Keeping the Listener Engaged: a Dynamic Model of Bayesian Persuasion

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Bayesian Persuasion

- **Classical question:** How (much) can a sender persuade a rational receiver to take a particular action? (e.g., *seller-buyer, media-voters, prosecutor-judge, entrepreneur-investor.....*)
- **An important assumption:** Commitment
  - Achieved by *instantaneous* and *unrestricted* experimentation.
- We relax the commitment assumption.
  - In our model, persuasion takes time and is costly:
    - To be informative takes real time.
    - Information is costly for sender to generate and for receiver to process.
  - No commitment to future actions: For instance, sender may not be able to credibly carry out sustained persuasion.
- **Key issue:** How to persuade the receiver to listen rather than walk away?
- **Questions:**
  - Is dynamic persuasion possible? What payoffs can be achieved?
  - Behavioral Implications: Dynamic Choice of Information Structures

Model — actions and preferences

- **Two States:** $\omega \in \{L, R\}$, **common prior** p_0 that $\omega = R$.

Receiver

- Takes irreversible action $a \in \{\ell, r\}$, or waits.
- Payoff from action a in state ω: $\boldsymbol{u_a^\omega}$
  - Prefers to "match" the state: $u_\ell^L > u_r^L, \quad u_r^R > u_\ell^R$
- Notation:

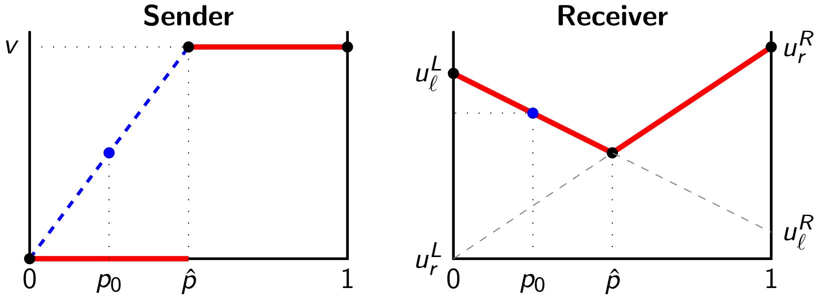
$$U_a(p) = pu_a^R + (1-p)u_a^L, \quad a \in \{\ell, r\}$$
$$U(p) = \max\{U_r(p), U_\ell(p)\}$$

Sender

- Receives state-independent payoff $v \cdot 1_{\{a=r\}}$, $v > 0$.
- Performs experiments over time to "persuade" receiver.

Static Benchmark: Kamenica-Gentzkow Model

- Sender picks an arbitrary Blackwell experiment, wlog: 2 signals.
- Notation: $\hat{p}$ solves $U_\ell(p) = U_r(p)$.



Observations

- "Fully-revealing of L" in case of L-signal
- R-signal sent excessively compared to full information.
- **The receiver enjoys no rents.**

Our Model: Dynamic Extension

- Continuous time, infinite time horizon.
- Flow cost $c > 0$ for sender and receiver.
- Payoffs if $a \in \{\ell, r\}$ is taken at time T when belief is p_T:
  - Receiver: $U_a(p_T) - c\,T$
  - Sender: $v \cdot 1_{\{a=r\}} - c\,T$

Timing

At each point t in time,

1. Sender picks an information structure (described later) at flow cost c or "passes" (=null information, costless).[1]
2. Receiver observes the information structure and its outcome, and either **takes an irreversible action** $a \in \{\ell, r\}$, or **waits**.
   - If she waits and listens to next experiment she incurs cost c.
   - No cost is incured if the sender passes.

[1] "partial passing" is possible

Our Model: Dynamic Extension

Comparison to KG

- In KG-benchmark: Arbitrary information structure can be implemented instantaneously without cost.

- Information structure in our model:
  - Information takes time to arrive.
  - No other restriction: Feasible information structures will allow to generate arbitrary distribution over posteriors.

- Important difference from KG:
  - Receiver can take game-ending action at any time.
  - Sender's information must make it worthwhile to wait. ($\Rightarrow$ commitment issue)

Feasible Information Structures: General Poisson Models

Basic Poisson Experimennt

The sender chooses arrival rates of a signal:

- $\lambda^L := \nu^L + \mu$ (in state L)
- $\lambda^R := \nu^R + \mu$ (in state R)

where $\nu^L + \nu^R \leq \lambda$ and $\mu \geq 0$.

Interpretation of (ν^L, ν^R) and μ:

- Real Signal: arrival rate constrained by λ.
- Noise ("inflation"): same arrival rate μ in each state.

Feasible Information Structures

Mixtures of experiments $(\lambda_i^L, \lambda_i^R)$ with weights $\sum_i \alpha_i \leq 1$ feasible.

