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CHANGES IN THE DISTRIBUTION OF MALE AND FEMALE WAGES ACCOUNTING FOR EMPLOYMENT COMPOSITION USING BOUNDS

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This paper examines changes in the distribution of wages using bounds to allow for the impact of nonrandom selection into work. We show that worst case bounds can be informative. However, because employment rates in the United Kingdom are often low, they are not informative about changes in educational or gender wage differentials. Thus we explore ways to tighten these bounds using restrictions motivated from economic theory. With these assumptions, we find convincing evidence of an increase in inequality within education groups, changes in educational differentials, and increases in the relative wages of women.

KEYWORDS: Inequality, earnings, selection, bounds.

1. INTRODUCTION AND MOTIVATION

ECONOMIES SUCH AS THE United States and the United Kingdom have seen large and unprecedented increases in wage inequality among workers over the last 30–40 years. This is illustrated in Figure 1, where we show the way that the interquartile ranges of male and female log hourly wages have evolved for those who work in the United Kingdom. These increases in inequality have been associated with increased returns to education, cohort effects, and increases in the returns to unobserved skill.[2] A variety of interpretations have been given as to why these events have occurred, including skill biased technical change, globalization induced increase in competition among low skill workers, and changes in the supply of graduates. Gosling, Machin, and Meghir (2000) showed that the increases in the United Kingdom can be attributed to permanent differences across cohorts and to changes in the returns to educa-

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[2]See Juhn, Murphy, and Pierce (1993) for the United States, and Gosling, Machin, and Meghir (2000) for the United Kingdom.

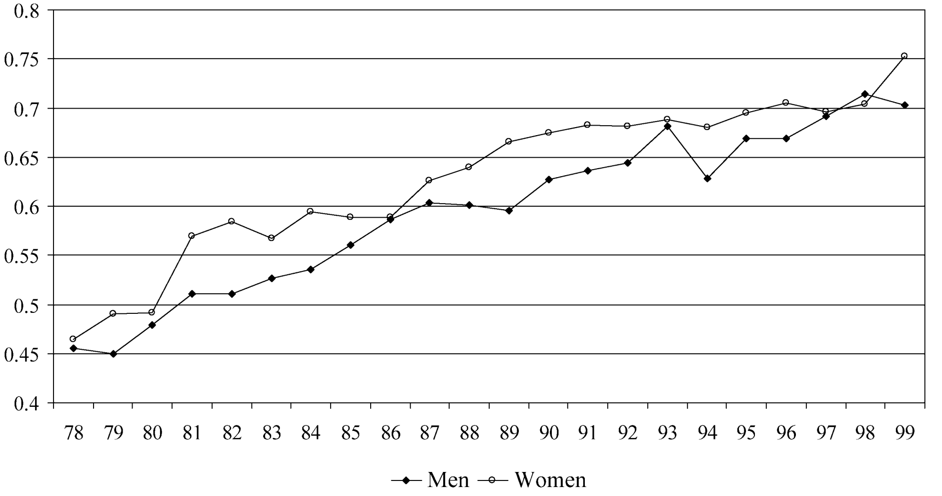


FIGURE 1.—Interquartile range of log wages.

tion. At the same time the gap between the wages of working men and working women has narrowed.[3]

In parallel with these momentous changes in the distribution of observed wages, employment rates for males and females and the composition of the work force have changed. Employment for men aged 22–59 has decreased from 93% in 1978 to 83% in 2000. This decline is not confined to older men and reflects more than the increase in early retirement. Women on the other hand, especially those below 30, saw an increase in their employment. In Figure 2 we show the age profile for employment for 1978 and for 2000 by gender.

The decline in employment since the late 1970s has its sources in a number of possible causes that interact with each other. The large demand shock of the early 1980s combined with the welfare system and the wage setting institutions at the time to cause persistent unemployment (see Jackman, Layard, and Nickell (1991)). A mechanism that propagated such persistence and related to older individuals was the steady increase in those who received sickness and disability benefits during the 1980s, from which there is little incentive to drop out and return to work (see Disney and Webb (1991)). Blundell, Reed, and Stoker (2003) also emphasized the role of the reforms with respect to the housing market and, in particular, the change since 1983 in rent setting for the public sector, leading to a large and steady increase in rents over the 1980s and 1990s. Rents for those with low incomes are subsidized through a housing benefit, which carries an implicit tax rate on earnings of 95% when combined with other welfare

[3]See Harkness (1996) for the United Kingdom and Blau (1998), and Blau and Kahn (1997) for the United States.

