Collective and unitary models: a clarification\textsuperscript{1}

Martin Browning  
CAM, Institute of Economics,  
University of Copenhagen

Pierre-André Chiappori  
Department of Economics,  
University of Chicago

Valérie Lechene  
Wadham College,  
Oxford University

October 29, 2004

\textsuperscript{1}We thank the Danish National Research Foundation for support through its grant to the Centre for Applied Microeconometrics. Comments are very welcome and should be directed to Browning at Martin.Browning@econ.ku.dk.
Collective and unitary models: a clarification

Keywords: unitary, collective, intrahousehold allocation, distribution factors, demand.
JEL code: D13.

1 Introduction

There is increasing consensus in the literature on household behaviour that we cannot model the decisions of a many person household as though the household had a set of stable and transitive preferences (the so-called ‘unitary’ model). A number of alternatives have been suggested, ranging from axiomatic bargaining models (Manser and Brown, (1980), McElroy and Horney, (1981), McElroy, (1990)) to (partly) non-cooperative models (see Leuthold, (1968), Bourguignon, (1984), Ulph, (1988), Chen and Woolley, (2001), Lundberg and Pollak, (1993)). At present there is no agreement on which model is appropriate and, indeed, it may be that different models are relevant in different contexts. For example, it may well be that different decision procedures are used for large, indivisible choices such as whether and when to have children or where to live than for less important decisions such how to spend the weekly budget. Certainly the sociology literature suggests that actual decision processes within the household depend on the context.

Amongst the many alternatives suggested, a good deal of recent attention has been focussed on the collective model, which posits that however decisions are made, the outcomes are Pareto efficient (see Chiappori (1988), (1992) and Browning and Chiappori (1998) for the basic theory). As is well known, under weak assumptions, this model can be implemented by assuming that the household has a welfare function that is a weighted sum of the individuals’ private utility functions. The Pareto weight in this welfare function may depend on prices and the household’s total expenditure on all
goods and on variables that do not enter the individual preferences (extra-environmental parameters (EEP’s) in the terminology of McElroy (1990) or distribution factors (DF’s) in the terminology of Browning et al (1994)). Exactly which variables enter the Pareto weight should depend on an explicit underlying model of the decision process but mostly informal justification is given for the inclusion of one or another variable. Examples that have been used in the empirical literature include the sex ratio in the surrounding population, the distribution of income within the household, the wealth contributed by each member at marriage and the level of single parent benefits.

When there are private goods, the collective model suggests the conceptual decentralization procedure of giving each person their own total expenditure and letting them buy their own private goods in the market.\(^1\) The notional sharing of total expenditure on private goods between the two partners is usually termed the sharing rule. Once again this can be dependent on non-preference factors as well as on prices and total expenditure. The sharing rule may also depend on the level of public goods chosen by the household but this is usually ruled out by a separability assumption.\(^2\)

In this note we identify and clarify a confusion that has arisen in the literature about the exact relationship between unitary and collective models and what enters the Pareto weight and the sharing function. Partly the confusion is terminological, so we have to be precise (pedantic) about our definitions. We shall say that a decision process leads to distribution factor independent (DFI) outcomes if the decisions depend only on factors that enter individual preferences and on prices and total expenditure; that is, if DF’s do not affect outcomes.

A particular variant of DFI, income pooling, obtains if household decisions do not depend on who receives the income within the household (hence

\(^1\)This is conceptual in the sense that we doubt that any household uses this for all private goods (the latter including much food that is consumed within the household). On the other hand, the concept may be used for some goods in the form of an ’allowance’ or ’pocket money’.

\(^2\)If we do have public goods and preferences over private goods are not separable from them, we can decentralise any allocation by giving each agent money to spend on both types of goods and then using Lindahl prices for the public goods (see Donni (2002)).
the term). From the point of view of policy, the income share is an important distribution factor, because policy can alter this variable. However, one can easily conceive of a model in which resources are pooled but the allocation of expenditures between household members depends on some DF’s other than income; examples include sex ratios in the marriage market, divorce laws and the age and education differences between household members. In such a case, outcomes are not DFI, but there is income pooling.

