 = 0$. This gives

$\lambda_i \setminus \mu_j$	8	7	4	5
0	$^{5}8$	6	7	$^{7}5$
-4	$^{2}4$	$^{6}3$	5	4
2	9	8	$^{10}6$	$^{1}7$

We check the constraints $\lambda_i + \mu_j \le c_{ij}$. If a constraint is not satisfied we put a cross and write how much we are off. This gives the table

$\checkmark$	$\times^{-1}$	$\checkmark$	$\checkmark$
$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$\times^{-1}$	$\times^{-1}$	$\checkmark$	$\checkmark$

We choose the largest number -1 and decide to include x_{12} in our basic variables. We decide to increase it to ϵ. This gives as new transport solution

$5-\epsilon$	ϵ	0	7
$2+\epsilon$	$6-\epsilon$	0	0
0	0	10	1

This is feasible as long as $\epsilon \leq 5$ and we choose this value to exit x_{11} from our basic variables. This gives as new basic feasible solution

0	5	0	7
7	1	0	0
0	0	10	1

To check whether it is optimal we find dual variables λ_i and μ_j satisfying complementary slackness. We choose $\lambda_1 = 0$. This gives

$\lambda_i \setminus \mu_j$	7	6	4	5
0	8	$^{5}6$	7	$^{7}5$
-3	$^{7}4$	$^{1}3$	5	4
2	9	8	$^{10}6$	$^{1}7$

We check the constraints $\lambda_i + \mu_j \leq c_{ij}$. If a constraint is not satisfied we put a cross and write how much we are off. This gives the table

$\surd$	$\surd$	$\surd$	$\surd$
$\surd$	$\surd$	$\surd$	$\surd$
$\surd$	$\surd$	$\surd$	$\surd$

So this solutions satisfies all the dual constraints and complementary slackness, so it is optimal. The cost for this solution (minimal cost) is

$$5 \cdot 6 + 7 \cdot 5 + 7 \cdot 4 + 1 \cdot 3 + 10 \cdot 6 + 1 \cdot 7 = 163.$$