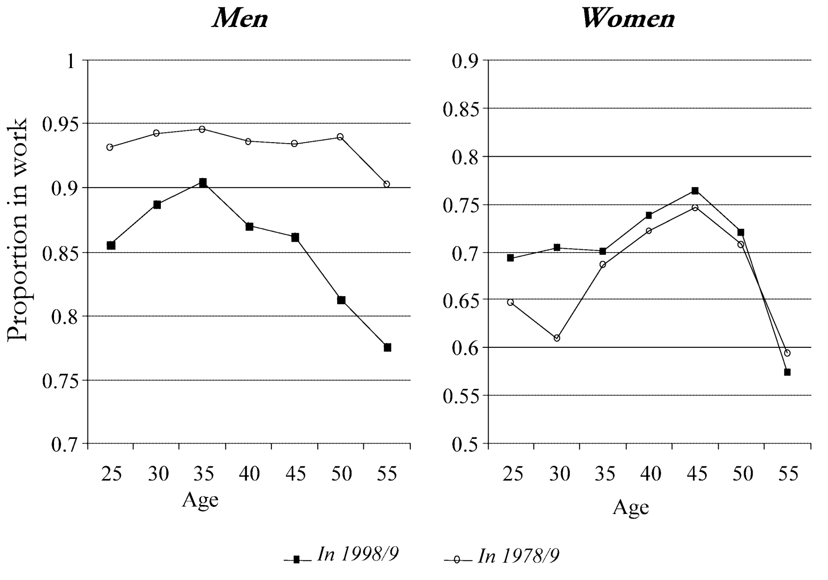


FIGURE 2.—Employment by age in 1978 and 1998.

programs.[4] The reform inadvertently led to a steady increase over time in the range of earnings over which some groups of people faced high marginal tax rates. This would have affected both the level and the composition of participants.[5] These institutional changes affected older cohorts disproportionately because they were overrepresented in public sector housing, partly explaining the more rapid decline in employment for older people. The preceding factors affected older and unskilled individuals more than the rest, although not exclusively, because out-of-work benefit income increased for all.

Figure 3 illustrates that the change in employment has been heavily skill biased. In this paper we define skill by three education groups: those leaving full time education at or before 16 (statutory schooling), those who completed education sometime between 17 and 18 (high school graduates), and those who completed full time education after 18 (some college). For women most of the increase in employment can be accounted for by the increase in employment of women with more than the minimal level of education. The employment rate of the statutory schooling group has shown a slight decline over the entire time period. Blundell, Duncan, and Meghir (1988) documented changes in the incentive structure. Moreover, it is possible that the gradual implementation and enforcement of antidiscrimination practices, formally introduced in the

[4]Housing benefit on its own has a marginal withdrawal rate of 65%.

[5]Those participants who have children and those in high rent areas have experienced large relative increases in out-of-work benefit income.

