Revealing Preferences Graphically: An Old Method Gets a New Tool Kit

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Because uncertainty is endemic in a wide variety of economic circumstances, models of decision making under uncertainty play a key role in every field of economics. The standard model of decisions under uncertainty is based on von Neumann–Morgenstern Expected Utility Theory (EUT), so it is natural that experimentalists should want to test the empirical validity of the Savage axioms on which EUT is based. Empirical violations of EUT provoke intriguing questions about the rationality of individual behavior and, at the same time, raise criticisms about the status of the Savage axioms as the touchstone of rationality. These criticisms have generated the development of various theoretical alternatives to EUT, and the investigation of these theories has led to new empirical regularities.

For the most part, these experimental investigations use several pairwise choices, à la Allais, to test EUT and its various alternatives such as weighted utility, implicit expected utility, and prospect theory, among others. Each of these theories gives rise to indifference curves with distinctive shapes in some part of the Marschak-Machina probability triangle, so each theory can be tested against the others by choosing alternatives that the various theories rank differently. In these studies, the criterion used to evaluate a theory is the fraction of choices it predicts correctly. Generally speaking, experimental work has, on the one hand, collected only a few decisions from each subject and, on the other, presented subjects with a series of choices designed to compare the predictive abilities of competing theories or discover violations of specific axioms (Colin F. Camerer 1995 and Chris Starmer 2000 review the experimental and theoretical work that focuses on evaluating non-EUT theories).

Although this practice is understandable given the purposes for which the experiments were designed, critics say that it limits the usefulness of the data for other purposes. First, while these experiments reveal that violations exist, they give us little sense of how important they are or how frequently they occur. Most important, the typical analysis builds on ad hoc assumptions about an error-generating process and lacks any substantive econometric methodology designed to compare the predictive power of different theories. Second, designing choices to reveal violations does not necessarily tell us very much about how choices are made in economic settings that are more often encountered in practice. A variation of this criticism is that the choice scenarios typically used are not very important to the applications of theories of choice under uncertainty in economics. Third, the small datasets generated for each subject force experimenters to pool data, thus ignoring individual heterogeneity and precluding the possibility of statistical modeling at the level of the individual subject.

These limitations highlight the importance of developing refined techniques and larger samples in order to enable a more rigorous test of choice under uncertainty. While considerable headway has been made in this endeavor, the problem remains a difficult one. New experimental results and theoretical advances can provide much richer guidance for understanding the preferences underlying decisions under uncertainty and the choices that implement them. Developing such methods will have far-reaching implications in many areas of economic theory and policy.

I. A New Experimental Design

Our objective of producing a general account of choice under uncertainty has led us to develop an experimental design that is innovative in a

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couple ways. In the experimental test we study, subjects are presented with a standard economic decision problem that can be interpreted either as a portfolio choice problem (the allocation of wealth between two risky assets) or a consumer decision problem (the selection of a bundle of contingent commodities from a standard budget set). These decision problems are presented using a novel graphical interface that was developed for this purpose, where subjects see a graphical representation of the budget lines on a computer screen and make choices through a simple point-and-click action. This intuitive and user-friendly interface allows for the quick and efficient elicitation of many decisions per subject under a wide range of budget sets.

In our concurrent paper, Choi, Fisman, Gale, and Kariv (2006, henceforth CFGK), each experiment consists of 50 independent decision problems. In each decision problem, a subject is asked to allocate tokens between two accounts labeled *x* and *y*. The *x* account corresponds to the *x*-axis and the *y* account corresponds to the *y*-axis in a two-dimensional graph. Each choice involves choosing a point of possible token allocations on a budget line. Each decision problem starts by having the computer select a budget line randomly from the set of lines that intersect at least one axis at or above the 50-token level and intersect both axes at or below the 100-token level. To choose an allocation, subjects use the mouse or the arrows on the keyboard to move the pointer to the desired allocation on the computer screen. At the end of each decision problem, the computer randomly selects one of the accounts, *x* or *y*, and the subject receives the number of tokens allocated to the account that was chosen[1].

There are several advantages of this experimental design. First, the choice of a portfolio subject to a budget constraint provides more information than a binary choice. Second, it allows us to test a wider range of choices than can be tested using a pencil-and-paper experimental questionnaire. We may thus apply statistical models to estimate preferences at the level of the individual subject rather than pooling data or assuming homogeneity across subjects. Third, the experimental technique allows us to confront subjects with choice problems that span a broad range of common economic problems, both in theory and in empirical applications, rather than, as in existing methods, with stylized choices designed to test a particular theory.

