

System-size coherence resonance in coupled FHN neurons with α -stable Lévy noise

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ARTICLE INFO

Keywords:

α -stable Lévy noise
System-size coherence resonance
FitzHugh–Nagumo neuron
Small-world network

ABSTRACT

In this paper, we investigate the system-size coherence resonance of a network of identical FitzHugh–Nagumo neurons driven by α -stable Lévy noise and coupled through small-world network. System-size coherence resonance is a resonance-like phenomenon that in ensembles of noise-driven dynamical systems a maximum order (the coefficient of variation of interspike intervals has a maximum) can appear at a certain system size, while the noise intensity and coupling strength remain constant. It is easy to adjust noise to achieve the maximal system order in experiments but not obvious how this adjustment takes place in nature. The study of system-size coherence resonance provides a method to improve the system order through changing the number of elements. We focus on the effects of the parameters of the small-world network (the rewiring probability and coupling range) and α -stable Lévy noise (the stability parameter and noise amplitude) on the system-size coherence resonance. Using numerical simulations, we find that the noise amplitude and the rewiring probability determine the emergence of system-size coherence resonance, and the stability parameter and the coupling range just affect the value of the coefficient of variation.

1. Introduction

Coherence resonance has attracted significant attention in nonlinear biological systems exposed to noise, as it represents a fundamental dynamical phenomenon. The concept was popularized by Hu et al. who found that in a noise-driven excitable system without external inputs, the system output becomes more ordered when an appropriate level of noise is applied. This phenomenon is known as “coherence resonance” or “autonomous stochastic resonance” [1–4]. In other words, the order of a noise-driven system can reach a maximum at a particular noise level, even in the absence of periodic driving forces.

With the development of complex networks, coherence resonance has been observed in diverse coupled network structures, including multiplex networks, dual neural populations in ring topologies, and stochastic SIR epidemic models [5–9]. Research increasingly emphasizes coupled nonlinear systems, likely due to the amplification effects that coupling can introduce. Wu et al. studied the temporal coherence of spiking behavior and time-delayed coherence resonance in coupled networks [10]. Masoliver investigated coherence resonance in time-lagged coupled networks composed of FitzHugh–Nagumo elements in excitable states [11].

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<https://doi.org/10.1016/j.chaos.2025.116985>

Received 13 May 2025; Received in revised form 30 July 2025; Accepted 31 July 2025

Available online 14 August 2025

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Wang et al. explored noise-induced chimera states in Hindmarsh-Rose (HR) neuronal networks, where chimera states reflect the coexistence of coherent and incoherent dynamics [12,13]. Pikovsky and Zaikin proposed a resonance-like phenomenon, namely, the **system-size coherence resonance**, that in ensembles of noise-driven dynamical systems a maximum order can appear at a certain system size, while the noise intensity and coupling strength remain constant [14].

System-size coherence resonance focuses on the influence of system size on system order, in contrast to traditional coherence resonance that emphasizes noise level [15,16]. Gong introduced the concept of non-Gaussian noise and system-size-induced coherence resonance in calcium oscillations within an array of interconnected cells [17]. Coherence resonance has been shown to enhance the detection of communication signals, increase the sensitivity of weak neuronal systems, and aid in recognizing subtle visual cues [18–20]. Pisarchik and Hramov demonstrated through numerous experimental and theoretical studies that coherence resonance in the brain's response to cognitive tasks not only facilitates neural activity in specific brain regions but also supports communication between distant areas of the brain [21]. One can easily adjust noise to achieve the maximal sensitivity to an external signal, it may not be obvious how this adjustment takes place in nature. However, the study of system-size resonance provide a new method to increase the sensitivity through changing the number of elements.

Most of the existing studies on system-size coherence resonance are based on Gaussian noise. However, this does not mean that the noise in neuronal systems is always a Gaussian one. In various cases, the system encounters sudden and large external disturbances, such that Gaussian noise is no longer applicable to model this type of perturbations. Therefore, we would like to employ α -stable Lévy noise [22–25], a class of non-Gaussian noise characterized by heavy-tailed distributions. This type of noise captures large fluctuations more realistically and has been widely applied across various disciplines. Rybalova studied coherence resonance in map-based neural models under Lévy noise and analyzed how different noise parameters influence oscillatory behavior [26]. Xu investigated the averaging principle for slow-fast stochastic Burgers equations driven by Lévy processes, and later introduced path-integral solutions for general n-dimensional stochastic difference equations with α -stable Lévy noise [27,28]. Korneev investigated the influence of Lévy noise characteristics on coherence resonance using both numerical simulations and experiments [29].

In this paper, based on coupled FHN neurons, we primarily investigate the role of system size in coherence resonance and explore the effects of α -stable Lévy noise and network topology on system-size coherence resonance. The remainder of this paper is organized as follows: Section 2 introduces the coupled FHN neurons with small-world network and the α -stable Lévy noise. In Section 3, we examine the influences of the parameters of α -stable Lévy noise (noise amplitude and stability parameter) and the small-world network (rewiring probability and coupling range) on system-size coherence resonance. Finally, Section 4 presents our main findings and conclusions.

2. Model description

We consider here N coupled α -stable noise-induced FHN neurons in a small-world network. The dynamical system can be described by the following equations:

$$\begin{aligned} \varepsilon \dot{x}_i &= x_i - \frac{x_i^3}{3} - y_i + \frac{\sigma}{2R} \sum_{j=1}^N A_{ij}(x_j - x_i), \\ \dot{y}_i &= x_i + a_i + \xi_i(t). \end{aligned} \quad (1)$$