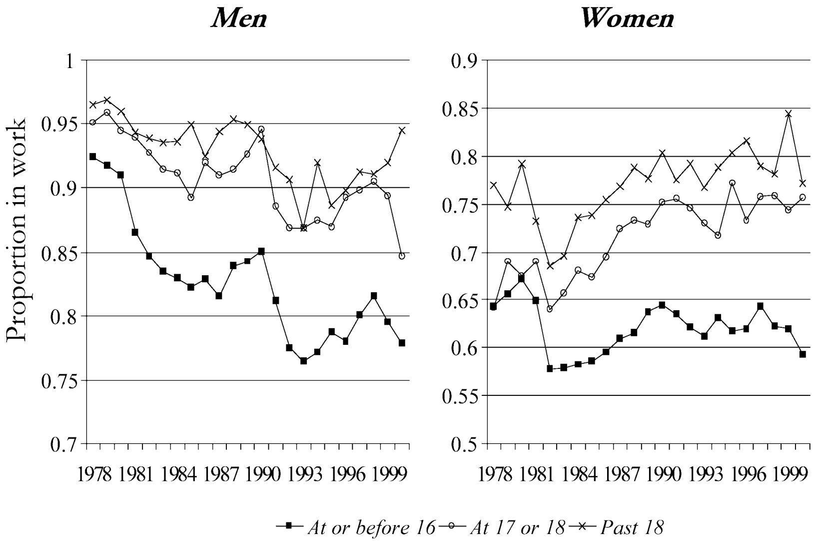


FIGURE 3.—Employment by education level for men and women over time.

1970s, may have made career progression for skilled women more of a reality over this time period (see Stewart and Greenhalgh (1984)).

As the composition of the work force changes, so will in general the observed distribution of wages for workers, whether the aggregate participation rate changes or not. This obscures the changes in the actual/uncensored distribution of wages, preventing us from understanding the nature of the change in inequality and the associated changes in educational, age, and gender wage differentials. A recent example that shows how important such selection issues can be for wages is the paper by Blundell, Reed, and Stoker (2003).

Selection effects have been central to labor economics ever since the pioneering work of Gronau (1974) and Heckman (1974, 1979). Approaches are either parametric[6] or semi/nonparametric.[7] Unfortunately, structural economic models fall short of delivering the assumptions required for identification. Taking a stand on the labor market paradigm is generally not sufficient to point-identify the underlying distribution of wages without further stronger assumptions, see Heckman and Honoré (1990). The theoretical restrictions can, however, deliver informative *bounds* to the distribution of wages. Thus our approach is to exploit restrictions that can be justified from an economic perspective to bound changes in within- and between-group inequality, including changes in educational and gender wage differentials.

[6]See Heckman (1979) or Heckman and Sedlacek (1985).

[7]See, for example, Powell (1987), Newey (1988), Das, Newey, and Vella (2003), Heckman (1990), Choi (1990), Ichimura and Lee (1991), Coslett (1991), and Ahn and Powell (1993).

This paper has two objectives. The first is methodological: to develop bounds for the distribution of wages, which is censored by nonparticipation, using restrictions motivated by theory and building on the existing literature on bounds; in particular, on the papers by Manski (1994) and Manski and Pepper (2000). We introduce a restrictions imposing positive selection into work; this is expressed as first-order stochastic dominance of the distribution of wages of nonworkers by that of workers or as a weaker version that imposes that the median wage of workers is higher than the median wage of nonworkers. We also derive sharp bounds for the interquartile range, which is our measure of inequality. Furthermore, along the lines of using additional information, we also explore the use of exclusion restrictions, where some variables are assumed to affect labor force participation but not the distribution of wages, as well as a much weaker version, the monotonicity restriction. In this case we allow for the instrument to affect wages as well as participation, with the restriction that its impact on the distribution of wages is monotonic, and we show how this restriction can be used to tighten the bounds to the distribution.

Some of the restrictions we impose have testable implications, because when they fail they can lead to the bounds crossing. We develop tests for the null hypothesis that the bounds do not cross. We use a combination of the parametric and nonparametric bootstrap to derive critical values for our tests, as well as to estimate confidence intervals for parameters of interest. We present Monte Carlo evidence that indicates that our tests are powerful for reasonable sample sizes. Our work is related to the now growing literature on inference for partially identified models. (See Imbens and Manski (2004) or Chernozhukov, Hong, and Tamer (2004), for example.)

The second objective of this paper is a substantive empirical analysis of the changes in the wage distribution. To characterize the changes, we provide bounds to changes in overall inequality and in within-group inequality as measured by the interquartile range of log real wages. We then provide bounds on how educational, age, and gender wage differentials have changed. The key distinctive characteristic of these results is that we allow for the effects of selection and we provide alternative bounds with weaker or stronger restrictions. Our main data source is the UK Family Expenditure Survey from 1978 to 2000. We also use the British Household Panel Survey to provide circumstantial evidence on positive selection into the labor market.

