Spectral theory of differential operators: what's it all about and what is its use
Part III

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Spectral theory for systems

There are two main types of systems.

Type 1 Self-adjoint and semi-bounded system of m PDEs, each of even order $2n$, on a compact d-dimensional manifold M with boundary ∂M. Requires mn boundary conditions.

Type 2 Self-adjoint system of m PDEs, each of odd order $2n-1$, on a compact d-dimensional manifold M without boundary. No requirement for system to be semi-bounded. In fact, the most interesting and physically meaningful examples are when the system is not semi-bounded. Think particle/antiparticle.

For type 1: second Weyl coefficient comes from the boundary ∂M.

For type 2: second Weyl coefficient comes from M itself.

For both types of systems the principal symbol is an $m \times m$ Hermitian matrix-function on $T^*M \setminus \{0\}$.

I will always assume that the eigenvalues of the principal symbol have constant multiplicity.

Moreover, for type 2 systems I will assume that the eigenvalues of the principal symbol are simple.