

Dirac equation as a special case of Cosserat elasticity

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The basic assumption in the classical theory of elasticity is that the deformation of a continuum is described by a vector function, the field of displacements, which is the unknown quantity in the system of equations. There is, however, a more general model of an elastic continuum, called *Cosserat elasticity*, in which points of the continuum are given the freedom to rotate: each point can rotate independently and these “microrotations” are not related to the “macrorotations” induced by the displacements. The Cosserat theory of elasticity has been in existence for almost a century [1] and appears under various names in modern applied mathematics literature such as *oriented medium*, *asymmetric elasticity*, *micropolar elasticity*, *micromorphic elasticity*, *moment elasticity* etc. It is also closely related to the theories of ferromagnetic materials and liquid crystals.

In our talk we investigate a special case of Cosserat elasticity when there are no displacements, only rotations. This special case is known as *teleparallelism* (Einstein’s terminology) or *absolute parallelism* (Cartan’s terminology), see [2].

We choose a particular conformally invariant Lagrangian and derive the Dirac equation from Cosserat elasticity. The talk is a development of [3].

The mathematical model presented in the talk is similar to that suggested by Einstein [4]. Einstein wrote down a general quadratic teleparallel Lagrangian and attempted to analyse the resulting Euler–Lagrange equations. However he and subsequent authors neglected to examine the particular case of the Lagrangian “axial torsion squared” which we use. Another difference is that in teleparallelism it is traditional to view the metric as a dynamical variable whereas in our model the metric is fixed, up to a conformal transformation.

References

- [1] E. Cosserat and F. Cosserat, *Théorie des Corps Déformables*, A. Hermann et Fils, Paris, 1909.
- [2] *Elie Cartan and Albert Einstein: Letters on Absolute Parallelism*, Princeton University Press, 1979.
- [3] D. Vassiliev, *Phys. Rev.* **D75**, 025006 (2007).
- [4] A. Unzicker and T. Case, *Translation of Einstein’s Attempt of a Unified Field Theory with Teleparallelism*, <http://arxiv.org/abs/physics/0503046>.