Wages is not the only area where bounds and restricted bounds find applicability. Growth in the use of bounds in industrial organization is reflected in the work of Berry and Tamer (2005) and Pakes, Porter, Ho, and Ishii (2006), who also developed results on inference in set-identified models. Bajari, Benkard, and Levin (2007) developed estimation methods for dynamic models of competition, in which point identification often fails but set identification leads to useful results. Honoré and Lleras-Muney (2006) used restrictions on bounds to identify a competing risks model in the context of cancer research. Heckman and Vytlacil (2005) as well as Shaikh and Vytlacil (2005) used restrictions on bounds to identify treatment effects. Honoré and Tamer (2006) examined the identifiability of dynamic discrete choice models and showed how parameters that may not be point-identified can be bounded tightly. Finally, Angrist, Bettinger, and Kremer (2004) used bounds to identify the effect of school vouchers on test scores, where the selection problem arises because not all treated pupils opt to take the test, while Lee (2004) employed the Imbens and Angrist (1994) monotonicity restriction to provide bounds on treatment effects in the absence of exclusion restrictions.

Our work also relates to a number of other papers, including Koenker and Bassett (1978), who developed the use of quantiles; Buchinsky (1994, 1998) and Gosling, Machin, and Meghir (2000), who used quantiles to analyze the distribution of wages; Heckman, Smith, and Clements (1997) and Heckman, LaLonde, and Smith (1999), who investigated bounds for the joint distribution when only the marginals are identified; and Heckman and Vytlacil (2001, 2005), who considered the use of instruments combined with index restrictions. Our approach allows us to obtain very tight bounds on parameters of interest and, in some cases, close to point estimates, while preserving the robustness of a nonparametric approach and imposing only relatively weak and transparent restrictions.

In the next section we show how we derive bounds for the censored wage distribution when restrictions are available. We complete the discussion on bounds by discussing a wage determination model that could be used to interpret our results and we explain the identification issue that led us to use bounds rather than attempt to point-estimate the wage distribution. We then discuss estimation of these bounds, as well as relevant specification tests. Finally, we discuss the data and our empirical results.

2. WORST CASE BOUNDS AND BOUNDS WITH RESTRICTIONS

Let W and X denote the random variable and the conditioning vector, respectively. In our case the dependent variable W should be taken to be the log wage and X should be understood to include gender, age, education, and year. When W is observed, the indicator variable E equals 1 and when W is not observed, E equals 0. In our case E indicates whether the person is employed. The probability of $E = 1$ given $X = x$ is written as $P(x)$. In our analysis this is the employment probability for individuals with characteristics x. We write the cumulative distribution function (CDF) of W given $X = x$ by $F(w|x)$, that given $X = x$ and $E = 1$ by $F(w|x, E = 1)$, and that given $X = x$ and $E = 0$ by $F(w|x, E = 0)$. Although $F(w|x)$, the object of interest, is not identified (because of nonrandom sample selection), we can write

$$F(w|x) = F(w|x, E = 1)P(x) + F(w|x, E = 0)[1 - P(x)]. \tag{1}$$

Given that the data identify both $F(w|x, E = 1)$ and $P(x)$, the problem can be respecified as one in which only $F(w|x, E = 0)$ is unknown. In our application this should be understood as the distribution of wages rejected by those who do not take up employment and $F(w|x)$ is an equilibrium distribution at a point in time that is not invariant to changes in $P(x)$.

Our starting point for the analysis is the work by Manski (1994), who noted that once the inequality

$$0 \leq F(w|x, E=0) \leq 1$$

is substituted into Equation (1), the bounds to the cumulative distribution function become

$$F(w|x, E=1)P(x) \leq F(w|x) \leq F(w|x, E=1)P(x) + [1 - P(x)]. \tag{2}$$

The bounds can then be translated to give the worst case bounds on the conditional quantiles (Manski (1994, Proposition 3, p. 152)). Denoting by $w^q(x)$ the qth quantile of $F(w|x)$, we have