In the remainder of the paper, we illustrate how this experimental design enables us to thoroughly analyze behavior under uncertainty at the individual level. The analysis builds on revealed preference techniques to determine whether the choices of hypothetical subjects are consistent with utility maximization and to recover their underlying preferences. Previous work consists primarily of a series of observed violations of specific axioms or presents evidence that challenges the assumption that choices are derived from well-behaved preferences, without quantifying the extent of violations. By contrast, we attempt to construct a metric for the consistency of individual behavior, and we have enough data to make such tests statistically useful.

II. Consistency

Suppose there are two equally likely *states of nature* denoted by $s = 1, 2$ and two associated *Arrow securities*. Let x_s denote the demand for the security that pays off in state s and let p_s denote its price. Without essential loss of generality, we normalize the prices so that $p \cdot x = 1$. Let $\{(p^i, x^i)\}$ be the data generated by some individual's choices, where p^i denotes the i-th observation of the price vector and x^i denotes the associated portfolio. Then, a *nondegenerate* utility function $u(x)$ is said to *rationalize* the observed behavior if $u(x^i) \geq u(x)$ for all x such that $p^i \cdot x^i \geq p^i \cdot x$ (u achieves the maximum on the budget set at the chosen portfolio). Following Sidney N. Afriat (1967), we can employ the Generalized Axiom of Revealed Preference (GARP) to test whether the data $\{(p^i, x^i)\}$ may be rationalized by a utility function. GARP requires that if x^i is indirectly revealed preferred to x^j, then x^j is not *strictly* directly revealed preferred to x^i ($p^j \cdot x^i \geq p^j \cdot x^j$). Afriat's (1967) theorem tells us that if a *finite* dataset satisfies GARP, then the data can be rationalized by a well-behaved (piecewise linear, continuous, increasing, and concave) utility function.

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[1] The computer program dialog window is shown in the experimental instructions reproduced at http://ist-socrates.berkeley.edu/~kariv/CFGK_1.pdf.

