Schedule for the Workshop on Geometric Measure Theory and Applications, UCL 25-27 July 2022

	Monday 25	Tuesday 26	Wednesday 27	
10:00-10:15	Registration		Z. Zhao	10:00-10:15
10:15				10:15
10:30	Welcome	Y. Tonegawa	*Almgren's regularity of area-minimizing currents and multi-valued functions*	10:30
10:45	coffee/tea			10:45
11:00-11:15			Coffee/tea	11:00-11:15
11:15	P. Minter			11:15
11:30	*Excess decay and the blow-up method*	Coffee/tea		11:30
11:45			Z. Zhao	11:45
12:00-12:15				12:00-12:15
12:15		P. Minter		12:15
12:30				12:30
12:45				12:45
13:00-13:15	Lunch		Lunch	13:00-13:15
13:15		Lunch		13:15
13:30				13:30
13:45				13:45
14:00-14:15				14:00-14:15
14:15	P. Minter		Y. Tonegawa	14:15
14:30		P. Minter		14:30
14:45				14:45
15:00-15:15	Coffee/tea	Coffee/tea	Coffee/tea	15:00-15:15
15:15				15:15
15:30				15:30
15:45	Y. Tonegawa		Y. Tonegawa	15:45
16:00-16:15		N. Wickramasekera		16:00-16:15
16:15	*Introduction to the Brakke flow and its existence theory*			16:15
16:30		*Allen--Cahn equation and The existence of prescribed-mean-curvature hypersurfaces*		16:30
16:45				16:45
17:00-17:15				17:00-17:15
17:15				17:15
17:30				17:30
17:45				17:45
18:00-18:15	Reception/			18:00-18:15
18:15	Refreshments			18:15
18:30				18:30
18:45				18:45
19:00-19:15				19:00-19:15
19:15				19:15

Talks and mini courses: UCL Department of Mathematics, 25 Gordon st, 5th floor, room 505. See UCL map: 25 Gordon Street Maths 505 LT | UCL Maps

Registration, coffee/tea breaks: UCL Department of Mathematics, 25 Gordon st, 5th floor, room 502.

Lunches, reception: UCL Japanese Garden. See UCL map: Japanese Garden Pavilion | UCL Maps

Titles and abstracts

Paul Minter: *Excess decay and the blow-up method*
Abstract: The aim of this course is to provide an introduction to some of the key techniques within the regularity theory of geometric measure theory, with a particular focus on excess decay and blow-up methods. We will start by proving Allard's regularity theorem, first in the case of Lipschitz graphs and then the general case of stationary integral varifolds; this will be done by establishing an excess decay lemma based on the fact that coarse blow-ups of stationary integral varifolds relative to a multiplicity one plane are harmonic functions. Then, we will discuss adaptations of this method to some of the following settings (time permitting and depending on the inclination of the participants): (i) the regularity theory of stable codimension one integral varifolds by N. Wickramasekera (in particular the role of the Hardt—Simon inequality and fine blow-ups); (ii) L. Simon's multiplicity one triple junction regularity theorem (in particular the role of density gaps); (iii) the study of branch points in stable codimension one stationary integral varifolds with certain classical singularities by Minter—Wickramasekera (in particular the role of fine varifold regularity theorems and frequency monotonicity in classifying homogeneous degree one blow-ups).

Yoshihiro Tonegawa: *Introduction to the Brakke flow and its existence theory*
Abstract: I plan to give an introductory mini course on the mean curvature flow in the setting of geometric measure theory called the Brakke flow, starting from the definition and a brief description of the basic properties. Then I focus on the existence theory based on my works with Lami Kim and Salvatore Stuvard. Some familiarity with Radon measures and rectifiable sets (for example, Part 1 of Maggi's book, Sets of Finite Perimeter and Geometric Variational Problems) will be helpful, but not essential; statements of basic background results needed will be provided.

Neshan Wickramasekera: *Allen--Cahn equation and the existence of prescribed-mean-curvature hypersurfaces*
Abstract: The lecture will be based on the speaker's recent joint work with Costante Bellettini, and will showcase the role of recent advances in Geometric Measure Theory in the resolution of a classical problem in Differential Geometry. We will start with a discussion of an abstract varifold regularity theorem, which will build on parts of the material covered in Minter's lectures earlier in the workshop. This theorem provides $C^{1, \alpha}$ regularity for a class of codimension 1 varifolds admitting generalised mean curvature in the presence of integer multiplicities possibly > 1. An important aspect of this theorem is that it is non-variational in the sense that the varifolds are not assumed to be critical points of a functional. This will be followed by a brief discussion of the limiting behaviour of solutions to a class of singularly perturbed Allen--Cahn equations with right hand side a given smooth positive function, including the Roger--Tonegawa theorem on limit varifolds and the Schoen--Tonegawa inequality for stable limit varifolds (i.e. limit varifolds arising from energy bounded stable solutions to the Allen--Cahn equation). We will then discuss how to apply the above $C^{1, \alpha}$ varifold regularity theorem, capitalising on its non-variational character, to obtain a $C^{2, \alpha}$ estimate for sufficiently flat parts of Allen--Cahn stable limit varifolds. In the last part of the lecture, we will outline how to combine this estimate with a simple PDE min-max argument and certain basic principles of semi-linear parabolic PDE theory to obtain existence of prescribed-mean-curvature hypersurfaces in a compact Riemannian manifold. Standard results from second order elliptic and parabolic PDE theory and a standard PDE mountain pass theorem will be assumed (and stated without proof), but it is not essential that the attendees have prior knowledge of these results or any GMT background beyond what is covered in Minter's lectures.

Zihui Zhao: *Almgren's regularity of area-minimizing currents and multi-valued functions*
Abstract: In Almgren's ground-breaking work in the 1980s, he proved that for an area-minimizing integral currents (of dimension n) in higher codimensions, its singular set is at most (n-2)-dimensional. The major difficulty and difference from the case of codimension one is branched singularity. I will explain a simplified model near a branched singular point, which are multi-valued functions that minimize the Dirichlet energy, and introduce tools to study its regularity.