We now turn to the relationship between the Slutsky conditions\(^3\), DFI and whether a particular model is ‘unitary’ or not. As is well known, if household demands satisfy the Slutsky conditions then we can recover a utility function for the household that depends only on quantities and that rationalises the data (Hurwicz and Uzawa (1971)). Given this, we suggest that the terminology ‘unitary’ should be used for any model that leads to demands that satisfy the Slutsky conditions, \textit{whether or not they also satisfy DFI} (or income pooling).\(^4\) This suggested definition allows that there may exist unitary models which do not satisfy DFI. It is important to emphasize that there is no consensus in the literature on this point, and many researchers would characterise such models as non-unitary. We propose that the models that satisfy Slutsky but fail DFI be called \textit{DF dependent unitary models}. Conversely, we could have a collective model that fails Slutsky but satisfies DFI which we term an \textit{DFI collective model}. These term do not exactly roll off the tongue, but we believe that they have the merit of capturing the important features of the objects they seek to describe. The important point is that, contrary to a great number of assertions in the literature, empirical rejections of DFI (or income pooling) do \textit{not} imply 'rejections' of the unitary model. The next section formalises these suggestions and the following section considers some associated issues.

\(^3\)In this note we shall always assume adding up and homogeneity so these conditions are symmetry and negative semidefiniteness of the Slutsky matrix.

\(^4\)Browning and Chiappori (1998) show that in general the demands from a non-trivial collective model will fail the Slutsky conditions and are hence non-unitary, by this definition. Collective demands will satisfy the condition that the Slutsky matrix is the sum of a symmetric and negative semidefinite matrix and a rank one matrix.
2 Unitary and collective models.

We consider a two person household with members $A$ and $B$ who spend a given total $x$ on $n$ private goods $q$ and $m$ public goods $Q$. Private goods are market goods that are bought and consumed in a rival way by the two partners. The market purchase of private good $i$ is denoted $q_i$ and is purchased at price $p_i$ with $p$ denoting the $n$-vector of prices. The purchase of any good $i$ is divided between the two agents with $A$ receiving $q_i^A$ and $B$ receiving $q_i^B$ so that $q_i^A + q_i^B = q_i$ and $q^A + q^B = q$. Public goods are market goods that are bought and consumed in a non-rival way by the two partners. The market purchase of public good $i$ is denoted $Q_i$ and is purchased at price $P_i$. We will denote by $\pi$ the vector of prices of private and public goods. Initially we assume that each person is egoistic in the sense that their preferences are only defined over consumption of their own private goods and public goods (we discuss more general preference structures below). Preferences are conditional on a vector of personal factors $d$ that include individual specific preference factors such as age, labor force status and education and common preference factors such as location and household size. The utility functions representing these preferences are denoted $\nu^A(q^A, Q; d)$, and $\nu^B(q^B, Q; d)$. Finally we must specify the household budget constraint. Assume that the two agents agree on a level of total expenditure $x$ so that the household budget constraint is $p^A q^A + P^B Q = x$.

As shown in Browning and Chiappori (1998), the most general collective model can be characterized by a generalised household utility function that takes the form:

$$u(q, Q; z, d, \pi, x) = \max_{q^A, q^B} \left\{ \mu(z, \pi, x) \nu^A(q^A, Q; d) + (1 - \mu(z, \pi, x)) \nu^B(q^B, Q; d) \right\}$$

subject to $q^A + q^B = q$.

The decision about how much to spend may itself be non-unitary in the sense that the two partners may have different preferences over intertemporal allocation; see Browning (2000) for such a model. An unexplored area is the interactions between intertemporal and intratemporal allocations within the household. For example, it might be that one partner agrees to extra total expenditure if the extra is spent in a particular way.
The Pareto weight $\mu(z, p, x)$ is bounded between zero and unity and gives the influence of person $A$ on market demands. For values of $(z, \pi, x)$ such that $\mu = 1$, person $A$ is an effective dictator whilst $\mu = 0$ gives that $B$ is a dictator. Generally we would expect that the weight is strictly between zero and unity. The important point to note here is that we allow that the Pareto weight depends on DF’s, $z$, and prices and total expenditure. Thus the generalised household utility function includes prices and total expenditure and variables that do not enter preferences directly.\textsuperscript{6} It is also worth noting that locally we may have that $\mu(.)$ is independent of one or more of its arguments but for other values it does depend on the value taken by these arguments. For example, consider the following Pareto weighting function that only depends on one DF, $z \in [0,1]$, and takes values between zero and unity:

$$
\mu(z) = \begin{cases} 
2z & \text{if } z \leq 0.25 \\
0.5 & \text{if } 0.25 < z < 0.75 \\
0.5 + 2(z - 0.75) & \text{if } z \geq 0.75 
\end{cases}
$$

(1)

Although this function does vary with $z$, for intermediate values we have that the Pareto weight are (locally) constant.

