A non-geometric representation of the Dirac equation in curved spacetime

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The talk is an attempt at developing a relativistic field theory based on the concepts from the analysis of partial differential equations as opposed to geometric concepts. The long-term goal is to recast quantum electrodynamics in curved spacetime in such 'non-geometric' terms. The potential advantage of formulating a field theory in 'analytic' terms is that there might be a chance of describing the interaction of different physical fields in a more consistent, and, hopefully, non-perturbative manner.

Consider a formally self-adjoint first order linear differential operator acting on pairs (two-columns) of complex-valued scalar fields over a four-manifold without boundary. We examine the geometric content of such an operator and show that it implicitly contains a Lorentzian metric, Pauli matrices, connection coefficients for spinor fields and an electromagnetic covector potential. This observation allows us to give a simple representation of the massive Dirac equation as a system of four scalar equations involving an arbitrary two-by-two matrix operator as above and its adjugate. The point of the talk is that in order to write down the Dirac equation in the physically meaningful four-dimensional hyperbolic setting one does not need any geometric constructs. All the geometry required is contained in a single analytic object — an abstract formally self-adjoint first order linear differential operator acting on pairs of complex-valued scalar fields.

The talk is based on the paper [1].

References

[1] Y.-L. Fang and D. Vassiliev, Analysis as a source of geometry: a non-geometric representation of the Dirac equation. *J. Phys. A: Math. Theor.* **48** (2015), 165203. This is an Open Access paper and it can be downloaded at `http://iopscience.iop.org/1751-8121/48/16/165203/`.

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