$$w^{q(\mathrm{l})}(x) \leq w^q(x) \leq w^{q(\mathrm{u})}(x),$$

where $w^{q(\mathrm{l})}(x)$ is the lower bound and $w^{q(\mathrm{u})}(x)$ is the upper bound that, respectively, solve the equations

$$q = F(w|x, E=1)P(x) + [1 - P(x)] \quad \text{and} \quad q = F(w|x, E=1)P(x) \tag{3}$$

with respect to w.[8] Unless there are restrictions on the support of W, we can only identify the lower bound to quantiles $q \geq 1 - P(x)$ and upper bounds for quantiles $q \leq P(x)$. In general, we cannot identify bounds to means and variances or higher order moments unless we impose further restrictions. We thus focus on quantiles to characterize the bounds to the distribution.

2.1. *Imposing Restrictions to Tighten the Bounds*

A higher $P(x)$ implies tighter bounds on quantiles. With the employment rates observed in our data, the worst case bounds can be informative about certain aspects of wages, such as life-cycle wage growth for men. However, in many cases they are uninformative. We seek to tighten the bounds on quantiles by imposing restrictions motivated by theoretical considerations about the employment process.

2.1.1. *Stochastic dominance and the median restriction*

In the standard labor supply model individuals with higher wages will be more likely to work unless the difference between wages and reservation wages

[8]The approach of Brown (1984) is quite similar to this, but it is less general. Essentially Brown compared the evolution of the observed median with the worst case lower bound.

is negatively associated with wages.[9] We express such positive selection into the labor market by assuming that the wages of those observed working will first-order stochastically dominate those of the nonworkers. More formally, when $0 < P(x) < 1$, we assume that

$$F(w|x, E=1) \leq F(w|x, E=0) \quad \forall w, \forall x \tag{4}$$

for each w with $0 < F(w|x) < 1$ or, equivalently,

$$\Pr(E=1|W \leq w, x) \leq \Pr(E=1|W > w, x)$$

for each w with $0 < F(w|x) < 1$. The equivalence follows from the equality

$$\begin{aligned} &F(w|x, E=1) - F(w|x, E=0) \\ &\quad = \big\{F(w|x)(1-F(w|x)) \\ &\qquad \times (\Pr(E=1|W \leq w, x) - \Pr(E=1|W > w, x))\big\} \\ &\qquad \times \big[P(x)(1-P(x))\big]^{-1}, \end{aligned}$$

which is a consequence of the Bayes rule. Under this assumption the bounds to the distribution of wages become

$$F(w|x, E=1) \leq F(w|x) \leq F(w|x, E=1)P(x) + [1-P(x)]. \tag{5}$$

For some groups there may be a strong enough positive relationship between wages and reservation wages to undermine the stochastic dominance assumption. This could be induced in part by the fact that we do not condition on assets in the probability of participation and these are likely to be positively related to past earnings. Whereas asset income is more likely to be an issue for those who have very high wages, we also consider a weaker restriction, namely that for individuals with observed characteristic x, the median wage offer for those not working is not higher than the median observed wage $w^{50(E=1)}(x)$. This implies that

$$\begin{aligned} &0 \leq F(w|x, E=0) \leq 1, \quad \text{if } w < w^{50(E=1)}(x), \\ &0.5 \leq F(w|x, E=0) \leq 1, \quad \text{if } w \geq w^{50(E=1)}(x). \end{aligned} \tag{6}$$

This is equivalent to assuming that the probability of observing someone working who has a wage above the median is higher than if their wage is below the

[9]We can expect individuals with higher preference for work and low reservation wages to have invested more in human capital in the past and thus to end up with higher wages.

median, conditional on x. The restriction implied by Equation (6), which we call the *median restriction*, sets the bounds as

$$
\begin{aligned}
&F(w|x, E=1)P(x) \\
&\quad \leq F(w|x) \\
&\quad \leq F(w|x, E=1)P(x) + (1 - P(x)) \quad (\text{if } w < w^{50(E=1)}(x)) \\
&F(w|x, E=1)P(x) + 0.5(1 - P(x)) \\
&\quad \leq F(w|x) \\
&\quad \leq F(w|x, E=1)P(x) + (1 - P(x)) \quad (\text{if } w \geq w^{50(E=1)}(x)).
\end{aligned}
\tag{7}
$$

As Equation (7) shows, this restriction provides tighter bounds to every quantile at the median and above.

