TELEPARALLEL REFORMULATION OF THE WEYL EQUATION

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The talk deals with the Weyl equation (massless Dirac equation).

We work on a 4-manifold equipped with prescribed Lorentzian metric g. As the unknown quantity we use the coframe — quartet of real covector fields ϑ^j, $j = 0, 1, 2, 3$, satisfying the constraint

$$g = o_{jk}\, \vartheta^j \otimes \vartheta^k \tag{1}$$

where $o_{jk} = o^{jk} := \mathrm{diag}(1, -1, -1, -1)$.

In the first part of the talk we give a teleparallel reformulation of the Weyl equation. We introduce the Lagrangian

$$L = l \wedge T^{\mathrm{axial}} \tag{2}$$

where $T^{\mathrm{axial}} := \frac{1}{3}\, o_{jk}\, \vartheta^j \wedge d\vartheta^k$ is the axial (totally antisymmetric) piece of torsion of the teleparallel connection and $l := l_j \vartheta^j$ is a lightlike teleparallel covector field; here the l_j are real constants, not all zero, satisfying $o^{jk} l_j l_k = 0$. We prove [1] that the Lagrangian (2) is, up to a change of variable, the Weyl Lagrangian. In particular, variation of the resulting action with respect to the coframe ϑ^j subject to the metric constraint (1) produces a field equation which is, up to a change of variable, the Weyl equation.

In the second part of the talk we introduce the Lagrangian

$$L = \|T^{\mathrm{axial}}\|^2 * 1 \tag{3}$$

and, working in Minkowski space, argue [2] that our original Lagrangian (2) is the formal linearisation of the Lagrangian (3) about a plane wave solution with momentum l. Hence, one expects perturbations of these plane wave solutions to be described by the Weyl equation. We suggest using the Lagrangian (3) as an alternative model for the neutrino.

Lagrangians quadratic in torsion were first proposed by Einstein in his papers [3] on teleparallel gravity. However, he did not identify axial torsion as a separate irreducible piece and, consequently, did not consider the particular Lagrangian (3).

References

[1] D. Vassiliev, *Phys. Rev.* **D75**, 025006 (2007).

[2] D. Vassiliev, gr-qc/0702020.

[3] A. Unzicker and T. Case, Translation of Einstein's ..., physics/0503046.

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