The maximisation of the household utility function subject to the budget constraint gives (household) market demand functions:

$$
\hat{q} = \xi(z, d, \pi, x) = \arg\max_q \left\{ u(q, Q; z, d, \pi, x) \text{ subject to } p'q + P'Q = x \right\}
$$

(2)

and similarly for public goods $Q$. Thus prices and total expenditure enter market demands through both the budget constraint and the household utility function. Note however that the influence of the DF’s, prices and total expenditure in the utility function enters only through the function $\mu(z, \pi, x)$, which gives strong restrictions on how they can affect demands; see Browning and Chiappori (1998).

\textsuperscript{6}To ease notation we assume that preference factors are disjoint from distribution factors, but we can easily accommodate the case in which they overlap.
We will consider four sets of assumptions concerning the DF’s and prices and total expenditure in the Pareto weight:

Case I: \[ \mu = \mu (z, \pi, x) \]
Case II: \[ \mu = \mu (\pi, x) \]
Case III: \[ \mu = \mu (z) \]
Case IV: \[ \mu = \text{constant} \]  

where the dependence is taken to mean that the weight is not completely independent of the argument (but may be locally independent).\(^7\) There is general agreement that case I is a non-unitary, collective model. Equally, everyone agrees that case IV is a unitary model.\(^8\) Case II gives demands that fail the Slutsky conditions but are independent of any DF’s. In particular, demands in case II satisfy income pooling. We believe the model in case II should be classified as an \textit{DFI collective model}. Even when prices are normalised to unity - as in cross-section empirical work - we have the dependence of the Pareto weight on total expenditure, which is decidedly non-unitary. On the other hand, it is difficult for us to conceive of any decision process in which household members agree that the outcome of the bargaining will depend on prices and total expenditure exclusively, so case II may be irrelevant.

The problematic case is III. This gives demands that satisfy the usual Slutsky conditions but are not independent from DF’s; in particular if \( z \) includes the distribution of income, these demands do not satisfy income pooling. Case III is, moreover, the one most often used in the empirical intra-household literature. It is almost always referred to as a collective model, with an implicit assumption that it leads to outcomes that cannot be rationalized by a conventional household utility function. However, given that demands satisfy the Slutsky conditions, the latter is not the case. In-

\(^7\) We do not enter into a discussion of other possible cases (for example \( \mu (p, z) \) or \( \mu (x) \)) as it is difficult to imagine models corresponding to these cases.

\(^8\) By definition, any unitary model is also a collective model but we shall implicitly exclude the latter when we talk of a collective model.
Indeed, using the definition given above of the household utility function in a general collective model, we can write the household utility function corresponding to case III as follows:

$$u(q, Q; d, z) = \max_{q^A, q^B} \left\{ \mu(z) \nu^A(q^A, Q; d) + (1 - \mu(z)) \nu^B(q^B, Q; d) \right\}$$

subject to $$q^A + q^B = q$$

(4)

In the absence of a theoretical model of the constitution of preferences and power in the household, the distinction between DF’s, z, and preference shifters, d, is blurred. Indeed, for most demographics used in empirical demand models it is possible to argue, equally convincingly, that it should be in the Pareto weight or that it should condition preferences. In these circumstances, the utility function $$u(q, Q; d, z)$$ in (4) above is no different from the household utility function in any standard unitary model. Therefore, the case where the Pareto weight is assumed to depend on DF’s, but not on prices and total expenditure is equivalent to a standard unitary model if the DF’s are relabelled preference shifters. We therefore suggest the term *DF dependent unitary model* to cover this case.

The typology we suggest above has radical implications for empirical work. For example, consider the case in which the Pareto weight depends only on household income: $$\mu = \mu(Y)$$. Household income is one of the genuine candidates we have for an DF since it should be excluded from unitary demand equations if we have conditioned on total expenditure. We classify this model as a DF dependent unitary model. In empirical work, however, it will be extremely difficult to distinguish credibly between such a model and the DFI collective model that has a Pareto weight dependent on total expenditure: $$\mu = \mu(x)$$, given the strong correlation between income and total expenditure in cross-sections. Thus we may be in a position in which it is virtually impossible to empirically distinguish between an DFI collective model and an DF dependent unitary model.