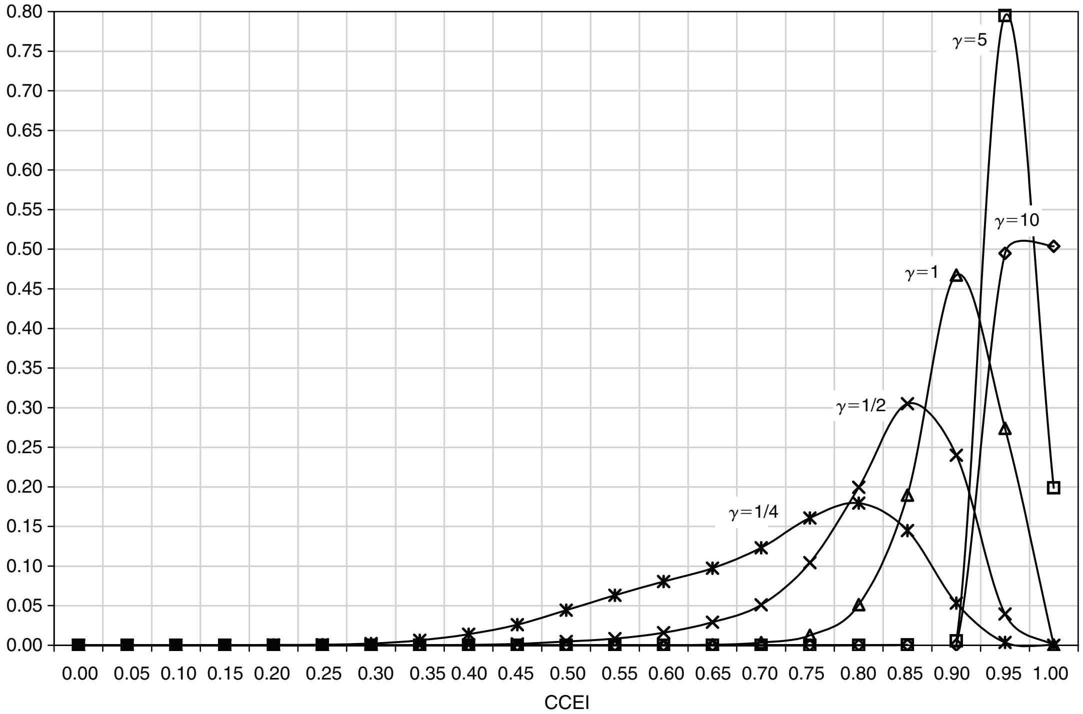

FIGURE 1. THE DISTRIBUTION OF CCEI SCORES $\rho = ½$ AND $\gamma = ¼, ½, 1, 5, 10$

Since GARP offers an exact test (either the data satisfy GARP or they do not) and choice data almost always contain at least some violations, we use Afriat's (1972) Critical Cost Efficiency Index (CCEI) to quantify the extent of violation. The CCEI measures the amount by which each budget constraint must be relaxed in order to remove all violations of GARP. Hence, the CCEI provides a summary statistic of the overall consistency of the data with GARP. It can be interpreted as measuring the upper bound of the fraction of his wealth that an individual is "wasting" by making inconsistent choices. The closer the CCEI is to one, the smaller the perturbation of budget sets required to remove all violations, and thus the closer the data are to satisfying GARP.

Next, we generate a benchmark level of consistency. To this end, we generate a random sample of hypothetical subjects who implement the power utility function $u(x) = x^{1-\rho}/(1 - \rho)$ commonly employed in the empirical analysis of choice under uncertainty with error. The likelihood of error is assumed to be a decreasing function of the utility cost of an error. More precisely, we assume an idiosyncratic preference shock that has a logistic distribution

$$\Pr(x^*) = \frac{e^{\gamma \cdot u(x^*)}}{\int_{x:p\cdot x=1} e^{\gamma \cdot u(x)}},$$

where the precision parameter γ reflects sensitivity to differences in utility. The choice of portfolio becomes purely random as γ goes to zero, whereas the probability of the portfolio yielding the highest expected utility approaches one as γ goes to infinity.

Figure 1 summarizes the distributions of CCEI scores generated by samples of 25,000 hypothetical subjects with $\rho = ½$, which is in the range of some recent estimates, and various levels of γ. Each of the 25,000 hypothetical subjects makes 50 choices from randomly generated budget sets in the same way as the human subjects do in CFGK. The data provide a clear illustration of the extent to which the

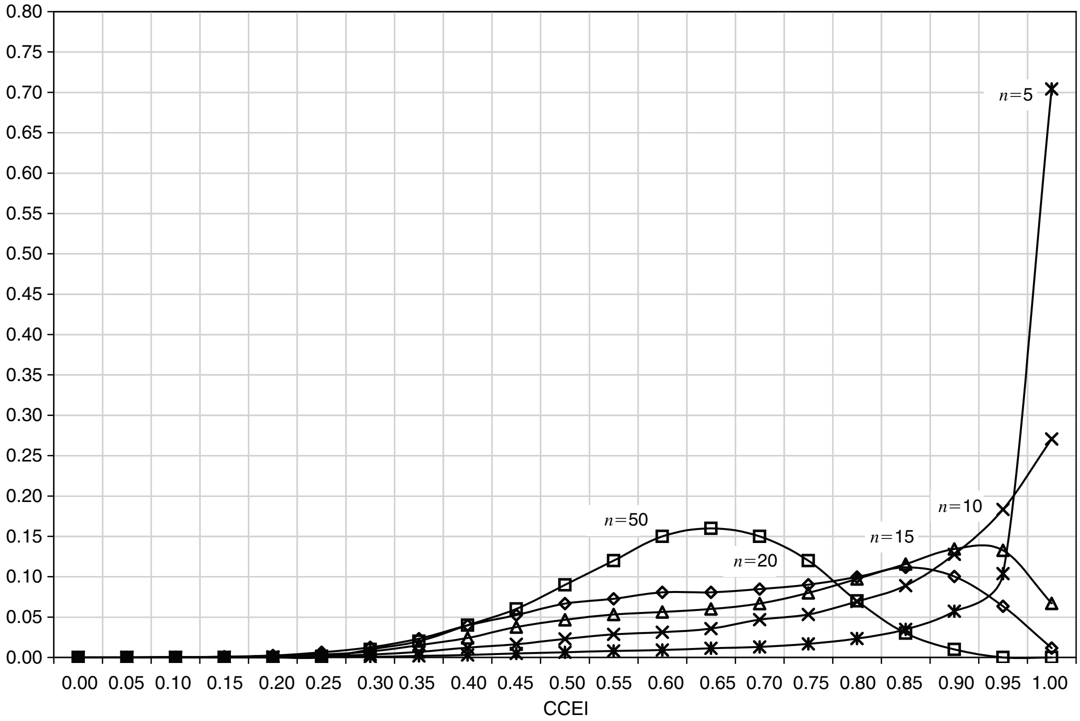

Figure 2. The Distribution of CCEI Scores $n = 5, 10, 15, 20, 50$ and $\gamma = 0$

