# Modelling the neutrino in terms of Cosserat elasticity

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## Subject of talk

Weyl equation (massless Dirac equation):

$$\sigma^{\alpha}{}_{\dot{a}b}\nabla_{\alpha}\xi^{b} = 0.$$

Here

- $\sigma^{\alpha}$, $\alpha = 0, 1, 2, 3$, are Pauli matrices,
- $\nabla_{\alpha}$ are covariant derivatives with respect to local coordinates $x^{\alpha}$, $\alpha = 0, 1, 2, 3$, and
- $\xi^{b}$, $b = 1, 2$, is the unknown spinor field.

Will construct an alternative model without spinors, Pauli matrices or covariant derivatives.

## Describing a 3D deformable medium

(a) Classical elasticity: displacements only.

(b) Cosserat elasticity: displacements and rotations. See

E. Cosserat and F. Cosserat, *Théorie des Corps Déformables*, 1909. Available from Amazon.

I will assume that there are no displacements, only rotations.

To describe rotations of material points mathematically I attach to each geometric point a *coframe*.

A coframe $\vartheta$ in 3D is a triplet $\vartheta^j$, $j = 1, 2, 3$, of orthonormal covector fields. Each covector field $\vartheta^j$ has a hidden tensor index: $\vartheta^j = \vartheta^j_\alpha$, $\alpha = 1, 2, 3$.

Same in plain English: a coframe is a field of orthonormal bases.

**NB.** Coframe lives separately from local coordinates (not aligned with coordinate lines).

The coframe $\vartheta$ is an unknown quantity (dynamical variable).

The other dynamical variable is a density $\rho$.

Choose potential energy from the condition of conformal invariance. Explicit formula:

$$P(x^0)=\int \|T^{\mathrm{ax}}\|^2\rho\,dx^1dx^2dx^3,$$

$$T^{\mathrm{ax}}=\frac{1}{3}(\vartheta^1\wedge d\vartheta^1+\vartheta^2\wedge d\vartheta^2+\vartheta^3\wedge d\vartheta^3).$$

Standard kinetic energy

$$K(x^0)=\int \|\dot\vartheta\|^2\rho\,dx^1dx^2dx^3,$$

$$\dot\vartheta=\frac{1}{3}(\vartheta^1\wedge\partial_0\vartheta^1+\vartheta^2\wedge\partial_0\vartheta^2+\vartheta^3\wedge\partial_0\vartheta^3).$$

My action (variational functional)

$$S=\int(P(x^0)-K(x^0))dx^0.$$

## Difference with existing models (teleparallelism)

1. I assume metric to be fixed, i.e. I do not vary metric.
2. My Lagrangian has never been considered.

## Solving Euler–Lagrange equations

Vary coframe and density to get Euler–Lagrange equations. Too complicated!

Switch to spinors:

$$\boxed{\text{coframe } \vartheta \text{ and density } \rho > 0}$$

$$\Updownarrow$$

$$\boxed{\text{nonvanishing spinor field } \xi \text{ modulo sign}}$$

My Lagrangian density $L(\xi)$ is a rational function of $\xi$, $\bar{\xi}$ and partial derivatives of $\xi$, $\bar{\xi}$.

## Stationary solutions

Look first for stationary solutions

$$\xi(x^0, x^1, x^2, x^3) = e^{-i\varepsilon x^0}\eta(x^1, x^2, x^3), \qquad \varepsilon \neq 0.$$

**Theorem 1** *In the stationary case my Euler–Lagrange equation is equivalent to a pair of Weyl equations.*

**Proof** Turns out my Lagrangian factorises as

$$L(\eta) = \frac{L^{+}_{\mathrm{Weyl}}(\eta) L^{-}_{\mathrm{Weyl}}(\eta)}{L^{+}_{\mathrm{Weyl}}(\eta) - L^{-}_{\mathrm{Weyl}}(\eta)}\,.$$

Result follows from factorisation. □

See `http://arxiv.org/abs/0902.1268`.

*Question 1.* Can I handle the non-stationary case, i.e. when time dependence is arbitrary? Yes, by means of perturbation theory. I look at perturbations of plane wave solutions. Again I get a pair of Weyl equations.

*Question 2.* Can I make my model relativistically invariant? Yes, by viewing $(1+3)$-dimensional spacetime as a Cosserat continuum. At perturbative level there is no difference between the nonrelativistic and relativistic models.

*Question 3.* Can I incorporate mass and external electromagnetic field into my model? Yes, by means of a Kaluza–Klein extension, i.e. by adding an extra coordinate $x^4$. See `http://arxiv.org/abs/0812.3948`.

**Theorem 2** *In the special case with no dependence on $x^3$ (i.e. for electron in dimension $1+2$) the massive version of my model is equivalent to the massive Dirac equation.*

## Summary

New mathematical model for fermions.

- Spacetime viewed as Cosserat continuum.

- Lagrangian chosen from condition of conformal invariance.

- Mass and electromagnetic field incorporated via Kaluza–Klein extension.