In the neural context, x_i represents the membrane potential of the i -th neuron, y_i is related to the time-dependence conductance of the potassium channels in the membrane, a_i is the system parameters, $i = 1, \dots, N$, and all indices are modulo N . The dynamics of x_i is much faster than that of y_i , as indicated by the small time-scale-ratio parameter ε . $\sigma > 0$ is the coupling strength, and $r = R/N$ is the coupling range. A_{ij} is the adjacency matrix of the network, if the neurons i and j are coupled to each other, $A_{ij} = 1$; otherwise, $A_{ij} = 0$. $\xi_i(t)$, $i = 1, \dots, N$ are independent α -stable random variables with the same distribution, and inserted in the slow-variable equation, as in most studies of coherence resonance [4,30,31]. $\xi_i(t)$ as the formal derivative of Lévy stable motion $L_i(t)$, is able to characterize the appearance of large jumps, which benefits from its distribution described by the characteristic function $\Phi_\alpha\{\beta, \mu, \theta\}(k)$,

$$\Phi_\alpha\{\beta, \mu, \theta\}(k) = \begin{cases} \exp[-\theta^\alpha |k|^\alpha (1 - i\beta \operatorname{sgn}(k) \tan(\pi\alpha/2)) + i\mu k], & \alpha \neq 1, \\ \exp[-\theta |k| (1 - i\beta \operatorname{sgn}(k)(2/\pi) \ln |k|) + i\mu k], & \alpha = 1. \end{cases} \quad (2)$$

where $\alpha (0 < \alpha \leq 2)$ is the stability parameter, determining the thickness of the tail, $\beta (-1 \leq \beta \leq 1)$ is the skewness parameter to measure the symmetry of the distribution, $\mu \in \mathbb{R}_1$ is the mean of the distribution, θ is the scale parameter and $D = \theta^\alpha$ denotes the noise amplitude (sometimes also called noise intensity). The detailed properties of the probability density function (PDF) of α -stable Lévy noise are given in [24,30]. With α decreasing, the tail of PDF is heavier and heavier. When $\alpha > 1$, the PDF is left-skewed for $\beta > 0$, right-skewed for $\beta < 0$ and symmetric for $\beta = 0$.

In this paper, we consider symmetric α -stable Lévy noise ($\beta = 0, \mu = 0$), and its characteristic function $\Phi_\alpha\{0, 0, \theta\}(k)$ has the following representation:

$$\Phi_\alpha\{0, 0, \theta\}(k) = \begin{cases} \exp(-\theta^\alpha |k|^\alpha), & \alpha \neq 1, \\ \exp(-\theta |k|), & \alpha = 1. \end{cases} \quad (3)$$

When $\alpha = 2$, $\exp(-\theta^2 k^2)$ is the Gaussian characteristic function; when $\alpha = 1$, $\exp(-\theta |k|)$ is the Cauchy characteristic function. Moreover, we consider $\alpha \in [1, 2]$ because the moments of order $\vartheta > \alpha$ diverge. The Chambers-Mallows-Stuck's algorithm [32] is

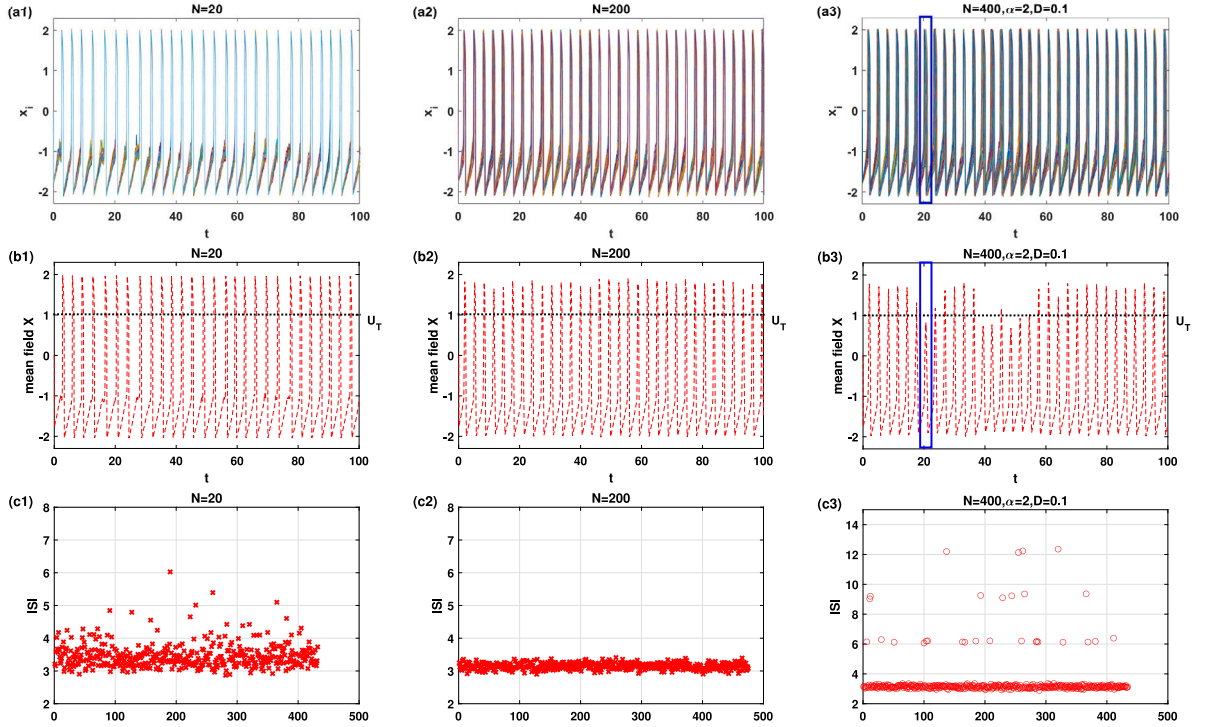


Fig. 1. (Top) The time series of x_i , (middle) the time dependence of the mean field $X(t)$ and (bottom) the series of interspike intervals of the mean field in the system (1) for $\alpha = 2$, $D = 0.1$, $p = 0$, $R = 4$, and different system sizes N . The left column: $N = 20$, the middle column: $N = 200$ and the right column: $N = 400$.

used to generate α -stable Lévy random number η_n , and the generated $|\eta_n|$ is limited to be less than M (if it is larger than M , it is reset to $\tilde{\eta}_n = \text{sgn}(\eta_n)M$) [30,31,33]. This restriction causes the tail of the original distribution to be cut off when $|\eta_n| > M$, reducing the extreme values and making the shape of the original distribution more concentrated. Since this article considers symmetric distributions, the symmetry and mean of the original distribution will not change, but the variance will reduce. The membrane potential of the FHN neuron is in the range $[-2.5, 2.5]$, so we set $M = 30$. This is more than ten times the normal range and is sufficient to describe the large disturbance suffered by the system. And counting the generated random numbers under all parameters in this paper, we find that it accounts for more than 99% for random numbers less than $M = 30$.