- Arrival rates scaled by weight: e.g. $(\alpha_1 \lambda, 0)$ and $(0, \alpha_2 \lambda)$
- Signals from different experiments can be distinguished

Feasible Information Structures: General Poisson Models

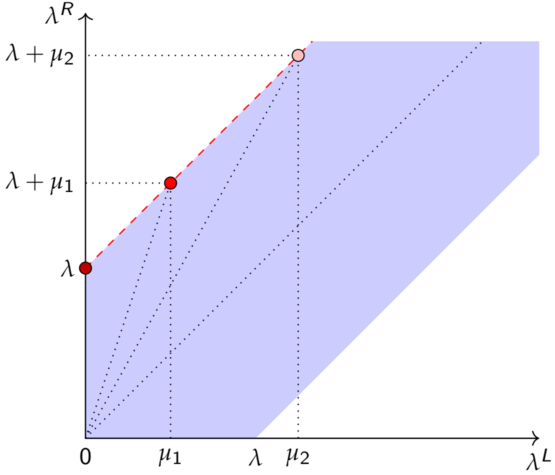


Figure: Arrival rates of feasible Poisson experiments.

Choosing "Jump Targets" for Posterior Beliefs

Reformulation in Terms of Beliefs

- Fix a current belief p and target posterior belief q.
- S can choose a feasible (λ^L, λ^R) so that, jump leads to posterior q.
  - $\Rightarrow$ Resulting arrival rate: $\frac{p(1-p)}{|q-p|}\lambda$.
- Important Feature: *Large jumps are preferred to small jumps.*

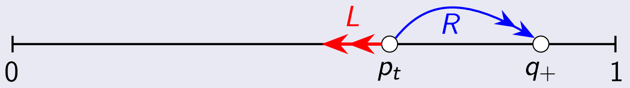
Summary: Feasible Information Structures

- Nests conclusive good news and conclusive bad news.
- Allows for any directionality and any degree of accuracy, and can mix different Poisson experiments.
- **Important feature**: *Real information takes time; the more precise the posterior q, the longer it takes for signals to arrive.*

Feasible Experiments: Three Building Blocks

L-drifting experiment (with right-jumps $q_+ > p_t$)

- R-signals: belief jumps to q_+, arrival rate: $\frac{p_t(1-p_t)}{|q_+-p_t|}\lambda$
- L-signals: belief drifts to the left: $\dot{p}_t = -\lambda p_t(1-p_t)$

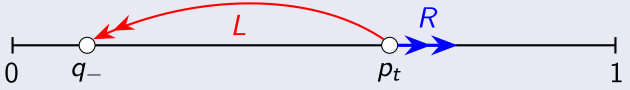


- Sender may choose the "precision" of R-evidence.
  - For example this allows to target $q_+ = \hat{p}$!

Feasible Experiments: Three Building Blocks

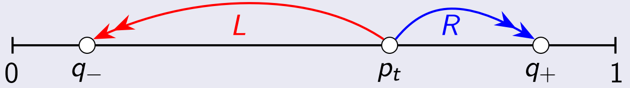
R-drifting experiment (with left-jumps to $q_- < p_t$):

- L-signals: belief jumps to q_-, arrival rate: $\frac{p_t(1-p_t)}{|q_- - p_t|}\lambda$
- R-signals: belief drifts toward right, $\dot{p}_t = \lambda p_t(1 - p_t)$



"Stationary" Experiment

- Splitting attention ($\alpha = 1/2$), we obtain **2 jumps and no drift**
- Jumps to q_- and q_+ at rates $\frac{\lambda p_t(1-p_t)}{2|q_\bullet - p_t|}$,—no drift.



Our Model: Dynamic Extension

Equilibrium

- We study: **Markov Perfect equilibria (MPE)**
  - Subgame Perfect Equilibrium in which
  - strategies only depend on payoff relevant state p.
- **Additional Restriction:** MPE should be a limit of discrete time equilibria.
  - Sender maximizes continuous time flow payoff even when receiver stops immediately.

- Continuous time game: admissible strategy profiles defined similarly to Klein and Rady (2011).

Literature

- Bayesian Persuasion: Kamenica and Gentzkow (2011,...), ..., Aumann/Maschler (1995)
- Wald Decision: Wald (1947), Arrow, Blackwell, and Girshick (1949), Moscarini and Smith (2001), Che and Mierendorff (2018), Nikandrova and Pancs (2018), Mayskaya (2017), Zhong (2018), Henry and Ottaviani (2019), McClellan (2017)
- Dynamic Persuasion: Brocas and Carrillo (2007), Kremer, Mansour and Perry (2014), Au (2015), Ely (2017), Renault, Solan and Vieille (2017), Bizzoto, Rudiger and Vigier (2017), Che and Hörner (2018), Henry and Ottaviani (2019), Ely and Szydlowski (2020), Orlov, Skrzypacz and Zryumov (2020).
- Repeated Persuasion/Communication: Margaria and Smolin (2018), Best and Quigley (2017), Mathevet, Pearce, and Stachetti (2018).