hypothetical subjects did worse than choosing consistently and the extent to which they did better than choosing randomly, and demonstrate that if utility maximization is not in fact the correct model, then our experiment is sufficiently powerful to detect it.

Finally, we note that there is a very high probability that even random behavior will pass the GARP test if the number of individual decisions is as low as it usually has been in experiments, thus underscoring the need to collect data on a large number of decisions per subject. To illustrate this point, we use the test designed by Stephen G. Bronars (1987) which employs the choices of a hypothetical subject who randomizes uniformly among all allocations on each budget set as a point of comparison. To this end, we calibrate the choices of 25,000 random subjects ($\gamma = 0$) over 5, 10, 15, 20, and 50 budgets. The results are presented in Figure 2, which shows that the distribution of consistency values is skewed to the left as the number of budget sets increases.

III. Recoverability

Since GARP imposes on the data the complete set of conditions implied by utility maximization, revealed preference relations in the data thus contain the information that is necessary for recovering the underlying preferences. Therefore, we turn to the problem of recovering underlying preferences using the revealed preference techniques developed by Hal R. Varian (1982). This approach is purely nonparametric and uses only information about the revealed preference relations in the data (the computer program and details of the algorithm are available from the authors upon request).

We next give a brief outline of Varian's algorithm, which provides the tightest possible bounds on indifference curves through an allocation x^0 that has not been observed in the previous data $\{p^i, x^i\}$. First, we consider the set of prices at which x^0 could be chosen and be consistent, i.e., does not add violations of GARP, with the previously observed data.