Although weaker than the first-order stochastic dominance assumption, the median restriction is not incontrovertible and may fail for some groups; in particular, for women and for older men. First, if high wage women are matched with high wage men, then the out-of-work income of women could be increasing their potential earnings and thus be associated with higher reservation wages (see Neal (2004)). Second, as more skilled women tend to delay rather than avoid childbirth, those in the older groups who have preschool children (and hence are less likely to work) could be the higher wage women. Last, older people who have high enough productivity may have saved sufficiently to retire early. If there are enough such individuals above the median in each of these groups, they could lead to violations of the assumption for that group. In the empirical section we use panel data to examine this assumption. Now we turn to restrictions that do not impose positive selection into the labor market.

2.1.2. *Using determinants of employment to tighten the bounds*

An exclusion restriction: Manski (1994) showed that if W is independent of Z conditional on X, that is,

$$F(w|x, z) = F(w|x) \quad \forall w, x, z,$$

then the bounds to the conditional distribution of W given X are given by

$$
\begin{aligned}
&\max_z \{F(w|x, z, E=1)P(x, z)\} \\
&\quad \leq F(w|x) \\
&\quad \leq \min_z \{F(w|x, z, E=1)P(x, z) + 1 - P(x, z)\}.
\end{aligned}
\tag{8}
$$

This formula can easily be modified to combine the exclusion restriction with the median restriction by replacing $F(w|x, z, E = 1)P(x, z)$ with the lower bound given in Equation (7).

In general, because $F(w|x, z, E = 1)$ depends on z, finding the minima and the maxima of $P(x, z)$ over z will not identify the tightest bounds. Note that we can rewrite the lower bound $F(w|x, z, E = 1)\Pr(x, z)$ as

$$F(w|x)\Pr(E = 1|W \le w, x, z)$$

when W and Z are independent given X. This implies that the lower bound is maximized at z that maximizes $\Pr(E = 1|W \le w, x, z)$. Analogously, the upper bound is minimized at z that *maximizes* $\Pr(E = 1|W > w, x, z)$. Thus, for the lower bound to be tightened at w, $\Pr(E = 1|W \le w, x, z)$ needs to vary over z and for the upper bound to be tightened at w, $\Pr(E = 1|W > w, x, z)$ needs to vary over z. If neither of these conditions is met, then the exclusion restriction does not help tighten the bounds. When $F(w|x, z) = F(w|x)$, a sufficient condition for the bounds to be tightened is that $P(x, z)$ depends on z.

There is nothing in the definition of the bounds obtained in the preceding text that forces the minimum of the upper bounds to lie above the maximum of the lower bounds if the exclusion restriction is false. Thus, in cases where W and Z are not conditionally independent, it is possible for the bounds to cross and for the upper bound for some values of Z to lie below the lower bounds. Later we construct a test of the null hypothesis that the bounds are equal against the alternative that the upper bound is lower than the lower bound. Rejection of the hypothesis is evidence against the exclusion restriction. However, it is possible that the restriction is false but the bounds do not cross.

Weakening the exclusion restriction—monotonicity: Strong exclusion restrictions of the previously discussed type may not always be credible. Following Manski and Pepper (2000), we might however, be prepared to assume the direction of the relationship between w and z. Thus we now derive bounds under the assumption that the distribution of wages decreases monotonically with the wage, that is,

$$F(w|x, z') \le F(w|x, z) \quad \forall w, x, z, z' \text{ with } z < z'. \tag{9}$$

This means that a higher value of the instrument Z will lead to a distribution of wages that first-order stochastically dominates the distribution of wages with lower values of Z.[10] To exploit this restriction, we can find the tightest bounds over the support of Z and then integrate out Z. For a value of $Z = z_1$, the best lower bound is the largest lower bound over $z \ge z_1$ in the support of Z. This is given by

$$F(w|x, z_1) \ge F^l(w|x, z_1) \equiv \max_{z \ge z_1}\{F(w|x, z, E = 1)P(x, z)\}. \tag{10}$$

[10]The reverse assumption is covered in the analysis because we can choose the sign of Z.