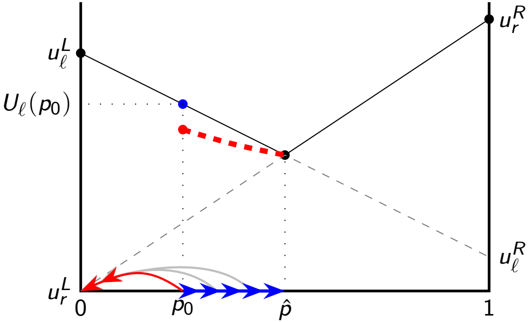
  Difference: Permanent state, MPE, slow learning.

Overview

1. **Illustration of the Commitment Problem**
2. Main result: Characterization of Equilibrium Payoffs
3. Equilibrium Construction and Persuasion Dynamics

Dynamic Implementation of Optimal Static Experiment

- Fix $p_0 < \hat{p}$.
- replicate KG: dynamic experiment that leads to beliefs 0 and $\hat{p}$
- For example: R-drifting experiment until belief reaches $\hat{p}$.

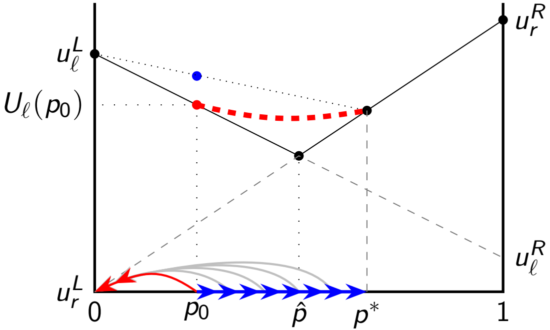


- **Problem:** Receiver does not wait if she does not get rent that compensates for flow cost.

$\Rightarrow$ **KG experiment does not keep receiver engaged.**

Fix: Dynamic Commitment

- *What if a dynamic commitment is possible?*
  - Example: Commit to R-drifting until the belief reaches $p^* > \hat{p}$.



- Similar to KG except to offer “rents” to compensate for Receiver’s flow cost.
- But will this work without commitment?

Is Persuasion Possible Without Commitment?

Persuasion Failure

- There is an MPE with total persuasion failure regardless of $c > 0$.

Persuasion

- Some dynamic commitment can be supported in MPE if cost is low enough.
- As $c \to 0$, a KG experiment as well as full revelation (and anything in between) is dynamically credible.
  $\Rightarrow$ **Folk Theorem**

Skip: No Persuasion

MPE: Persuasion Failure

Theorem (Persuasion Failure MPE)

For any $c > 0$, there exists a MPE in which no persuasion occurs.

Proof.

MPE strategy profile:

- Receiver never waits—he picks r if $p \geq \hat{p}$ and ℓ for $p < \hat{p}$.
- Sender passes if $p \geq \hat{p}$ (and if $p < \hat{p}$ is very low)
- Sender performs L-drifting experiment with jumps to $\hat{p}$ if $p < \hat{p}$ (not too low).
- Remark: satisfies refinement since sender maximizes flow payoff.

□

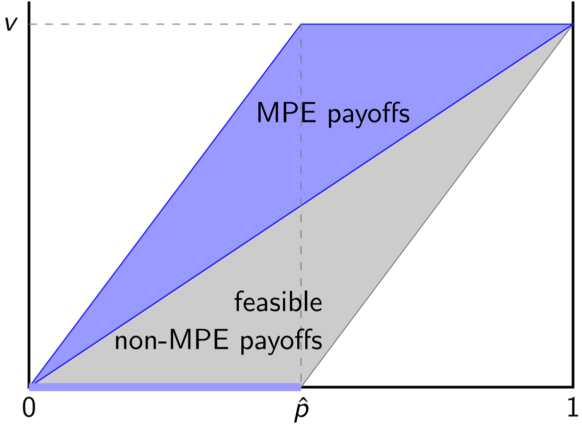
Folk Theorem

Theorem (Folk Theorem)

Any sender payoff between KG benchmark and "full revelation" is supported by an MPE for c sufficiently small.

Any receiver payoff between KG benchmark and "full revelation" is supported by an MPE for c sufficiently small.

Folk Theorem: Sender's Payoffs as $c \to 0$

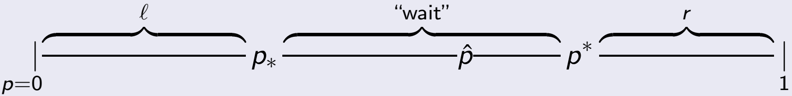


- Feasible payoffs outside "blue region" cannot be supported as MPE as $c \to 0$.