The coupling topology is small-world networks ($SW(N, p, 2R)$). N is the total nodes of the network, p is the rewiring probability and $2R$ is the number of nearest neighbors each node is connected to. In practical small-world networks, most nodes are not adjacent, but most of them can reach to each other within a few steps. Therefore, R is usually much smaller than the number of nodes. The rewiring probability p can change the connection structures of the network, while keeping the number of nodes and the number of edges unchanged. The precise procedure for generating such small-world networks is given in [30]. For $p = 0$, the network is a regular network, where each node only connects to its local nearest neighbors. For $p \in (0, 1]$, the network becomes more random with the increase of p . For $p = 1$, the network becomes a random network. It stipulates that there is at most one edge between any two different nodes, and each node should not connect to itself.

In an uncoupled deterministic FHN neuron system, there is only a stable fixed point corresponding to the quiescent state when $|a| > 1$, whereas a limit cycle appears when $|a| < 1$. The neuron generates firing spikes when the membrane voltage exceeds the threshold potential U_T . The m th spike firing time is identified by t_m [30]. If $|a|$ is slightly greater than 1, the system is in an excitable state. The parameters are chosen as $a_i \equiv 1.01$, $i = 1, \dots, N$ and $\varepsilon = 0.01$ so that the FHN model is in a monostable excitable regime in the absence of coupling. In the following, we numerically integrate the differential equations (1) using the Euler scheme with a fixed time step of 0.001, $\sigma = 1$, and mainly consider the effects of noise amplitude D , stability parameter α , the system size N , the coupling range r and the rewiring probability p on the system behaviors.

3. System-size coherence resonance

We introduce the average values of the activator and inhibitor variables as

$$X(t) = \frac{1}{N} \sum_{i=1}^N x_i(t), Y(t) = \frac{1}{N} \sum_{i=1}^N y_i(t). \quad (4)$$

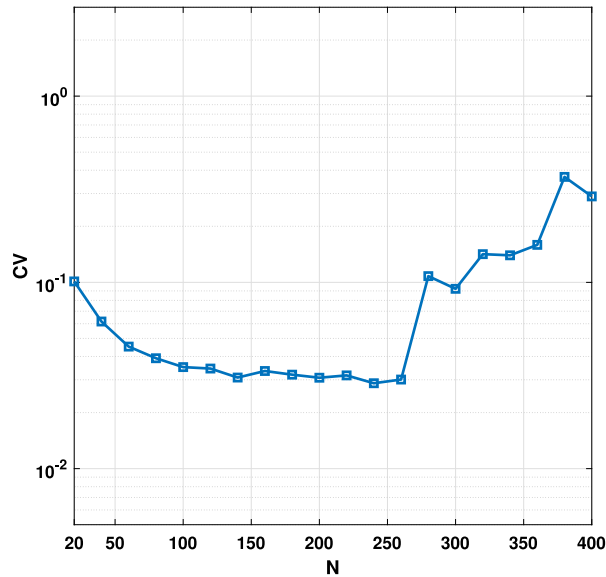


Fig. 2. The dependence of CV on the system size N keeping $\alpha = 2, p = 0, D = 0.1, R = 4$.

Then it is possible to reach an approximate effective equation for these average values of the form

$$\begin{aligned} \varepsilon \dot{X} &= F(X, \sigma, R) - Y, \\ \dot{Y} &= X + a + N^{\frac{1}{\alpha}-1} \xi(t). \end{aligned} \quad (5)$$

where $\xi(t)$ is α -stable random variable with the same distribution of $\xi_i(t)$ in the system (1). Although the exact form of the function $F(X, \sigma, R)$, which depends on the global variable X , the coupling strength σ and the number of nearest neighbors R , and the analysis of the approximations assumed in the derivation will be presented elsewhere, we need only to remark here that in the (exact) equation for $Y(t)$ the scale parameter θ appears rescaled as $N^{\frac{1-\alpha}{\alpha}} \theta$. Therefore, the noise intensity appears rescaled as $N^{1-\alpha} D$. This approximation suggests that the optimal effective noise intensity for the appearance of coherence resonance can be achieved by varying the number of coupled elements N [34].

To go beyond this approximation, we numerically integrate the Eqs. (1). And $X(t)$ presents firing spikes when $X(t)$ overtakes the threshold potential $U_T = 1$. The m th interspike interval of the mean field ISI_m is defined as the time difference between two consecutive spikes

$$ISI_m = t_{m+1} - t_m > 0,$$

where t_m is the time in which the m th firing spike occurs. In order to measure the regularity of firing spikes, we calculate the mean value of interspike intervals ($\overline{ISI}$), its standard deviation (σ_{ISI}), and the coefficient of variation (CV) [4] is given

$$CV = \frac{\sigma_{ISI}}{\overline{ISI}}. \quad (6)$$

A smaller CV corresponds to a smaller dispersive degree at the level of $\overline{ISI}$ and stands for more regular firing spikes and better system order. Therefore, the coefficient of variation measures the closeness of the pulse train to a periodic one (where CV obviously vanishes).

