Modelling the neutrino in terms of Cosserat elasticity

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Subject of talk

Weyl equation (massless Dirac equation):

$$\sigma^{\alpha}{}_{\dot{a}b}\nabla_{\alpha}\xi^{b} = 0.$$

Here

- σ^{α} , $\alpha = 0, 1, 2, 3$, are Pauli matrices,
- ∇_{α} are covariant derivatives with respect to local coordinates $x^{\alpha}, \; \alpha=0,1,2,3,$ and
- ξ^b , b = 1, 2, is the unknown spinor field.

Will construct an alternative model without spinors, Pauli matrices or covariant derivatives.

Describing a 3D deformable medium

(a) Classical elasticity: displacements only.

(b) Cosserat elasticity: displacements and rotations. See

E. Cosserat and F. Cosserat, *Théorie des Corps Déformables*, 1909. Available from Amazon.

I will assume that there are no displacements, only rotations.

To describe rotations of material points mathematically I attach to each geometric point a *coframe*.

A coframe ϑ in 3D is a triplet ϑ^j , j = 1, 2, 3, of orthonormal covector fields. Each covector field ϑ^j has a hidden tensor index: $\vartheta^j = \vartheta^j_{\alpha}$, $\alpha = 1, 2, 3$.

Same in plain English: a coframe is a field of orthonormal bases.

NB. Coframe lives separately from local coordinates (not aligned with coordinate lines).

The coframe ϑ is an unknown quantity (dynamical variable).

The other dynamical variable is a density ρ .

Choose potential energy from the condition of conformal invariance. Explicit formula:

$$P(x^{0}) = \int ||T^{ax}||^{2} \rho \, dx^{1} dx^{2} dx^{3},$$
$$T^{ax} = \frac{1}{3} (\vartheta^{1} \wedge d\vartheta^{1} + \vartheta^{2} \wedge d\vartheta^{2} + \vartheta^{3} \wedge d\vartheta^{3}).$$

Standard kinetic energy

$$K(x^{0}) = \int \|\dot{\vartheta}\|^{2} \rho \, dx^{1} dx^{2} dx^{3},$$
$$\dot{\vartheta} = \frac{1}{3} (\vartheta^{1} \wedge \partial_{0} \vartheta^{1} + \vartheta^{2} \wedge \partial_{0} \vartheta^{2} + \vartheta^{3} \wedge \partial_{0} \vartheta^{3}).$$

My action (variational functional)

$$S = \int (P(x^0) - K(x^0)) dx^0.$$

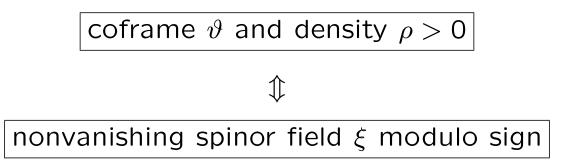
Difference with existing models (teleparallelism)

- 1. I assume metric to be fixed, i.e. I do not vary metric.
- 2. My Lagrangian has never been considered.

Solving Euler–Lagrange equations

Vary coframe and density to get Euler–Lagrange equations. Too complicated!

Switch to spinors:



My Lagrangian density $L(\xi)$ is a rational function of ξ , $\overline{\xi}$ and partial derivatives of ξ , $\overline{\xi}$.

Stationary solutions

Look first for stationary solutions

$$\xi(x^0, x^1, x^2, x^3) = e^{-i\varepsilon x^0} \eta(x^1, x^2, x^3), \qquad \varepsilon \neq 0.$$

Theorem 1 In the stationary case my Euler–Lagrange equation is equivalent to a pair of Weyl equations.

Proof Turns out my Lagrangian factorises as

$$L(\eta) = \frac{L_{\text{Weyl}}^{+}(\eta)L_{\text{Weyl}}^{-}(\eta)}{L_{\text{Weyl}}^{+}(\eta) - L_{\text{Weyl}}^{-}(\eta)}$$

Result follows from factorisation. \Box

See http://arxiv.org/abs/0902.1268.

Question 1. Can I handle the non-stationary case, i.e. when time dependence is arbitrary? Yes, by means of perturbation theory. I look at perturbations of plane wave solutions. Again I get a pair of Weyl equations.

Question 2. Can I make my model relativistically invariant? Yes, by viewing (1 + 3)-dimensional spacetime as a Cosserat continuum. At perturbative level there is no difference between the nonrelativistic and relativistic models.

Question 3. Can I incorporate mass and external electromagnetic field into my model? Yes, by means of a Kaluza–Klein extension, i.e. by adding an extra coordinate x^4 . See http://arxiv.org/abs/0812.3948.

Theorem 2 In the special case with no dependence on x^3 (i.e. for electron in dimension 1+2) the massive version of my model is equivalent to the massive Dirac equation.

Summary

New mathematical model for fermions.

- Spacetime viewed as Cosserat continuum.
- Lagrangian chosen from condition of conformal invariance.
- Mass and electromagnetic field incorporated via Kaluza–Klein extension.