Equilibrium Construction

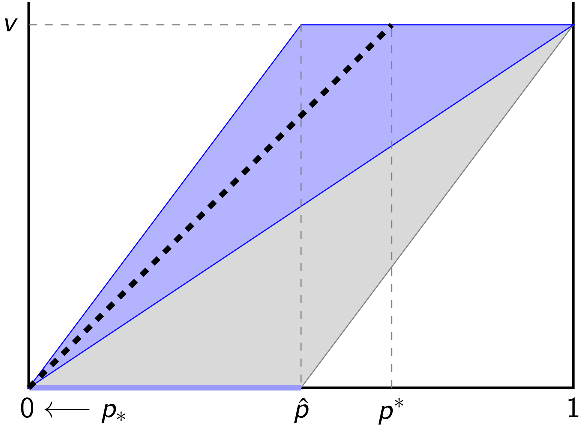
Definition (Simple Markov Perfect Equilibrium — SMPE)

In an SMPE the receiver waits if any only if $p \in (p_*, p^*)$:



▸ Feasible Payoff Vectors

Equilibrium Construction: Preview



- Dashed line: Equilibrium payoffs for fixed p^* as $c \to 0$
- Folk Theorem: Can choose $p^* \searrow \hat{p}$ or $p^* \nearrow 1$ as $c \to 0$

Equilibrium Construction

Construction Depends on Two Conditions

- How demanding is the "persuasion target" p^*?

$$p^* \leq \eta \approx 0.943 \tag{C1}$$

- Who benefits more from information revealed in equilibrium?

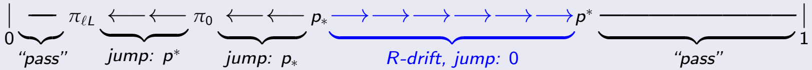
$$v > U_r(p^*) - U_\ell(p^*) \tag{C2}$$

  - Note: (C2) always holds in equilibria that approximate the KG benchmark ($p^* \approx \hat{p}$).

Equilibria when (C1) and (C2) hold

Proposition (Fix $p^* \in (\hat{p}, \eta]$ such that (C2) holds.)

If $c > 0$ is sufficiently small, then there exists a unique SMPE with p^*. The sender's strategy has the following structure:



- $p_* \to 0$ *as* $c \to 0$.

Proof

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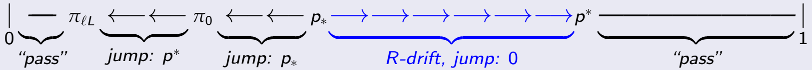

Power of Beliefs provides Incentives for Sender

- What happens if the Sender stops experimenting at $\hat{p}$?
- Receiver believes in continuation sender follows equilibrium.
- Facing "optimistic" receiver, sender does not benefit from deviating.

(reminiscent of Che and Sákovisz, ECMA, 2004)

Equilibria when (C1) and (C2) hold

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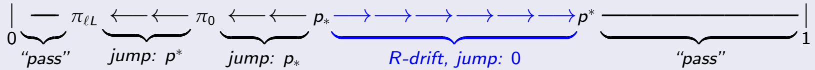

Persuasion Dynamics

- $p \in [p_*, p^*)$: receiver is already interested in listening.
  - $\Rightarrow$ Confidence building, try to rule out state L.
  - $\Rightarrow$ Persuasion backloaded.
- $p < p_*$: Receiver is skeptical

  ▸ Details
  - $\Rightarrow$ Sender is desperate: Throws a "Hail Mary"
  - $\Rightarrow$ Persuasion almost surely fails.

Equilibria when (C1) and (C2) hold

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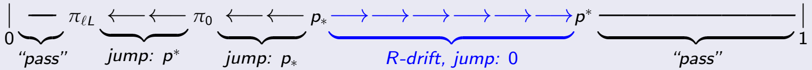


Why not opposite dynamics? (L-drifting until belief reaches p_*)

- Sender "spends confidence" while trying to get breakthrough.
- Leads to stopping at p_* when the confidence becomes too low.
  - Advantage: Avoids costly experimentation if $p \approx p_*$
  - Disadvantage: Lower persuasion probability
- (C2) $\implies$ Receiver stops early (at high p_*):

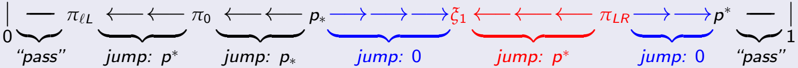
  $\Rightarrow$ Lower persuasion probability outweighs cost advantage.

Equilibria when (C1) fails and (C2) holds

- The persuasion target p^* is now more demanding.

Proposition (Fix any $p^* > \eta$ such that (C2) holds.)

If $c > 0$ is sufficiently small, there exists a unique SMPE with p^*. The sender's strategy has the structure:



At ξ_1: stationary strategy with jump targets $q_- = 0$, $q_+ = p^*$.

- $p_* \to 0$ *as* $c \to 0$
- $\pi_{LR} \to 1$ *and* $\xi_1 \to 1/2$ *as* $p^* \to 1$.

Equilibria when (C1) fails and (C2) holds

Proposition (Fix any $p^* > \eta$ such that (C2) holds.)

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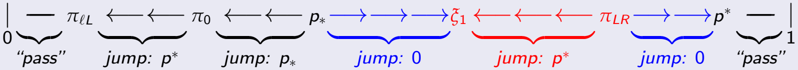


At ξ_1: stationary strategy with jump targets $q_- = 0$, $q_+ = p^*$.

