Mathematical foundations for Origins

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My track record in theoretical physics

General Relativity: space and time are inseparable, and should be viewed as *spacetime* — a 4-dimensional manifold (curved surface of dimension 4).

Spacetime is mathematically described using *differential geometry*.

Two basic quantities: *curvature* and *torsion*.

Curvature shows how much spacetime is bent.

Torsion shows how much spacetime is twisted.

Standard assumptions:

$$\text{curvature} \neq 0, \quad \text{torsion} = 0.$$

Alternative (metric-affine) theories of gravity:

$$\text{curvature} \neq 0, \quad \text{torsion} \neq 0.$$

Suggested by Einstein himself and developed by É.J.Cartan, Eddington, Higgs, Levi-Civita, Schrödinger, H.Weyl etc. Spacetime is more flexible, has more degrees of freedom and this gives hope for a unified field theory.

My works in alternative theories of gravity: found a solution which could be interpreted as an elementary particle (neutrino?).

Currently working in *teleparallelism*:

$$\text{curvature} = 0, \quad \text{torsion} \neq 0.$$

Opposite of standard General Relativity!

Simpler mathematical description of neutrino* and, possibly, electron. Hope of rewriting all Quantum Electrodynamics in teleparallel form.

* D. Vassiliev, Phys. Rev. **D75**, 025006 (2007).

Two big mathematical challenges

- Making sense of elementary particles.

- Making sense of cosmology.

Making sense of elementary particles

Quantum Electrodynamics makes sense of the most basic elementary particles (photons, electrons, neutrinos and corresponding antiparticles) whereas the *Standard Model* makes sense of the others.

Quantum Electrodynamics has a beautiful mathematical structure but suffers from the following defects.

- It gives a formal asymptotic expansion with respect to a small parameter (usually this parameter is the fine structure constant $\alpha \approx \frac{1}{137}$) but does not identify a partial differential equation (?) containing a small parameter which generates the asymptotic expansion in question. For a mathematician this is most unusual.

- It doesn't actually explain what photons, electrons, neutrinos really are. Waves, particles, or some sort of solitons? Etc.

- At a more basic level, it doesn't even explain what electromagnetism is.

Problems with the Standard Model

- Very complicated and lacks the beauty of Quantum Electrodynamics. To a mathematician the Standard Model appears to be an empirical construction in which agreement with experiment is achieved by the appropriate choice of sufficiently many physical parameters (29) and sufficiently many groups. There does not appear to be a coherent underlying mathematical structure.

- The Large Hadron Collider will in 2008 reach an energy regime above the electroweak symmetry-breaking scale. This means the Standard Model can no longer be treated as a low energy effective theory without the Higgs mechanism, and thus its mathematical form will be challenged in a way unprecedented in the last two decades.

- Does not incorporate gravity.

Possible ways of making sense of elementary particles

- String theory?
- Kaluza–Klein type theory?
- Noncommutative geometry?
- Supersymmetry?
- Loop quantum gravity?
- Metric-affine theory?
- Teleparallelism?

Personal belief: the way forward is to go back to basics and try understanding electromagnetism, neutrinos and electrons.

Making sense of cosmology

Solid mathematical basis: Einstein's equation. Works well on the scale of the solar system.

However, problems occur at large spatial scales and at late times. *Accelerated* expansion of the universe has to be explained. Does Einstein's equation work at large scales? Does Newton's law of gravitation work at large scales?

Possible ways of making sense of cosmology

- Dark energy. Mathematically it means introduction of the cosmological constant (i.e. the gravitational action now has a term proportional to the volume of spacetime).

- Dark matter.

- Modified Newtonian dynamics (MOND). Involves modification of Newton's Second Law.

- Modifications of gravity (for example, $f(R)$, Gauss–Bonnet, Chern–Simons, etc).

- Metric-affine models.

- Brane-world cosmology (extra dimensions).

Common ground between the two subjects, particle physics and cosmology

Neutrinos, for some reason, play a fundamental role in both subjects and provide a natural link. Experiments on solar neutrinos (neutrino oscillations) give hope for the measurement of the neutrino mass.

Neutrinos also interest mathematicians:

D. Vassiliev, *Teleparallel model for the neutrino*, Phys. Rev. **D75**, 025006 (2007).