

Spectral theory: what's it all about and what is its use

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<https://en.wikipedia.org/wiki/Pendulum>

Period of oscillation of a pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}} \text{ seconds,}$$

where

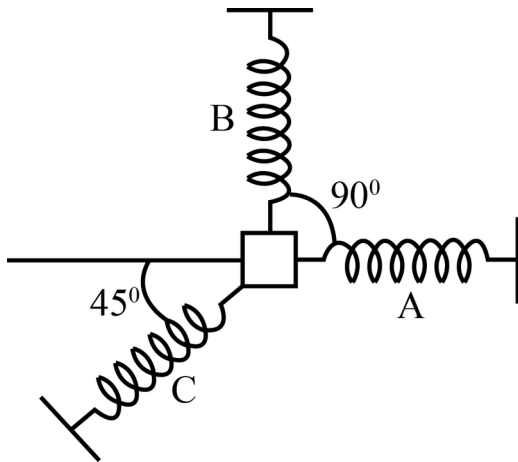
$$g \approx 9.8 \frac{m}{s^2}$$

is the acceleration of gravity (gravity of Earth) and L is the length of the pendulum.

Natural frequency of a pendulum:

$$f = \frac{1}{T} \text{ Hz.}$$

A system with 3 degrees of freedom.



It has 3 natural frequencies.

A string: system with infinitely many degrees of freedom.

https://en.wikipedia.org/wiki/String_vibration

It has infinitely many natural frequencies:

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}, \quad n = 1, 2, \dots,$$

where T is the tension of the string, μ is the linear density of the string and L is the length of the string.

The collection of all natural frequencies is called *spectrum*.

Lowest natural frequency is called *fundamental frequency*, whereas higher natural frequencies are called *overtones*.

Vibrations of a membrane. Think of a drum.

Things become more complicated: need to deal with *partial* differential equations. Here *partial* refers to the fact that we have more than one spatial coordinate.

Chladni Plate.

<https://www.youtube.com/watch?v=T04iLhJsars>

Ernst Florens Friedrich Chladni (1756 - 1827) was a German physicist and musician.

Acoustic resonator. Like a membrane, but in 3D.

Things get really tough, finding natural frequencies f_n , $n = 1, 2, \dots$, becomes very difficult.

A miracle: the Rayleigh–Jeans law (1905) states that

$$f_n \approx c \left(\frac{3n}{4\pi V} \right)^{1/3} \quad \text{as } n \rightarrow +\infty,$$

where c is the speed of sound and V is the volume of the resonator.

Jeans' contribution to the Rayleigh–Jeans law

“It seems to me that Lord Rayleigh has introduced an unnecessary factor 8 by counting negative as well as positive values of his integers”.

Lord Rayleigh replied with a paper consisting of one sentence as well: “I agree”.

1910: Lorentz visits Göttingen at Hilbert's invitation and delivers a series of lectures “Old and new problems in physics”. Lorentz states the Rayleigh–Jeans law as a mathematical conjecture. Hermann Weyl is in the audience.

1912: Weyl publishes a rigorous proof of Rayleigh–Jeans law. Almost incomprehensible.

Comprehensible proof: in R.Courant and D.Hilbert, *Methods of Mathematical Physics* (1924).

Weyl's Conjecture (1913)

One can do better and prove a two-term asymptotic formula for the acoustic resonator with hard walls (Neumann boundary conditions):

$$f_n \approx c \left(\frac{3n}{4\pi V} \right)^{1/3} - \frac{c S}{16V} \quad \text{as } n \rightarrow +\infty,$$

where S is the surface area of the boundary of the resonator.

Very difficult problem bordering on *analytic number theory*.

https://en.wikipedia.org/wiki/Gauss_circle_problem

Theorem (Victor Ivrii, 1980) Weyl's conjecture holds if periodic billiard trajectories have measure zero.

Theorem (Vassiliev, 1984) Under certain geometric assumptions on the branching Hamiltonian billiards, Weyl's conjecture holds for higher order operators. Furthermore, I have an explicit algorithm for the evaluation of the second Weyl coefficient.

Yu.Safarov and D.Vassiliev, *The asymptotic distribution of eigenvalues of partial differential operators*, American Mathematical Society, 1997 (hardcover), 1998 (softcover).

“In the reviewer's opinion, this book is indispensable for serious students of spectral asymptotics”. Lars Hörmander for the Bulletin of the London Mathematical Society.

I described my path into spectral theory, but of course there's much more to spectral theory than that.

Most important application of spectral theory: atomic physics.