Intuition:

- Sender uses strategy that ...
  - ... leads to "optimal posteriors" 0 and p^* and
  - ... minimizes cost of experimentation.
- Buildup of "confidence" up to p^* takes long time.
- Seeking breakthroughs (jump to p^*) has lower average delay.
- Gradual loss of reputation stops at ξ_1.
- Persuasion less backloaded compared to $p^* < \eta$.

Summary: Main Contributions

1. Introduce sequential information production into Bayesian Persuasion model:
   - Relax commitment power.
   - Power of beliefs allows to sustain persuasion.

2. Folk Theorem yields large set of equilibrium outcomes:
   - Any outcome between KG and full revelation can arise.
   - Despite sender's control over information, sender optimal information structure is not unique outcome.

3. Characterize Persuasion Dynamics.
   - Building confidence vs. spending confidence.
   - Persuasion dynamics depend on type of equilibrium.

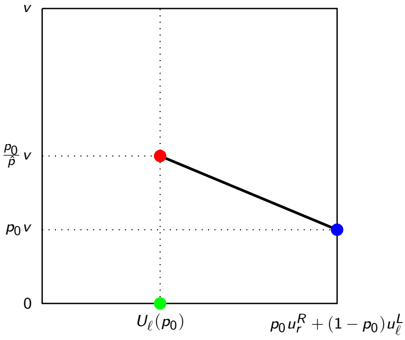
4. Tractable model of dynamic strategic information choice.

Thank you!

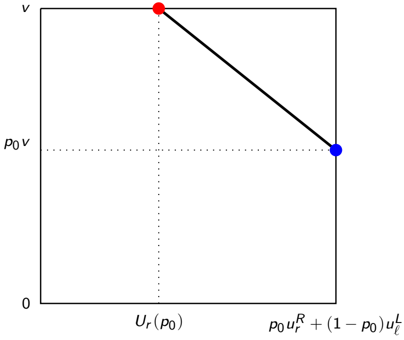
Persuasion MPE: Folk Theorem — Feasible Payoff Vectors

- Consider equilibria with convex persuasion region.
- Feasible payoff vectors in the limit as $c \to 0$ are:

$p_0 < \hat{p}$



$p_0 > \hat{p}$



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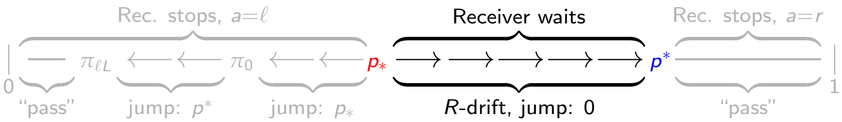
Equilibrium Construction (C1 & C2 hold) — IR

The Persuasion Region: Sender and Receiver Individual Rationality



Determine p_* for given $p^* \in (\hat{p}, \eta)$

- Receiver's utility from R-drifting strategy for $p < p^*$:

$$U_R(p) = \frac{p}{p^*} U_r(p^*) + \left(1 - \frac{p}{p^*}\right) u_\ell^L - C(p, p^*)$$

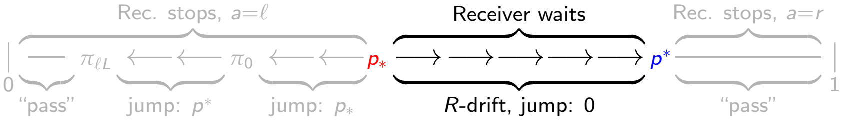
- Derive p_* from indifference condition:

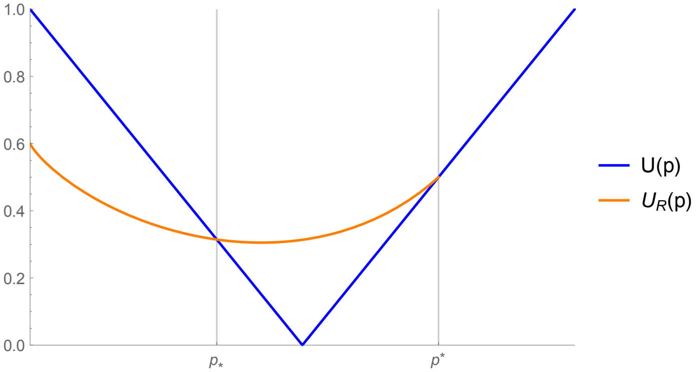
$$U_\ell(p_*) = U_R(p_*)$$

Back

Equilibrium Construction (C1 & C2 hold) — IR

The Persuasion Region: Sender and Receiver Individual Rationality





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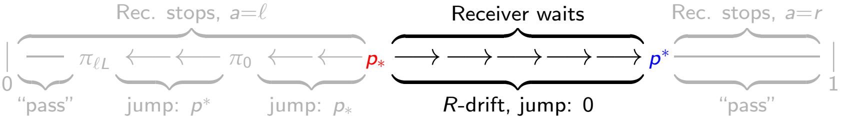
Equilibrium Construction (C1 & C2 hold) — IR