Similarly, we can obtain a best upper bound at $Z = z_1$ by choosing the smallest possible upper bound over the support of Z such that $z \leq z_1$:

$$(11) \qquad F(w|x, z_1) \leq F^{\mathrm{u}}(w|x, z_1) \equiv \min_{z \leq z_1}\{F(w|x, z, E = 1)P(x, z) + 1 - P(x, z)\}.$$

The bounds on the distribution of $F(w|x)$ may then be obtained by integrating over the distribution of Z given $X = x$, that is,

$$(12) \qquad E_Z[F^{\mathrm{l}}(w|x, Z)|x] \leq F(w|x) \leq E_Z[F^{\mathrm{u}}(w|x, Z)|x].$$

For the bounds on the distribution of wages $F(w|x)$ to be tightened using the monotonicity restriction at some value of $W = w$, either the lower or the upper bound has to be increasing over some range of the support of the instrument Z, subject to them not crossing at any value of Z. To interpret what this means, observe that

$$F(w|x, z, E = 1)P(x, z) = F(w|x, z) \Pr(E = 1|W \leq w, x, z)$$

and that $F(w|x, z)$ is *decreasing* in z by the monotonicity assumption. Thus, for the lower bound function to be increasing in z, $\Pr(E = 1|W \leq w, x, z)$ needs to be increasing for some z. Analogously, for the upper bound function to be increasing in z, $\Pr(E = 1|W > w, x, z)$ needs to be *decreasing* for some z.

Neither the exclusion restriction nor the monotonicity imposes positive selection.

2.1.3. *Bounds to within-group inequality*

Let $q_1 < q_2$ with $P(x) < q_1$ and $q_2 < 1 - P(x)$, and denote corresponding quantiles given x by $w^{q_2}(x)$ and $w^{q_1}(x)$. To measure inequality within our framework, we will be estimating bounds to the differences between quantiles:

$$(13) \qquad D(x) = w^{q_2}(x) - w^{q_1}(x).$$

An example is the interquartile range. To obtain the bounds, note that

$$F(w|x, E = 0) = \frac{F(w|x) - P(x)F(w|x, E = 1)}{1 - P(x)}.$$

Because $F(w|x, E = 0)$ is nondecreasing in w, the equality places a restriction on $F(w|x)$: $F(w|x)$ cannot increase slower than $P(x)F(w|x, E = 1)$. This provides the upper bound on $D(x)$.

To be more precise, let $w^{q_1(\mathrm{u})}(x)$ and $w^{q_1(\mathrm{l})}(x)$ be the upper and lower bounds to the q_1th quantile of $F(w|x)$. For any w_0 between $w^{q_1(\mathrm{l})}(x)$ and $w^{q_1(\mathrm{u})}(x)$, $F(w|x)$ with $F(w_0|x) = q_1$ is a candidate CDF. The slowest it can increase is by

$P(x)F(w|x, E = 1)$ and when it does, the implied CDF lies entirely between the bounds. This class of CDFs can be denoted by

$$P(x)[F(w|x, E = 1) - F(w_0|x, E = 1)] + q_1.$$

Any $F(w|x)$ that is "parallel" to $P(x)F(w|x, E = 1)$ must be one of these. The q_2th quantile of this CDF is $F^{-1}(F(w_0|x, E = 1) + (q_2 - q_1)/P(x)|x, E = 1)$ and thus the upper bound of $D(x)$ is

$$\sup_{w_0 \in [w^{q_1(l)}(x), w^{q_1(u)}(x)]} \bigl\{F^{-1}\bigl(F(w_0|x, E = 1) + (q_2 - q_1)/P(x)|x, E = 1\bigr) - w_0\bigr\}.$$

Clearly the lower bound is $\max(0, w^{q_2(\mathrm{l})}(x) - w^{q_1(\mathrm{u})}(x))$.

It turns out that imposing the implication that $F(w|x, E = 0)$ is a CDF on obtaining the bounds makes them considerably tighter in practice.