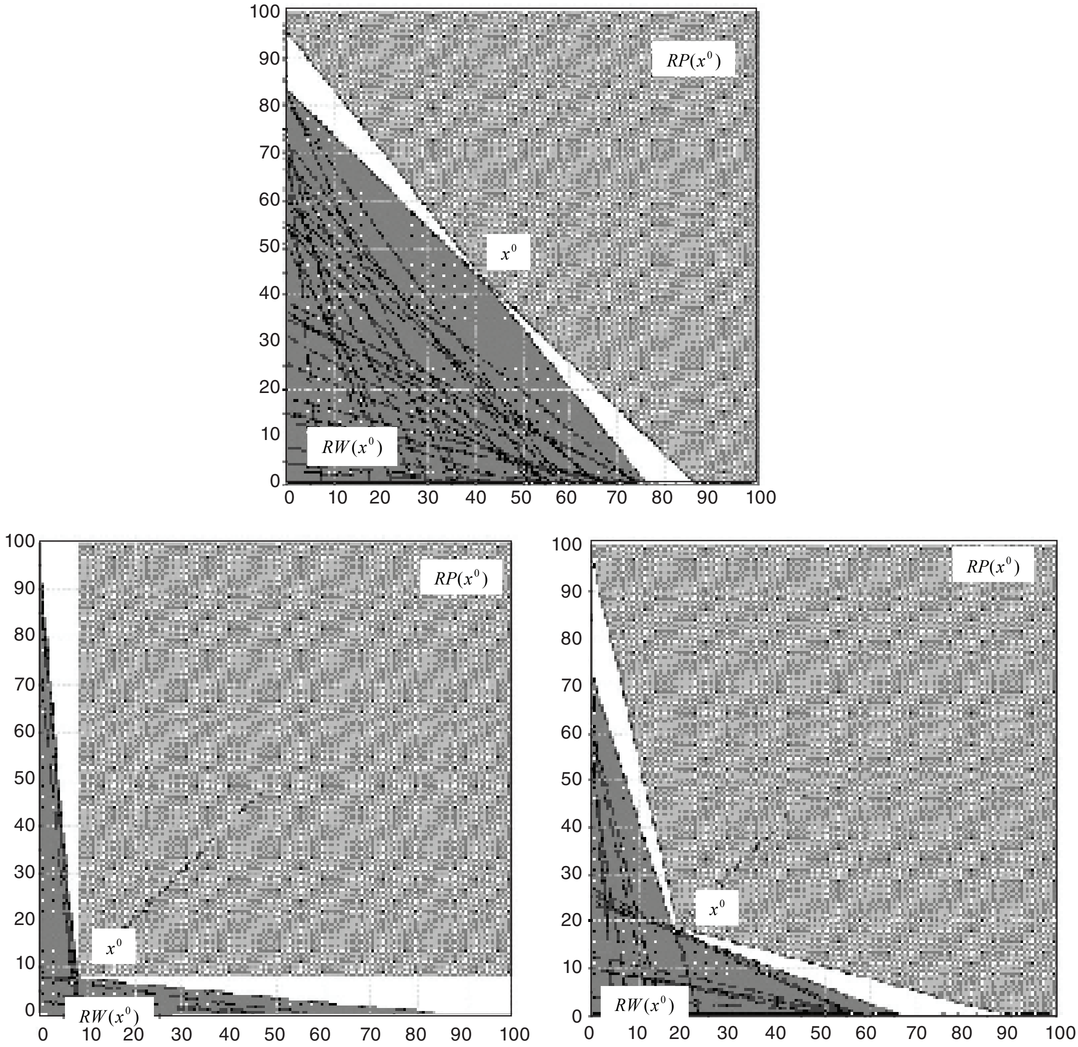

FIGURE 3. ILLUSTRATION OF RECOVERABILITY FOR SELECTED PROTOTYPICAL PREFERENCE TYPES

This set of prices is the solution to the system of linear inequalities constructed from the data and revealed preference relations. Call this set $S(x^0)$. Second, we use $S(x^0)$ to generate the set of observations, $RP(x^0)$, revealed preferred to x^0 and the set of observations, $RW(x^0)$, revealed worse than x^0.

It is not difficult to show that $RP(x^0)$ is simply the convex monotonic hull of all allocations revealed preferred to x^0. To understand the construction of $RW(x^0)$, note that if x^0 is directly revealed preferred to some portfolio x for all prices $p^0 \in S(x^0)$ ($p^0 \cdot x^0 \geq p^0 \cdot x$), then it is indirectly revealed preferred to any allocation in the budget set on which x was chosen. Similarly, it is indirectly revealed preferred to all observations that x is revealed preferred to and so on. Hence, the two sets $RP(x^0)$ and the complement of $RW(x^0)$ form the tightest inner and outer bounds on the set of allocations preferred to x^0. Similarly, $RW(x^0)$ and the complement of $RP(x^0)$ form the tightest inner and outer bounds on the set of allocations worse than x^0.

Figure 3 depicts the construction of the bounds described above through some allocation x^0 for hypothetical types whose choices fit with some canonical utility functions. In addition to the $RW(x^0)$ and $RP(x^0)$ sets, we show the choices as well as the budget sets used to construct $RW(x^0)$. The top panel shows the bounds on the indifference curve for risk neutrality where the revealed worse and preferred sets closely bound a linear indifference curve with slope of about -1. The bottom-left panel shows the bounds for a utility function with infinite risk aversion where the bounds suggest a near–right angled indifference curve. Finally, the bottom-right panel shows the bounds for loss aversion preferences (where the safe portfolio $x_1 = x_2$ is taken to be the reference point) where the bounds imply there is a distinct "kink" at the certainty line.

We have shown a small subset of possible choices, and have chosen them to highlight the merits of our approach and to illustrate the power of the experimental methodology. While these cases generate a particularly close fit, we may generally provide reasonably precise bounds as long as x^0 is chosen within the convex hull of the data. Since the nonparametric approach makes no assumptions about the form, parametric or otherwise, of the underlying utility function, it provides relatively little information about the structure of preferences. For this, standard parametric tools from demand analysis can also be brought to bear in order to understand preferences. Econometric analyses based on experimental data of the type we discuss here may be found in CFGK.

IV. Concluding Remarks

The experimental and analytical techniques described above also serve as a foundation for studying decision making in other choice scenarios. For example, Fisman, Kariv, and Daniel Markovits (2006) employ a similar experimental methodology to study social preferences. While the papers share a similar experimental methodology, they address very different questions and produce very different behaviors. We believe that our experimental apparatus will ultimately prove to be a useful tool in a range of disciplines interested in examining individual choices, and extensions of our method will continue to improve our understanding of choice under uncertainty.

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