The Persuasion Region: Sender and Receiver Individual Rationality



Lemma (Receiver's Individual Rationality)

The Receiver prefers waiting to stopping for all $p \in [p_*, p^*]$ *if*

$$p^* < \overline{p} = 1 - \frac{1}{u_\ell^L - u_\ell^R} \frac{c}{\lambda}.$$

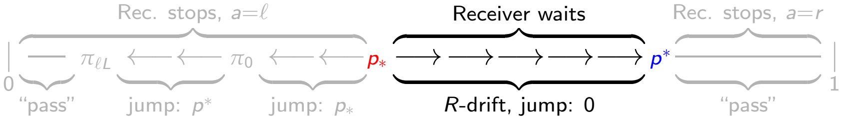
and p_* *is given by the indifference condition* $U_\ell(p_*) = U_R(p_*)$.

- $\overline{p} \to 1$ as $c \to 0$.

Back

Equilibrium Construction (C1 & C2 hold) — IR

The Persuasion Region: Sender and Receiver Individual Rationality



Lemma (Sender's Individual Rationality)

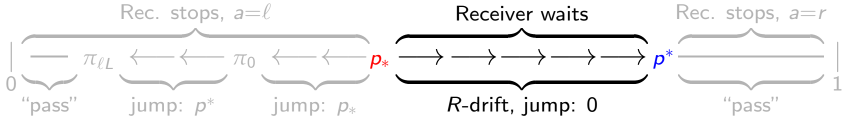
The Sender's payoff is positive for $p \in [p_*, p^*]$, if

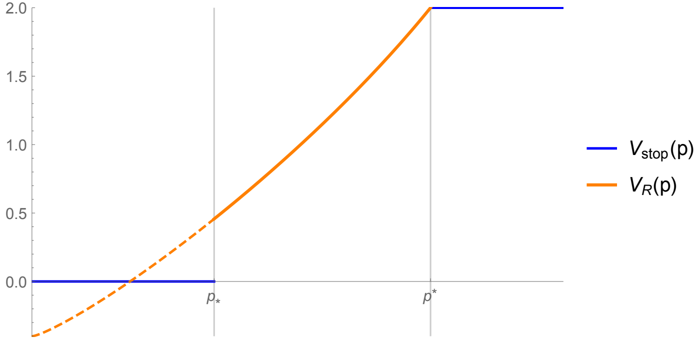
$$v > U_r(p^*) - U_\ell(p^*). \tag{C1}$$

Back

Equilibrium Construction (C1 & C2 hold) — IR

The Persuasion Region: Sender and Receiver Individual Rationality





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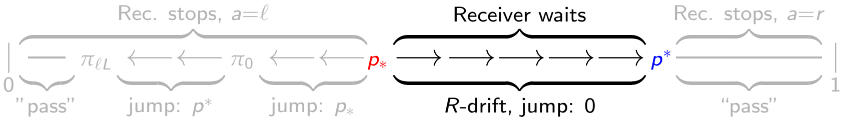
Equilibrium Construction (C1 & C2 hold) — Experiments

The Persuasion Region: Sender's Optimal Experiment



Consider HJB Equation

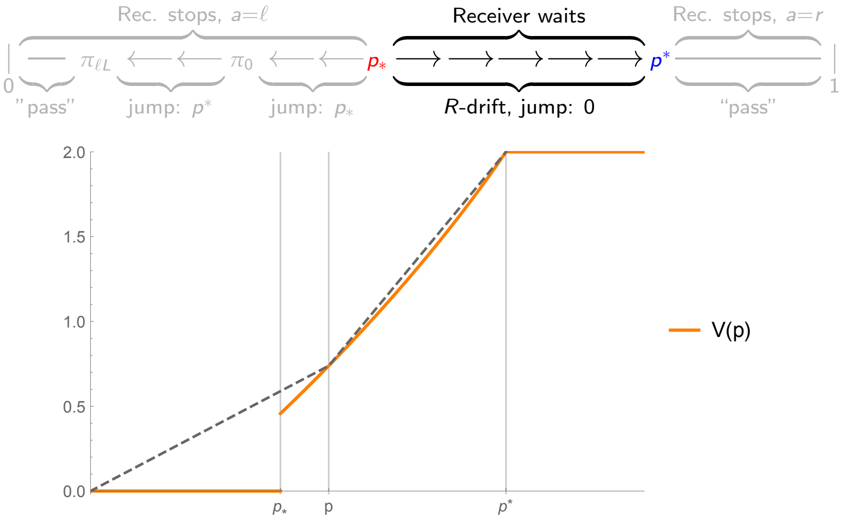
$$c = \max_{\alpha, q_-, q_+} \left\{ \lambda p(1-p) \left[\alpha \frac{V(q_+) - V(p)}{|q_+ - p|} + (1-\alpha) \frac{V(q_-) - V(p)}{|q_- - p|} - (2\alpha - 1) V'(p) \right] \right\}$$

- Optimal downward jump minimizes $\frac{V(p) - V(q_-)}{p - q_-}$
- Optimal upward jump maximizes $\frac{V(q_+) - V(p)}{q_+ - p}$
- Optimal jumps are $q_- = 0$ and $q_+ = p^*$.