Fig. 1 presents the time series of x_i (top row), the time dependence of the mean field $X(t)$ (middle row) and the series of interspike intervals (bottom row) in the system (1) for $\alpha = 2, D = 0.1, p = 0, R = 4$, and different system sizes N . The left column: $N = 20$, the middle column: $N = 200$ and the right column: $N = 400$. To give us a better visualization, the time series of x_i and the time dependence of the mean field only capture the results during 0–100 unit time. And the series of interspike intervals demonstrate the results during entire computational time (0–1500 unit time). As can be seen in the top row of Fig. 1, the number of firing spikes for the single neuron with $N = 200$ or $N = 400$ is 32, while for $N = 20$ it is 28. This suggests that increasing the number of elements in the network encourages the single neuron to produce more firing spikes. Next, we will discuss the phenomena that arise from changes in the number of elements in the mean field. For $N = 20$, each neuron fired nearly synchronously. Thus, the series of interspike intervals of the mean field $X(t)$ is the same as that of each single neuron x_i . For $N = 200$ and $N = 400$, since the firing times of each neuron deviate slightly, this leads to a decrease in the spiking value of the mean field $X(t)$. This decrease is even greater as N increases. For $N = 200$, all the spiking values of the mean field $X(t)$ overtake $U_T = 1$, the series of interspike intervals of the mean field $X(t)$ is also the same as that of each single neuron x_i . For $N = 400$, some spiking values of the mean field $X(t)$ do not overtake $U_T = 1$, as shown in the blue rectangles. Therefore, the mean field $X(t)$ may present no firing spike even if each x_i

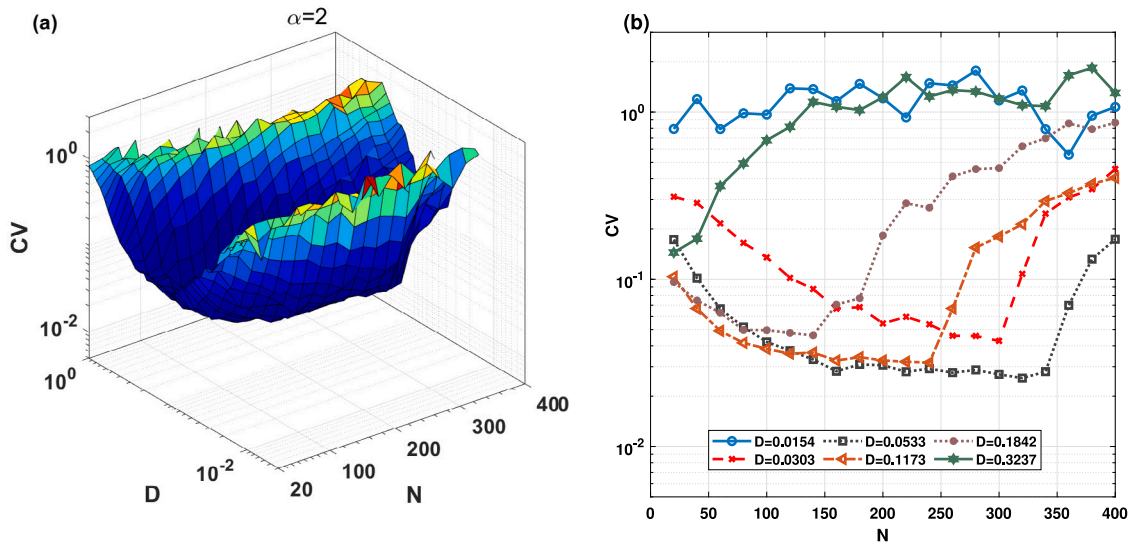


Fig. 3. (a) The dependence of CV on the system size N and D , (b) The dependence of CV on the system size N for different noise amplitudes keeping $\alpha = 2, p = 0, R = 4$.

presents firing spike, and the number of firing spikes for each single neuron is larger than that for mean field $X(t)$. Compared the series of interspike intervals of the mean field of three different system sizes, we can see that the middle system size ($N = 200$) leads to more firing spikes and better order than both small ($N = 20$) and large ($N = 400$) system sizes. It is important to note that the choice of U_T has an effect on the number of firing spikes in mean field. In Fig. 1, on the blue rectangle of mean field oscillations at $N = 400$, a spike is visible slightly below the level $U_T = 1$. So, it is not taken into account. If we set $U_T = 0.95$, it will be taken into account. Thus, the number of firing spikes in mean field depend significantly on the choice of threshold U_T . However, the choice of U_T does not affect the trend of the number of firing spikes with N in mean field. This is because, when N increases to a certain level, the number of firing spikes of individual neurons remains constant. However, an increase in N leads to a decrease in the spike value in the mean field. Therefore, as N increases, the number of firing spikes in the mean field first increases and then decreases.

To further investigate the effect of N on the order of the system, we present the dependence of the CV on the system size N keeping $\alpha = 2, p = 0, D = 0.1, R = 4$ in Fig. 2. There is a nonlinear relationship between CV and N : as N increasing, CV decreases first, then reaches a minimum at $N \approx 240$, and finally rises again. In other words, an optimal response is obtained for an appropriate value of the number N of coupled systems in the presence of a constant amount of noise. This resonance-like phenomenon dependence on the system size is system-size coherence resonance. When the system size N is small, the independent noise of individual units cannot be effectively suppressed by averaging (the law of large numbers has a weak effect). The overall effective noise intensity is high, leading to severe disruption of coherent oscillations and low coherence. When the system size N is too large, although individual noise is significantly suppressed through averaging (the law of large numbers is effective, resulting in low overall effective noise intensity), the coordination cost required to maintain high synchronization among all units in the large network increases sharply. Even minor fluctuations or inherent heterogeneity are more likely to trigger propagating asynchrony or desynchronization, leading to destructive interference and similarly impairing overall coherent oscillations. The optimal system size N_{opt} for system-size coherence resonance precisely occurs at the optimal trade-off point between these two competing effects. At this point, the noise reduction effect from noise averaging (reducing effective noise intensity) is just sufficient to significantly enhance coherence, while the system size has not yet grown large enough to be dominated by destructive asynchronous effects. This shares a similar principle with the “a matching between noise amplitude and system timescale enhances signal transmission” balance concept in classical stochastic resonance or coherence resonance. In the following, we will further discuss the effects of α -stable Lévy noise and network topology on the system-size coherence resonance.