The different assumptions on the Pareto weight have implications for

---

9For most’, but not for all. For example, household income or gender specific windfall incomes would not usually be considered preference factors and hence their significant presence in an empirical demand equation would signal a failure of DFI.
the sharing rule. To simplify notation, we shall assume in all that follows that there are no public goods (so that $\pi = p$). As mentioned in the introduction, one way to implement any collective decision is through a decentralization procedure that divides total expenditure, $x$, between the two agents, $x_A$ and $x_B$ such that $x_A + x_B = x$, and then to allow them to buy their own (private) goods in the market. The function $\rho(z, d, p, x) = x_A/x$ that achieves this decentralization is known as the sharing rule. In many empirical applications it is the sharing rule that is the focus of interest rather than the Pareto weight, if only because its definition depends on the particular cardinalisation adopted for individual preferences. Thus we must ask what are the implications of the four cases considered above for the sharing rule. Denote the mapping from demographics, prices and total expenditure to $A$’s consumption bundle by $\xi^A(z, d, p, x)$. By definition, the sharing rule is $\rho = p'\xi^A(z, d, p, x)/x$. From this we see that the sharing rule always depends on prices and total expenditure, through the budget constraint and the Pareto weight. The sharing rule depends on preference shifters $d$ through preferences and finally, it will depend on the DF’s $z$ through the Pareto weight, if and only if the latter depends on $z$. Empirical tests of the dependence of the sharing rule on total expenditure, prices or preference shifters $d$ do not discriminate between the four models. In particular, a finding that total expenditure enters the sharing rule does not necessarily imply a non-unitary model. Finally, dependence of the sharing rule on DF’s does not discriminate between collective and unitary models; it only indicates whether demands depend on DF’s.

Table 1 summarises matters for the four cases discussed (assuming no public goods). The second column shows the Pareto weight in each case considered and our suggested terminology. The third column indicates the functional dependence for the corresponding demand functions. The next two columns indicate whether demands are independent from DF’s and satisfy the Slutsky conditions. The final column gives the form for the sharing rule.
3 Some special cases

3.1 Perfect aggregation

In general a collective model leads to demands that do not satisfy Slutsky nor DFI. There are, however, exceptions. The most important is if the preferences of the two partners satisfy the Gorman aggregation conditions. That is, preferences are quasi-homothetic with identical marginal propensities to spend so that demands take the form:

\[
\begin{align*}
q^A &= \alpha^A(d, p) + \beta(d, p) x^A = \alpha^A(d, p) + \beta(d, p) \rho(z, d, p, x) x \\
q^B &= \alpha^B(d, p) + \beta(d, p) x^B = \alpha^B(d, p) + \beta(d, p) (1 - \rho(z, d, p, x)) x
\end{align*}
\]

In this case, of course, household demands take the form:

\[
q = q^A + q^B = \left(\alpha^A(d, p) + \alpha^B(d, p)\right) + \beta(d, p) x
\]

If the individual demands are derived from a constrained maximisation problem then the \(\alpha^A(\cdot), \alpha^B(\cdot)\) and \(\beta(\cdot)\) functions satisfy integrability conditions. In this case, the household demands are integrable and there is a
household utility function depending only on \((q, d)\) which rationalises the household demands. In this case we have a trivial DFI collective model with, for example, \(\mu = 1\) and \(A\)’s utility function being set equal to the household utility function. Of course, the utility function imputed to \(A\) in this case is not her actual utility function.

The converse of this is that even if the two partners have identical ordinal egoistic preferences that are not quasi-homothetic then households demands will not satisfy either Slutsky or DFI.

In the set of all pairs of utility functions, the set of pairs of quasi-homothetic utility functions with identical marginal propensities to spend is, of course, ‘thin’. However, suppose that there is assortative mating on the marriage market, so that the preferences of individuals in couples are identical or almost identical. Then it may be that locally household demands are approximately integrable and satisfy DFI. When we consider cross-section variation, however, this will not be the case.

### 3.2 Caring preferences

In the analysis above we have assumed egoistic preferences but this is hardly realistic for couples living together. The most general form of preferences are given by \(u_A = u_A(q_A, q_B, Q)\) and similarly for \(B\). With these general preferences we cannot generally decentralise collective decisions by the use of a sharing rule. Generally, however, the literature follows Becker in assuming caring preferences:

\[
\begin{align*}
U_A &= F^A(v^A(q^A, Q), v^B(q^B, Q)) \\
U_B &= F^B(v^A(q^A, Q), v^B(q^B, Q))
\end{align*}
\]

so that individual \(A\) derives utility from \(B\)’s consumption only in so far as \(B\) derives utility from it. All of the results for egoistic preferences hold for this case. To see this most easily, consider the following special case of caring:

