Math 7502

Homework 6

Due: February 28, 2008

1. Consider the two problems
$$\begin{array}{rr} \text{minimize} & c^t \cdot x \\ \text{subject to} & Ax \geq b \\ & x \geq 0, \end{array} \tag{1}$$
and
$$\begin{array}{rl} \text{minimize} & c^t \cdot x \\ \text{subject to} & Ax \geq b + v \\ & x \geq 0, \end{array} \tag{2}$$
where v is a vector is $\mathbf{R}^m$ of small 'size'. We consider the problem (2) as a 'perturbation' of the problem (1). For instance, if (1) is the diet problem with daily need prescribed by the vector b, then in the problem (2) we are allowing a small change in the dietary needs. The question is how does this affect the cost of the diet. Let y_0 be the optimal for the dual problem to (1). Show that the optimal (minimal) cost of the (diet) problem (2) is
$$f = (b + v)^t y_0.$$
Hint: Use duality.

Remark: The vector $-y_0$ appeared in the last row of the last simplex tableau. This problem explains why the last row gives information on the *shadow prices*.

2. (a) Suppose that the problem
$$\begin{array}{rr} \text{maximize} & c^t \cdot x \\ \text{subject to} & Ax \leq b \\ & x \geq 0 \end{array}$$
has a finite optimal solution. Here A is an $m \times n$ matrix, $b \in \mathbf{R}^m, c \in \mathbf{R}^n$. Show that, no matter what the vector $b' \in \mathbf{R}^m$ might be, the problem
$$\begin{array}{rr} \text{maximize} & c^t \cdot x \\ \text{subject to} & Ax \leq b' \\ & x \geq 0 \end{array}$$
cannot be unbounded.

Hint: Use duality.

3. (a) Suppose that a two-person zero-sum game with payoff matrix A has a saddle-point. Show that all saddle points have the same value.

   (b) Show that, if a_{ij} is a saddle point then the row i is a maximin row and the column j is a minimax column and

$$\max_k \min_l a_{kl} = \min_l \max_k a_{kl} = a_{ij}.$$

   (c) If

$$\max_k \min_l a_{kl} = \min_l \max_k a_{kl}$$

   then the intersection of the maximin row and the minimax column is a saddle point.