3.1. The effect of α -stable Lévy noise on system-size coherence resonance

First, we explore the effect of D on the system-size coherence resonance. We present the dependence of the CV on the system size N and D keeping $\alpha = 2, p = 0, R = 4$ in Fig. 3(a). It is evident that there is a valley for CV with the change of D , which is consistent with the typical characteristics of coherence resonance. That is to say, there is a nonlinear relationship between CV and D : CV initially decreases, then reaches a minimum, and subsequently rises again as D increases. Moreover, the surface of CV reaches a minimum $min_{CV} = 0.0251$ at optimal system size $N_{opt} \approx 340$ and optimal noise amplitude $D_{opt} \approx 0.0596$. The value of CV increases gradually as it moves away from the minimum. It increases relatively slowly along N and relatively quickly along D . For different noise amplitudes, the curves of CV with the system size N are shown in Fig. 3(b). One can observe that CV decreases initially then increases gradually with the system size N increasing, and there is clearly a minimum value for the dependence on the

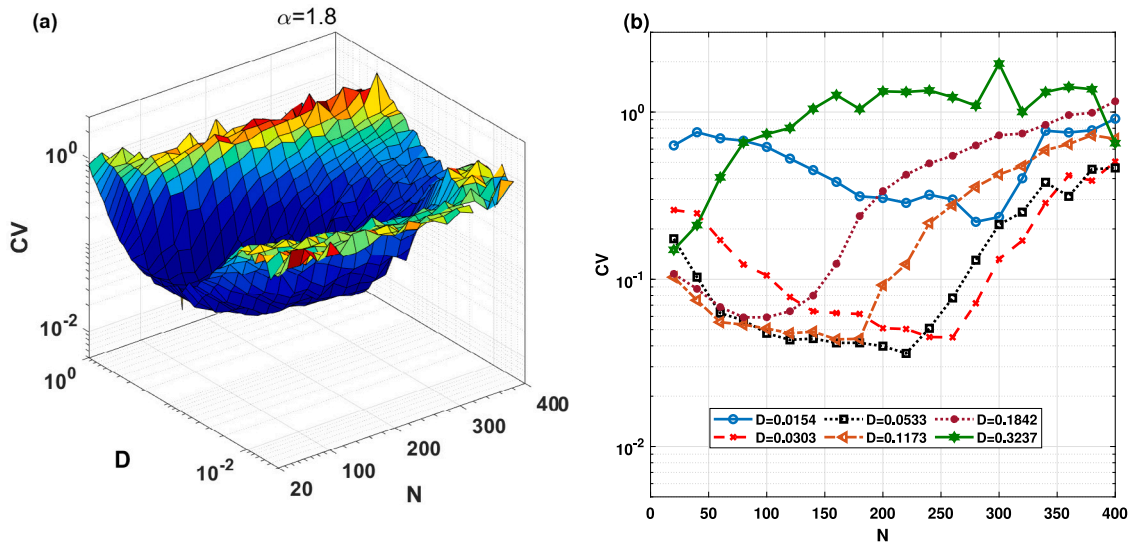


Fig. 4. (a) The dependence of CV on the system size N and D , (b) The dependence of CV on the system size N for different noise amplitudes keeping $\alpha = 1.8$, $p = 0$, $R = 4$.

system sizes N , which indicates the system-size coherence resonance occurs for $D = 0.0154, 0.0303, 0.0533, 0.1173, 0.1842$. Besides, for $D = 0.3237$, noise amplitude is so large that CV has been maintaining an upward trend as N increases, which indicates the system-size coherence resonance disappears. This implies that the occurrence of system-size resonance fundamentally hinges on the magnitude of the noise amplitude. Therefore, the system is able to achieve a better order (that is, the system can produce coherent resonance) through the presence of noise in the coupled FHN neurons under small-world network.

Next, to describe the joint influence of the stability parameter α and noise amplitude D on system-size coherence resonance, surfaces of CV as a function of system size and noise amplitude for $\alpha = 1.8, 1.6, 1.4, 1.2$ and 1 are shown in Figs. 4(a), 5(a), (b), (c) and (d), respectively. Fig. 4(b) presents the dependence of CV on the system size N for different noise amplitudes at $\alpha = 1.8$. Comparing Figs. 3(b) and 4(b), we can see that a change in the stability parameter will cause a change in the properties of the system. When $\alpha = 2$ becomes $\alpha = 1.8$, the system size N corresponding to the minimum CV decreases for $D = 0.0154, 0.0303, 0.0533, 0.1173, 0.1842$. Moreover, for $D = 0.0154$, the curve of the coefficient of variation CV with the system size N for $\alpha = 1.8$ is more obvious system-size coherence resonance than that for $\alpha = 2$, which is attributed to the heavy tail of α -stable Lévy noise that enhances the spikes of the system so that the system enters into coherence resonance even at lower noise intensities. Comparing Figs. 3(a), 4(a), 5(a), (b), (c) and (d), these are similar in the case of $\alpha = 2$, there are also valleys for CV with the change of D . The table below shows the $\min_{CV}$ for different α , along with the corresponding N_{opt} and D_{opt} . From the table, we can see that when α increases from 1 to 1.8, $\min_{CV}$ slowly increases, while when α increases from 1.8 to 2, CV decreases. As the α increases, the N_{opt} first increases, then decreases, and then increases again. The D_{opt} first decreases and then increases as the α increases.

α	1	1.2	1.4	1.6	1.8	2
$\min_{CV}$	0.0249	0.0258	0.0287	0.0329	0.0360	0.0251
N_{opt}	280	300	300	260	220	340
D_{opt}	0.2584	0.0936	0.0422	0.0533	0.0533	0.0596

Furthermore, in order to give more details of the effects of stability parameter α on the system-size coherence resonance, Fig. 6(a) presents the dependence of CV on the stability parameter α and the system size N keeping $p = 0$, $D = 0.1$, $R = 4$. Clearly, for any given α , as N changes, the system exhibits significant system-size coherence resonance. However, for any given N , as α changes, the trend of changing CV cannot be intuitively deduced from Fig. 6(a). Moreover, comparing Fig. 6(a) with Figs. 3(a), 4(a) and 5, we can find that changing D has the greatest, changing N has a greater, and changing α has the least effects on system order. Fig. 6(b) and (c) present the dependence of $\min_{CV}$ and N_{min} on α , respectively. For each fixed α , $\min_{CV}$ and N_{min} are respectively the minimum CV and the N corresponding to $\min_{CV}$. We can see that as α increases, $\min_{CV}$ roughly shows an increasing then decreasing trend, while N_{min} roughly shows a decreasing then increasing trend, opposite to $\min_{CV}$.