Back

Equilibrium Construction (C1 & C2 hold) — Experiments

The Persuasion Region: Sender's Optimal Experiment



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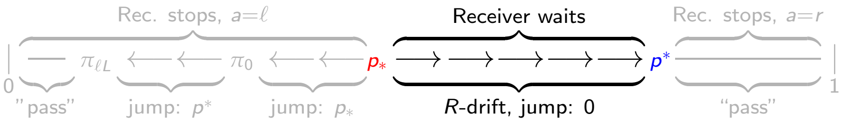
Equilibrium Construction (C1 & C2 hold) — Experiments

The Persuasion Region: Sender's Optimal Experiment



- Let $V_S(p)$ denote the sender's value of the stationary strategy with jumps to zero and p^*.

Lemma (Unimprovability)

If $V_R(p) \geq V_S(p)$, then $V_R(p)$ satisfies the (HJB) equation.

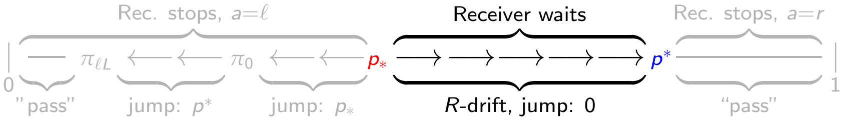
Lemma (R-drifting Experiment is optimal if p^* not too high)

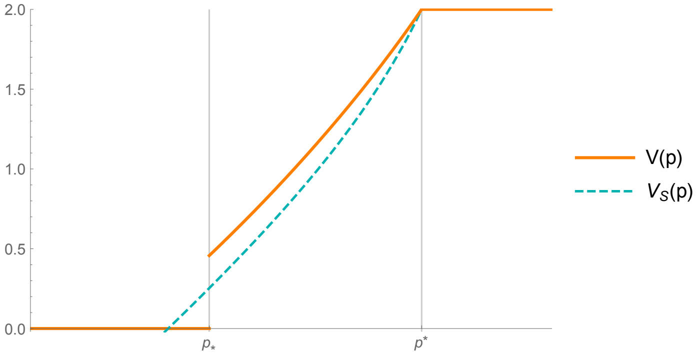
If $p^* < \eta \approx 0.943$, then $V_R(p) > V_S(p)$ for all $p < p^*$.

Back

Equilibrium Construction (C1 & C2 hold) — Experiments

The Persuasion Region: Sender's Optimal Experiment





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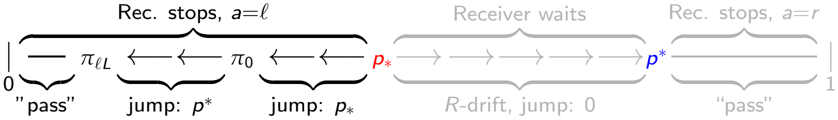
Equilibrium Construction (C1 & C2 hold) — Stopping

The Stopping Region: Receiver must have incentive to stop



Construction of Sender's strategy

- Sender knows receiver will stop: **throws "Hail Mary"**.
- Optimal upward jump maximizes $\frac{V(q_+)-V(p)}{q_+-p}$.
- $p > \pi_0$: Jump to p_*
- $p < \pi_0$: Jump to p^*
- Will the receiver stop?

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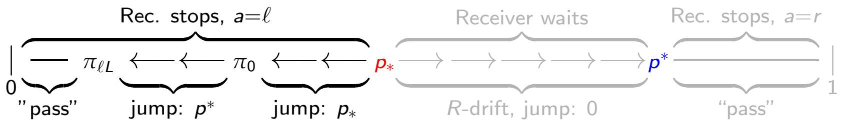

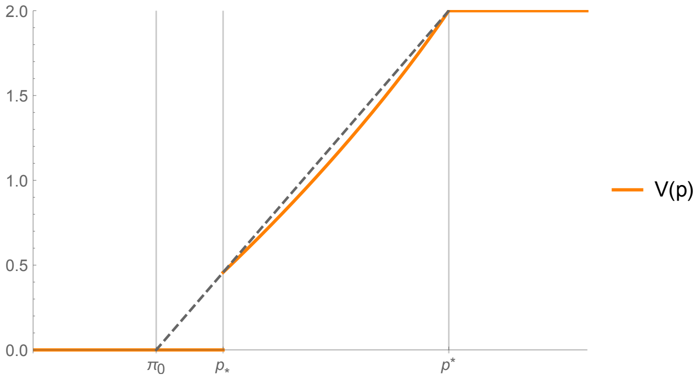

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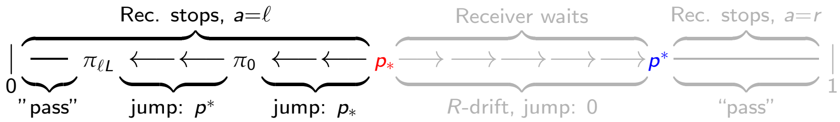

Receiver's incentives for $p \in (\pi_0, p_*)$

- Waiting yields strictly lower payoff than stopping since $U_\ell(p_*) = U_R(p_*)$
- We see: Definition of p_* is crucial for Receiver's incentives.

