Math 7502

Homework 3

Due: January 31, 2008

1. $*$ Consider the region defined by the following constraints:

$$\begin{aligned} -x_1 + x_2 &\leq 2 \\ -x_1 + 2x_2 &\leq 6 \\ x_1, x_2 &\geq 0. \end{aligned}$$

   (i) Maximize $-4x_1 + x_2$ subject to the constraints above.

   (ii) Minimize $3x_1 - 4x_2$ subject to the constraints above.

   (iii) Maximize $-x_1 + 3x_2$ subject to the constraints above. You should find the maximal solution value is unbounded. Explain this carefully by (a) exhibiting feasible points with objective value increasing to infinity (b) showing the situation on the graph of the feasible region in the x_1, x_2-plane.

2. $*$ A nut packager has on hand 150 kg of peanuts, 100 kg of cashews, and 50 kg of almonds. The packager can sell three kinds of mixtures of these nuts: a cheap mix consisting of 80% peanuts and 20% cashews; a party mix with 50% peanuts, 30% cashews, and 20% almonds; and a deluxe mix with 20% percent peanuts, 50% cashews, and 30% almonds. If the 1 kg can of the cheap mix, the party mix and the deluxe mix can be sold for 0.9, 1.1 and 1.3 pounds respectively, how many cans of each type would the packager produce in order to maximize the return? Use the simplex method in tableau format and a hand-held calculator for the computations.

3. (Only for maths students) In analysis you saw the notion of a convex function. A function $f : [a, b] \to \mathbf{R}$ is called convex if its graph is below the secant segment between any two points of the graph $(x, f(x))$ and $(y, f(y))$, i.e. for all $t \in [0, 1]$ we have

$$f((1-t)x + ty) \leq (1-t)f(x) + tf(y).$$

   Consider the set

$$S = \{(x, y) | y \geq f(x), x \in [a, b]\}.$$

   Show that f is convex function if and only if S is a convex set in $\mathbf{R}^2$.