\[
\begin{align*}
U_A &= v^A(q^A, Q) + \tau_A v^B(q^B, Q) \\
U_B &= v^B(q^A, Q) + \tau_B v^A(q^B, Q)
\end{align*}
\]
where we generally take both $\tau_A$ and $\tau_B$ to be in the interval $[0, 1)$. In this case, the overall household utility function for the general collective model is given by:

$$\mu(z, \pi, x) U_A + (1 - \mu(z, \pi, x)) U_B =$$

$$(\mu + (1 - \mu) \tau_B) \nu^A(q^A, Q; d) + (\mu \tau_A + (1 - \mu)) \nu^B(q^B, Q; d)$$

which can be considered as a simple Pareto re-weighting with the weights still depending on $(z, \pi, x)$ or some subset of these variables.

### 3.3 Outcomes other than demand

The discussion so far has been in terms of demand outcomes, but other outcomes of household decision making are also of interest. For instance, individuals may value leisure, which is a somewhat specific good because its price for market participants is the wage, which is also an element of income. When we consider demands for commodities that are assumed separable from labour supply, the distinction between prices and incomes is clear. Then this distinction is crucial, as in that case incomes enter the Pareto weight as DF’s rather than as prices. If demands depend on wages in that case, then one has a DF dependent model, be it unitary or collective. If we do not assume separability, matters are more complicated as the wage is a price but it may also be a DF, in which case it is not possible to distinguish between its effect as one and as the other.

There is also a closely related issue concerning whether the Pareto weight can depend on ‘outcomes’, i.e. choices variables. For instance, Basu (2001) discusses a model of labor supply in which the Pareto weights depend on labor incomes (as opposed to wages). Such a setting, however, violates the efficiency assumption. People typically work more than is efficient in the sense that a negotiation in which agents decide to keep the Pareto weights unchanged and decrease bilaterally their labor supply would increase both members’ utility. This is a more general result, according to which outcomes are typically not efficient if the Pareto weight depends on choice
variables. Some empirical researchers consider labor income as distribution factors, particularly when analyzing demand functions; then they assume constrained labor supplies and separability between consumption and leisure.

3.4 Heterogeneity in household models

As we noted above, the distinction between elements of observable heterogeneity which are assumed to affect individual preferences on the one hand and the Pareto weight on the other hand is crucial, and identification in household models relies on the ability to distinguish between them. Thus the issues arising from observable heterogeneity are conceptually clear. This is not the case for unobserved heterogeneity. This important but tricky question has received little attention so far, and unobserved heterogeneity is most often introduced in an *ad hoc* fashion. Errors are usually introduced additively to parametric demand equations without attempt or possibility to trace their origin to the structural elements of the model. As is well established in the empirical (unitary DFI) demand literature this is an unsatisfactory state of affairs. In a collective setting, one justification for the procedure would be that preferences across households and the Pareto weight are identical conditional on observed heterogeneity and that the errors simply capture measurement error in demands. This is clearly unacceptable but to date very little has been done to assess the implications of allowing for unobserved heterogeneity in preferences and Pareto weights.

3.5 Static and dynamic efficiency

The discussion so far has been in terms of static models of household behaviour. Several authors consider the temporal aspect of household interactions explicitly within the framework of structural models of behaviour (see for instance, Konrad and Lommerud (2000), Lundberg and Pollak (2003) or Mazzocco (2003)). Mazzocco (2003) presents an extension of the collective model to an intertemporal setting. He shows that in such a setting, an important distinction arises depending upon whether household members
can make full or only limited commitment. If household members can commit to an allocation of resources at the beginning of the partnership, then intra-household allocation at all future dates should depend only on what determined the allocation initially, that is the initial DF’s or some function of the expected stream of values of the DF’s. If however, household members can only make limited commitment, by which it is meant that the allocation of resources is determined at each date, then outcomes depend on each period’s DF’s. Mazzocco shows that the difference between the two leads to differences in how DF’s enter the household Euler equations. However, even under the full commitment assumption, any unforeseen change in either prices, total expenditure or DF’s can lead to changes in the allocation of resources.

4 Conclusion

In this note, we have tried to clarify several issues in the use of models of intra-household behaviour. Our most controversial suggestion is that we should denote as ‘unitary’ any model that leads to outcomes that satisfy the Slutsky conditions whether or not these outcomes depend on distribution factors. In particular, income pooling is neither necessary nor sufficient for a unitary model. We also show that the presence of prices or total expenditure in the sharing rule cannot be used as a test for a unitary model.

References