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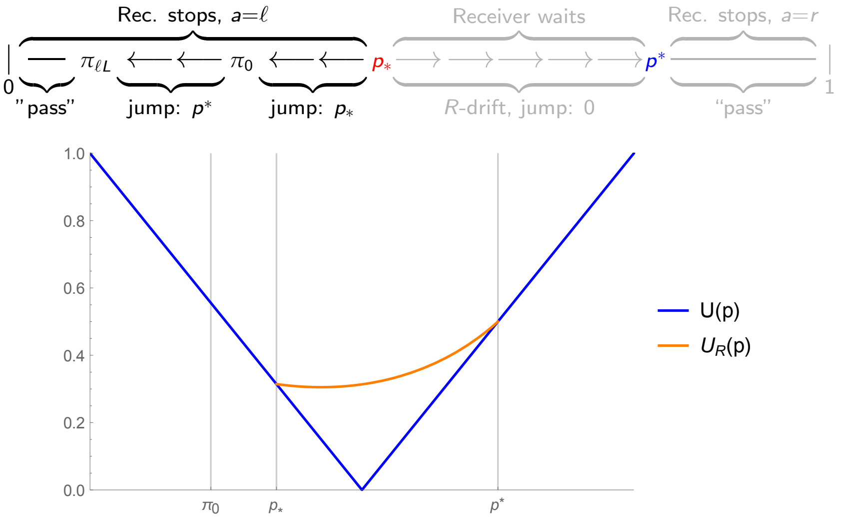

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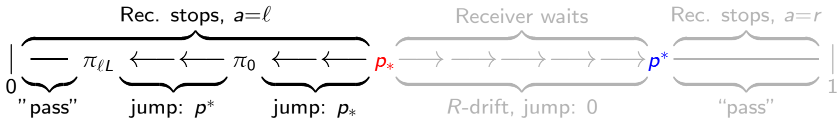

Lemma (Receiver's incentives for $p \leq \pi_0$)

If $v > U_r(p^*) - U_\ell(p^*)$ and c sufficiently small, then the receiver has no incentive to wait for any $p < \pi_0$

Intuition:

- $c \to 0$ implies $\pi_0 \to 0$
- For $c \to 0$, and $p \to 0$, the sender's value (of "hail mary") is zero.
- If $v > U_r(p^*) - U_\ell(p^*)$: receiver values "hail mary" less than the sender.

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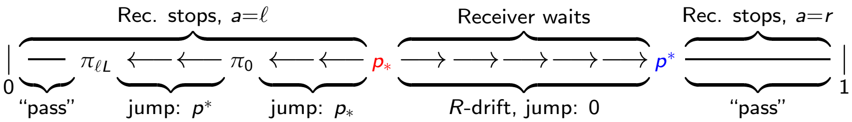
Equilibrium Construction (C1 & C2 hold) — Summary



Summary and Limit as $c \to 0$.

- Equilibrium exists if $p^* < \eta$ and c sufficiently small.
- As $c \to 0$, $p_* \to 0$.
- Payoffs converge to $\frac{p_0}{p^*} v$ and $\frac{p_0}{p^*} U_r(p^*) + \left(1 - \frac{p_0}{p^*}\right) u_\ell^L$.
- Can pick sequence $p^* \to \hat{p}$ as $c \to 0$.
- This concludes the proof for approximation of KG payoffs

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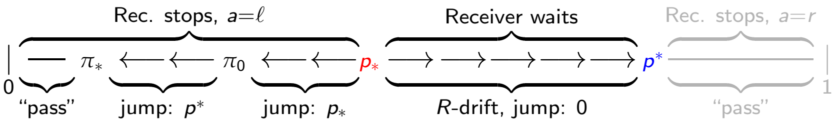
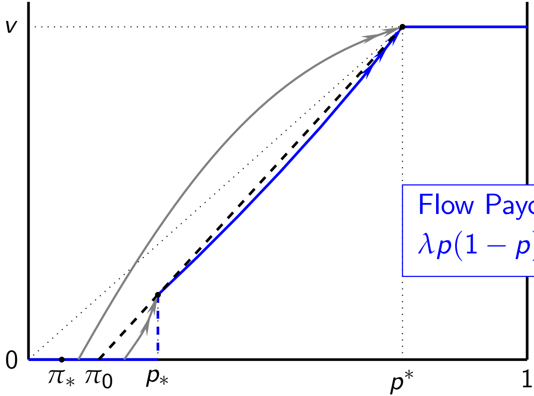
Sender Incentive





Flow Payoff:
$\lambda p(1-p)\frac{V(q)-0}{q-p} - c$

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