Since the law of evolution of CV with α is not well observed in Fig. 6. The curves of CV as a function of α for several different N are presented in Fig. 7. We can also notice that N has a significant effect on the value of CV . As N changes from 400 to 300, CV decreases for all α . As N changes from 300 to 200, CV basically decreases, except for a small α interval ($\alpha \in [1, 1.15]$) where CV increases a little. As N changes from 200 to 100, CV basically increases a little, except for a small α interval ($\alpha \in [1.8, 2]$). As N changes from 100 to 40, CV basically increases for all α . Overall, CV roughly decreases and then increases as N decreases. When N is fixed, the trends of CV with increasing α can be roughly classified into three types: CV decreases ($N = 40$), CV increases and then decreases ($N = 400, 300, 200$), CV decreases then increases and finally decreases again ($N = 100$).

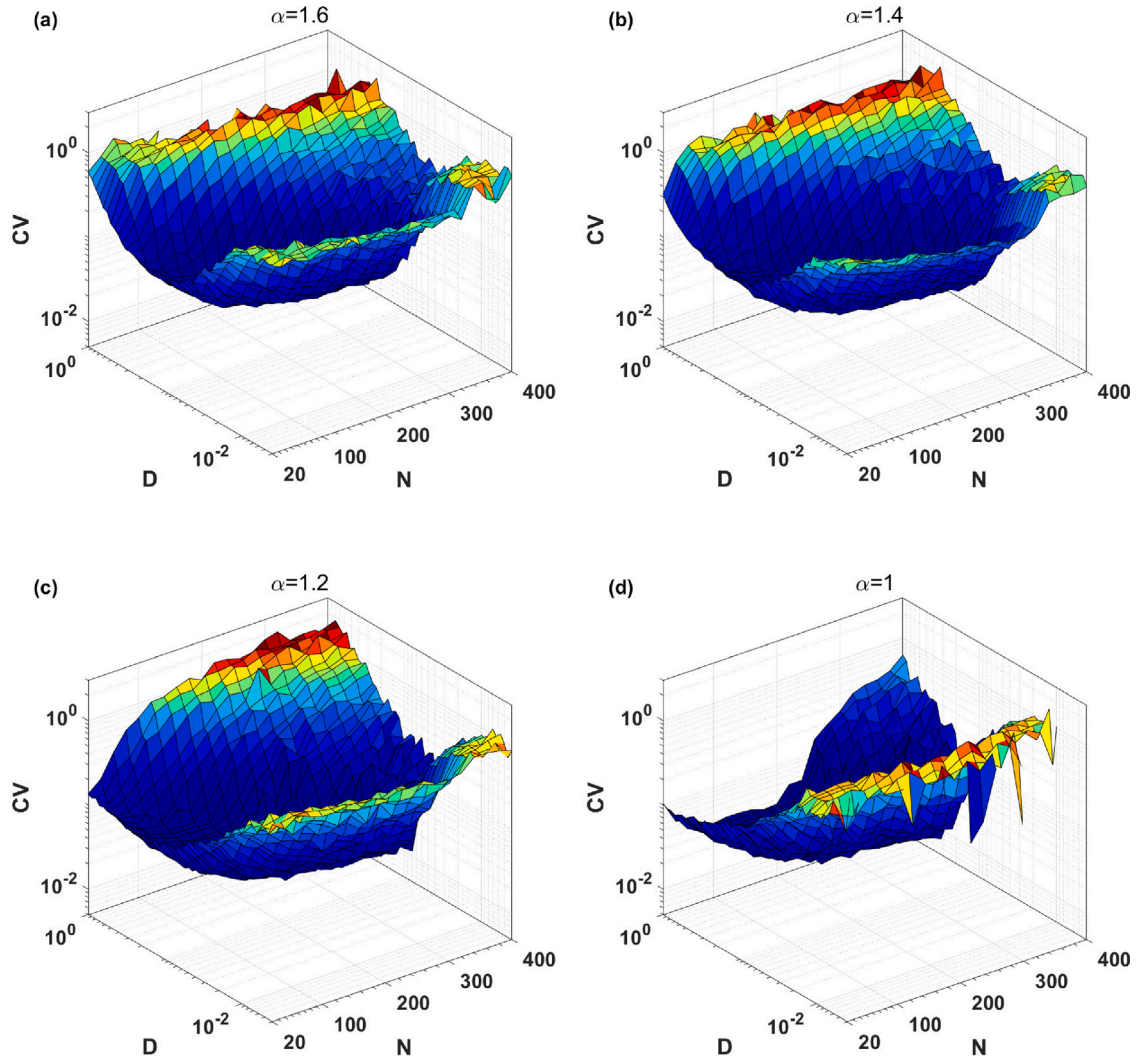


Fig. 5. The dependence of CV on the system size N and D for different α : (a) $\alpha = 1.6$, (b) $\alpha = 1.4$, (c) $\alpha = 1.2$, (d) $\alpha = 1$ keeping $p = 0$, $R = 4$.

3.2. The effect of network topology on system-size coherence resonance

First, we explore the effect of the rewiring probability p on the system-size coherence resonance. We present the dependence of CV on the system size N and p keeping $\alpha = 2$, $D = 0.04$, $R = 4$ in Fig. 8. We can see that CV decreases as p and N increases. An increase in N naturally lengthens the mean path length, but the rewiring probability p can offset this effect. And the reduction effect of p on the mean path length is stronger than the increase effect of N , resulting in a significant overall decrease. Therefore, the firing information of the neurons can be quickly transmitted to the entire network, and the coherence increases as p and N increases. Moreover, the system-size coherence resonance appears for $p = 0$, but disappears for $0 < p \leq 1$ that CV always increases as N increases. It can be concluded that the rewiring probability p can also determine whether the system will produce the system-size coherence resonance.

Then, we explore the effect of the coupling range r on the system-size coherence resonance. The dependence of CV on the system size N and r keeping $\alpha = 2$, $D = 0.04$, $p = 0$ is shown in Fig. 9. There is no system-size coherence resonance in Fig. 9, and CV tends to increase with increasing N and r . In other words, an increase in N or r makes the system low coherence. Especially note that coupling range r is related to the number of nearest neighbors R and N : $R = r * N$. In contrast to the previous fixed $R = 4$, R is not a fixed value, but increases as N and r changes. For more details, we can see Fig. 10. For a fixed $r = \frac{1}{20}$, as N increases, the number of firing spikes and the coherence for both the single neuron and the mean field decreases.