Electrons in the atom are described as waves governed by the Schrödinger equation or the Dirac equation. Natural frequencies are interpreted as energy states of the atom.

https://en.wikipedia.org/wiki/Energy_level

https://en.wikipedia.org/wiki/Emission_spectrum

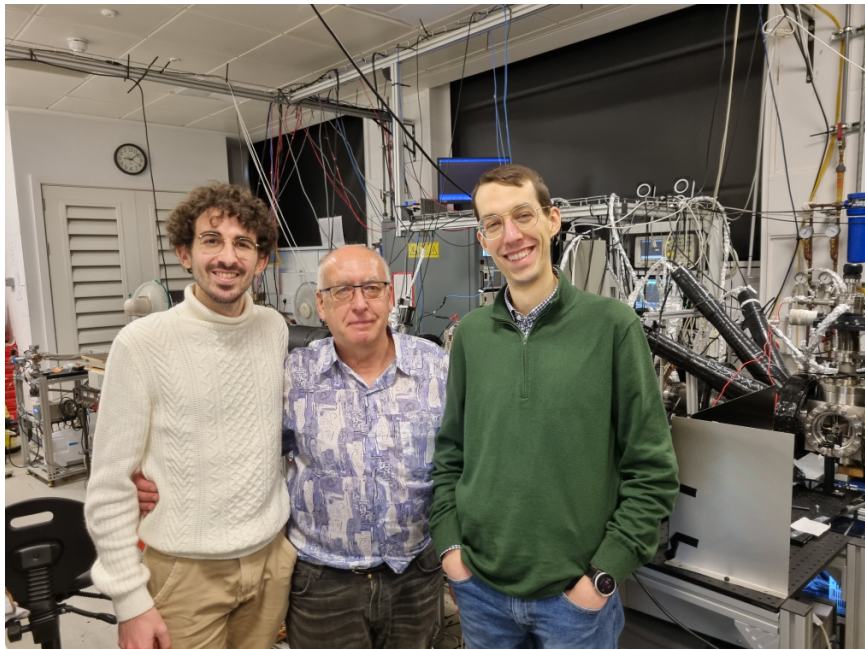
I am fascinated by the positronium atom

<https://en.wikipedia.org/wiki/Positronium>

Here the agreement between theory and experiment is 10^{-10} .
Triumph of spectral theory.

Problem with positronium: theoretical results are non-rigorous.

Bethe–Salpeter equation.



Ivan Vinogradov - Wikipedia

en.wikipedia.org/wiki/Ivan_Vinogradov

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Current & Upcoming Events

September 16-18, 2024, Space Research Institute (IKI) (новость размещена: 25.06.2024)



Conference "Cosmic Gas Dynamics" in memory of Professor V.B. Baranov dedicated to the 90th anniversary of his birth

October 23-25, 2024



Tenth International Scientific School for Young Scientists "Physical and Mathematical Modeling of Earth and Environment Processes".
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This is the new website of the Institute for Problems in Mechanics of the Russian Academy of Sciences. It was launched on 9th November 2015. The old website is available at <http://www1.ipmnet.ru>.

 [EqWorld – World of mathematical equations](#) is internet-portal devoted to equations and methods of their solution.

 [MechMath. Mechanics](#)

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Krylov State Research Center

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Coordinates: 59°49'30"N 30°21'38"E

The **Krylov State Research Center** (**Russian:** **Крыловский государственный научный центр**) is a Russian shipbuilding research and development institute, which operates as a federal state-owned **unitary enterprise**.

The institute is named after **Aleksey Krylov**, the Russian naval designer and mathematician who was one of its first superintendents, and is based in **Saint Petersburg**.^[1]

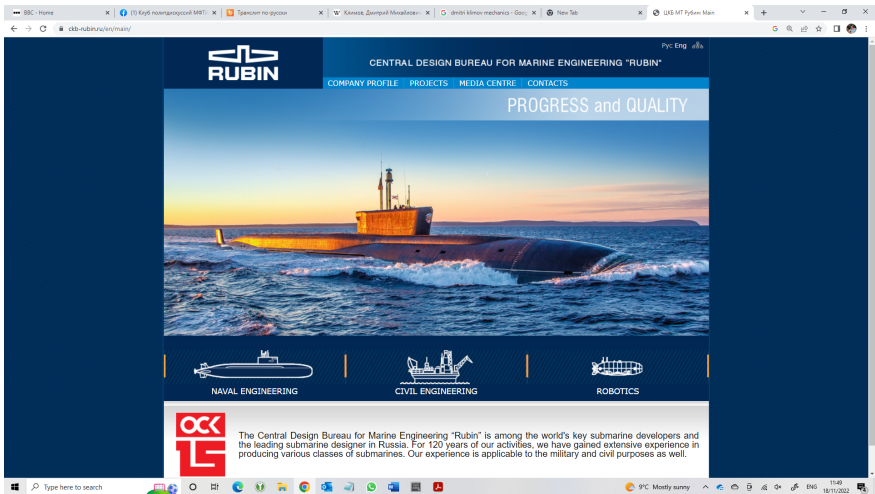
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Krylov State Research Center



