Math 7502

Homework 4

Due: February 7, 2008

1. $*$ Use the two phase simplex algorithm to solve the linear program

$$\begin{array}{lrcrcrcr} \text{maximize} & x_1 & + & x_2 & + & x_3 & & \\ \text{subject to} & -x_1 & - & x_2 & + & x_3 & \leq & -2 \\ & x_1 & + & 2x_2 & + & x_3 & \leq & 5 \\ & 3x_1 & + & x_2 & + & x_3 & \leq & 8 \\ & x_1 & , & x_2 & , & x_3 & \geq & 0. \end{array}$$

2. $*$ (a) Use the two phase simplex algorithm to solve the linear program

$$\begin{array}{lrcrcr} \text{minimize} & x_1 & + & x_2 & & \\ \text{subject to} & 4x_1 & + & x_2 & \geq & 4 \\ & x_1 & + & 6x_2 & \geq & 6 \\ & 6x_1 & + & 10x_2 & \geq & 23 \\ & x_1 & , & x_2 & \geq & 0. \end{array}$$

$*$ (b) Solve the same problem graphically and explain what the two phase simplex algorithm does geometrically (on the graph).

3. Let x_1, $x_2, \ldots, x_k$ be points in $\mathbf{R}^n$. We say that $y \in \mathbf{R}^n$ is a convex combination of $x_1, \ldots, x_k$ if we can find scalars $\lambda_1, \lambda_2, \ldots, \lambda_k$ such that

$$y = \sum_{j=1}^{k} \lambda_j x_j, \quad \lambda_j \geq 0, \quad j = 1, \ldots, k, \quad \sum_{j=1}^{k} \lambda_j = 1.$$

(a) Let S be the set of convex combinations of $x_1, \ldots, x_k$. Prove that S is a convex set. The set S is called the convex hull of $x_1, \ldots, x_k$.

(b) Let y be a convex combination of a and $b \in \mathbf{R}^n$. Assume also that a, b are convex combinations of $x_1, \ldots, x_k$. Prove that y is a convex combination of $x_1, \ldots, x_k$.

(c) Show that the convex hull of $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$ is the square $[0,1]\times[0,1]$. The difficult part is to show that every point in the square is a convex combination of the four extreme points of the square. Write (x,y) as a convex combination of $(0,y)$ and $(1,y)$ first and use (b).