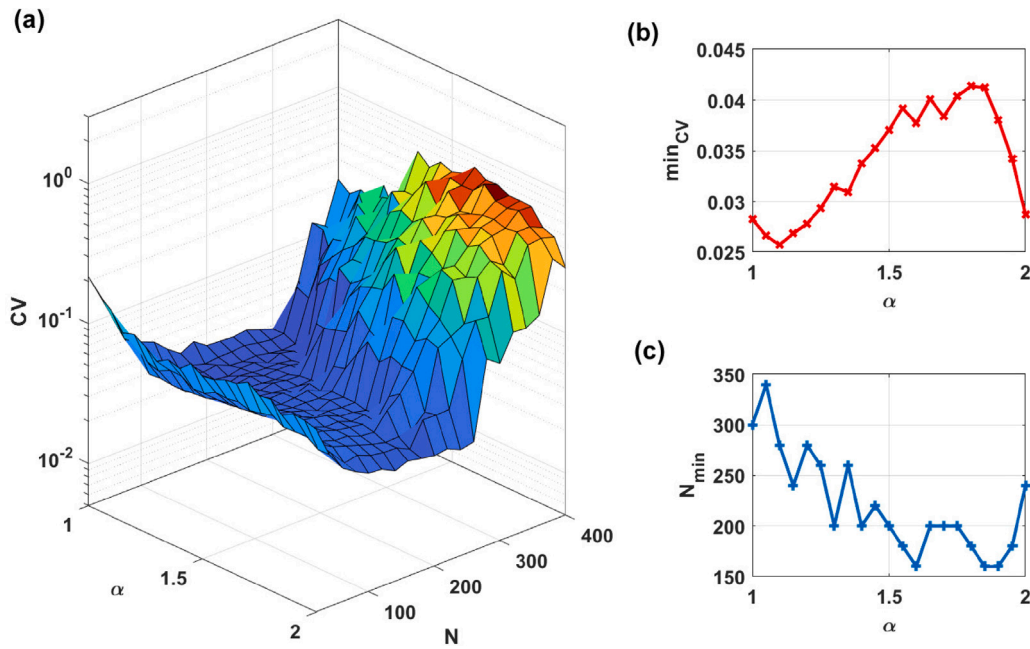


Fig. 6. (a) The dependence of CV on the stability parameter α and the system size N , (b) the dependence of $\min_{CV}$ on α , (c) the dependence of $N_{\min}$ on α for $p = 0, D = 0.1, R = 4$.

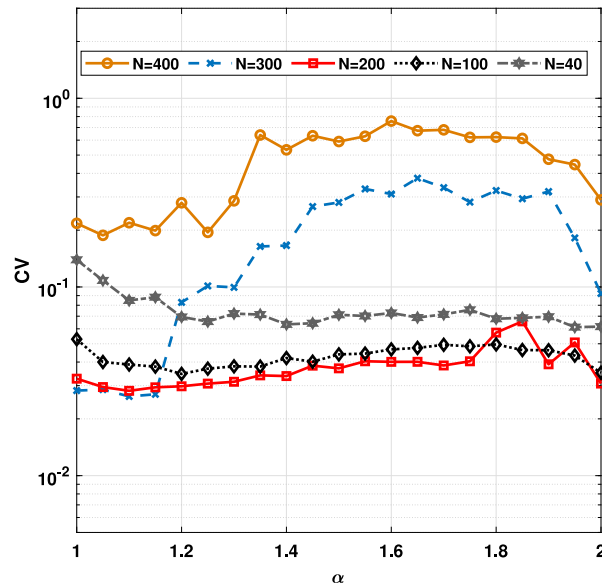


Fig. 7. The dependence of CV on the stability parameter α with the different system size N for $p = 0, D = 0.1, R = 4$.

4. Conclusion

In this paper, we have investigated the collective dynamical behaviors of a network of identical FHN neurons that are coupled under small-world schemes upon addition of α -stable Lévy noise. First, by comparing the time evolution of the mean field for different system sizes, we find that an intermediate system size ($N \approx 240$) leads to more frequent firing spikes and greater system order than either smaller or larger system sizes. This resonance-like behavior, characterized by a nonlinear relationship between the coefficient of variation CV and system size N that CV manifests a decrease in followed by an increase as N increases, known as **system-size coherence resonance**. Next, we investigate the effects of the α -stable Lévy noise parameters (namely the noise intensity and the stability parameter) on system-size coherence resonance. Our results show that the system can achieve a more

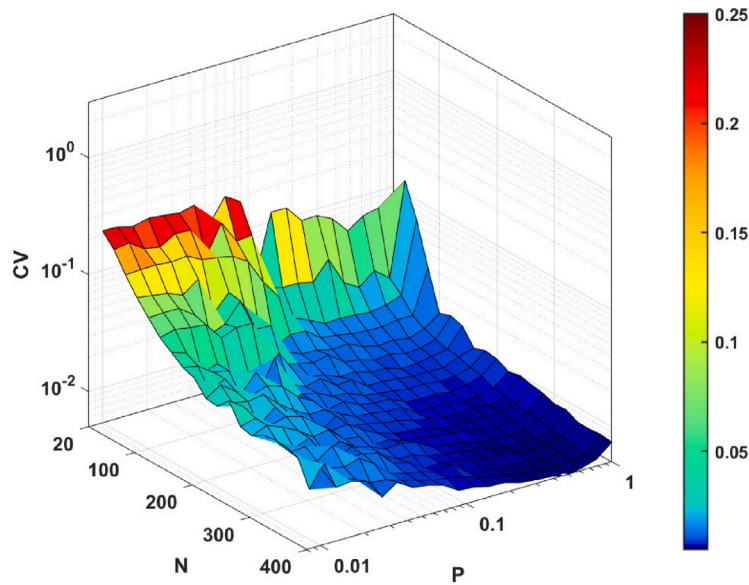


Fig. 8. The dependence of CV on the rewiring probability p and the system size N for $\alpha = 2, D = 0.04, R = 4$.