Native name	Крыловский государственный научный центр
Industry	Shipbuilding
Founded	March 8, 1894; 128 years ago
Headquarters	Saint Petersburg, Russia
Key people	Andrei Dutov



Russian state-controlled TV 'accidentally' broadcasts secret plans for nuclear torpedo system

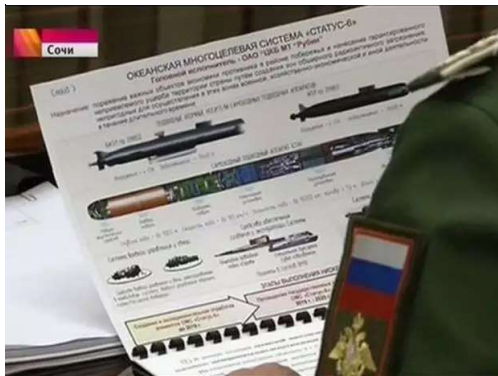
Vladimir Putin had announced the development of a system to defeat Nato defences

Lizzie Dearden @lizziedearden Thursday 12 November 2015 09:18 GMT 11 comments



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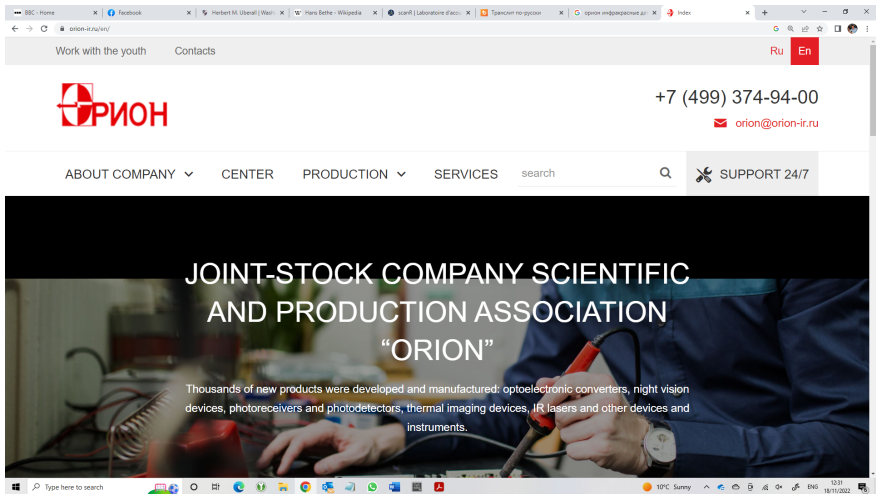
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World Scientific



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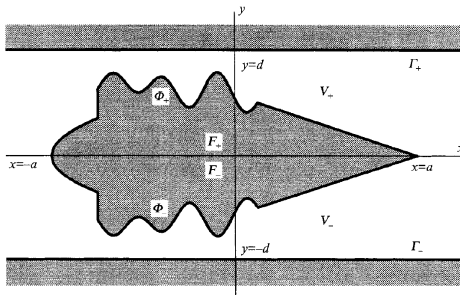
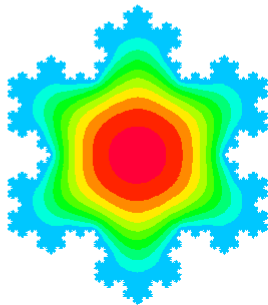
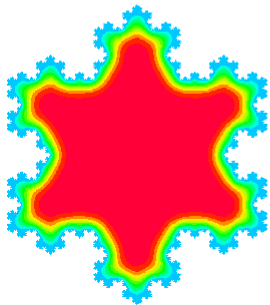
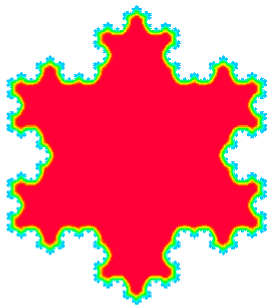
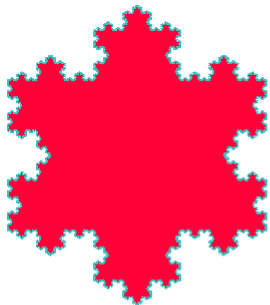


FIGURE 1. Waveguide with obstruction ($V(s) \not\equiv 0$).

not go through the same point twice). For technical reasons we also assume that



Spectral asymmetry.

Single elementary particle living in closed 3-dimensional universe.

Particle is described as a wave governed by the Dirac equation.

Positive natural frequencies are interpreted as energy states of a particle, whereas negative natural frequencies are interpreted as energy states of a antiparticle.

Contributors to this subject area: Sir Michael Atiyah, Isadore Singer, Nigel Hitchin, Peter Gilkey, Lev Pontryagin, Friedrich Hirzebruch, Shiing-Shen Chern, Jim Simons, Bob Seeley.