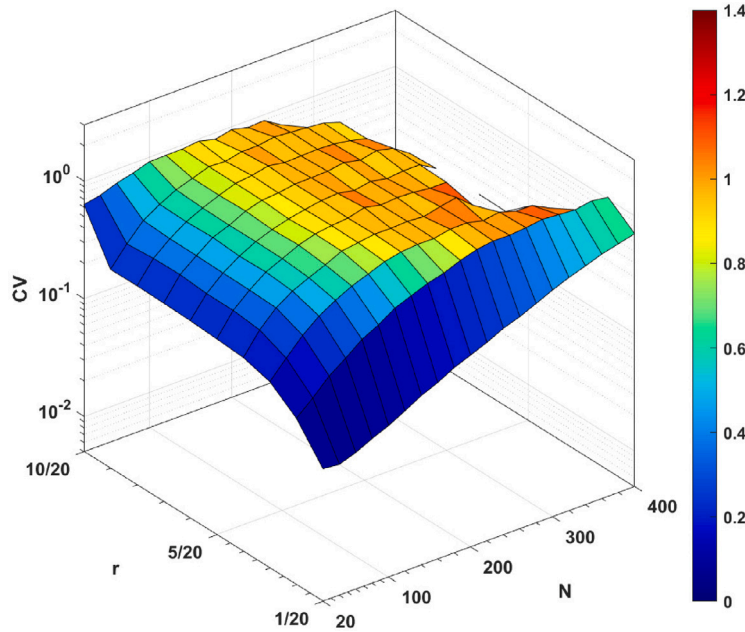


Fig. 9. The dependence of CV on the coupling range r and the system size N for $\alpha = 2, D = 0.04, p = 0$.

ordered state (i.e., exhibit coherence resonance) due to the presence of noise. Moreover, the occurrence of system-size coherence resonance fundamentally depends on the noise amplitude. Seen Figs. 3(b) and 4(b), the system-size coherence resonance occurs for $D = 0.0154, 0.0303, 0.0533, 0.1173, 0.1842$, and the system-size coherence resonance disappears for $D = 0.3237$. However, the occurrence of system-size coherence resonance does not depend on the stability parameter α . As can be seen in Fig. 6, for any given α , the system exhibits system-size coherence resonance as N changes. The stability parameter α only influences the value of CV . Furthermore, we explore the influence of the network topology on system-size coherence resonance. We find that the rewiring probability can also determine whether the system exhibits this resonance phenomenon. Additionally, increasing either the system size N or the coupling range leads to a reduction in system order.

All curves and time-series are drawn from what appears to be a single simulation run per parameter set except for Figs. 8 and 9, which appears 30 simulations. The results from 30 realizations are broadly similar to those from one realization but the surface

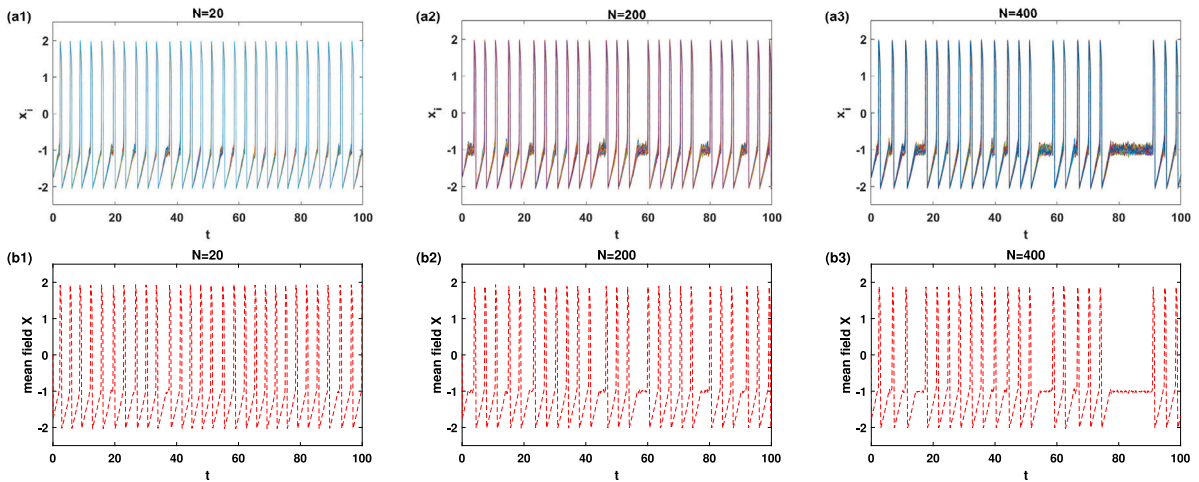


Fig. 10. (Top) The time series of x_i , (bottom) the time dependence of the mean field $X(t)$ in the system (1) for $\alpha = 2$, $D = 0.04$, $p = 0$, $r = \frac{1}{20}$, and different system sizes N . The left column: $N = 20$, the middle column: $N = 200$ and the right column: $N = 400$.

is smoother. Therefore, the Figures of the single simulation are also acceptable because they can reflect the trend of changes in the coherence, even though there are some mini errors in the value of the CV . Our model assumes the coupling strength $\sigma = 1$. However, we expect that the reported here also holds for both stronger and weaker coupling. A detailed investigation of the influence of the coupling strength would be of general physical interest, but is beyond the scope of this work. Additional general investigations should also consider various network configurations, such as scale-free and multilayer networks.

CRediT authorship contribution statement

Zhanqing Wang: Writing – original draft, Writing – review & editing. **Yu Gao:** Writing – original draft. **Alexey Zaikin:** Writing – review & editing. **Hua Li:** Writing – review & editing. **Yong Xu:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors express their gratitude to the editors and the reviewers for the helpful comments. Yong Xu was partially supported by the Key International (Regional) Joint Research Program of the National Science Foundation of China (12120101002). Zhanqing Wang thanks the National Natural Science Foundation of China (12202332). Hua Li thanks the National Natural Science Foundation of China (12202145). AZ acknowledges the Basic Research Program at the National Research University Higher School of Economics (HSE University) and its HPC facilities, Moscow, Russia.

Data availability

No data was used for the research described in the article.

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