2.1.4. *Bounding wage differentials between groups and their change over time*

We will present bounds to the difference in median wages across education groups, gender, cohort, and age. In contrast to the case where we bound differences in the quantiles of the same distribution, there is no restriction on the bounds to differences in the quantiles across different values of x. Consider the case with two conditioning variables, education and time. We are interested in $D_t^q = w^q(\mathrm{ed}_1, t) - w^q(\mathrm{ed}_0, t)$. This is given by

$$w^{q(\mathrm{l})}(\mathrm{ed}_1, t) - w^{q(\mathrm{u})}(\mathrm{ed}_0, t) \le D_t^q \le w^{q(\mathrm{u})}(\mathrm{ed}_1, t) - w^{q(\mathrm{l})}(\mathrm{ed}_0, t). \tag{14}$$

Similarly $\Delta D_{ts}^{q(\mathrm{l})}$, the lower bound to $D_t^q - D_s^q$, is given by

$$\{w^{q(\mathrm{l})}(\mathrm{ed}_1, t) - w^{q(\mathrm{u})}(\mathrm{ed}_0, t)\} - \{w^{q(\mathrm{u})}(\mathrm{ed}_1, s) - w^{q(\mathrm{l})}(\mathrm{ed}_0, s)\}$$

and $\Delta D_{ts}^{q(\mathrm{u})}$, the upper bound, is given by

$$\{w^{q(\mathrm{u})}(\mathrm{ed}_1, t) - w^{q(\mathrm{l})}(\mathrm{ed}_0, t)\} - \{w^{q(\mathrm{l})}(\mathrm{ed}_1, s) - w^{q(\mathrm{u})}(\mathrm{ed}_0, s)\}.$$

Thus, even if the bounds to the quantiles are tight, the bounds to differentials will be much larger and those of the *change* in differentials will be larger still. We therefore consider two types of restrictions that will make these bounds narrower.

The first is to assume that observables are independent of unobservables and they are log-additively separable. This is a fairly common assumption made in both parametric and semiparametric selection literature. We show how bounds can be derived under this condition in the Appendix. However, we do not report the results from exploiting this restriction because it leads the bounds to cross in most cases, implying the restriction is invalid.

The second restriction that helps tighten the bounds to the *change* in educational differentials is also on the functional form of log wages. Let $\Delta D_{ts}^{q}(a)$ denote the change in the educational differential between period t and period s at age a. We assume that the change in education differentials over time is the same across age groups within an age range A (e.g., below 40 versus above 40). Thus for a given quantile q, we have

$$\Delta D_{ts}^{q}(a) = \Delta D_{ts}^{q} \quad \forall a \in A. \tag{15}$$

This holds if the age effect on the qth log-wage quantile for the relevant group is additively separable and we thus call Equation (15) the additivity assumption. The bounds to the change in the education differentials are then

$$\max_{a\in A} \Delta D_{ts}^{(\mathrm{l})q}(a) \leq \Delta D_{ts}^{q} \leq \min_{a\in A} \Delta D_{ts}^{(\mathrm{u})q}(a).$$

We also impose this restriction when estimating bounds to the change in gender wage differentials. Deriving the bounds under additivity implies looking for the best bounds across age groups and in this sense is analogous to the bounds derived from independence, which is shown in the Appendix.

3. WAGE DETERMINATION AND SELECTION INTO THE LABOR MARKET

We lay out a framework of wage and employment determination that underlies our interpretation of the results and highlights the identification issues, motivating the use of bounds.

We assume that each individual i in period t possesses an amount of productive human capital $h(s_{it}, a_{it}, x_{it})$ that is determined by schooling s_{it} and unobserved ability a_{it}, which may be time varying, as well as other characteristics such as age and gender, summarized in x_{it}. The rental value of human capital of type s is denoted by p_t^s and is determined in a set of interrelated competitive labor markets for each schooling type s by equating labor supply to labor demand for each skill. Thus the equilibrium wage rate for an individual i in period t is

$$w_{it} = p_t^s h(s_{it}, a_{it}, x_{it}). \tag{16}$$

The object of the empirical analysis is to identify the distribution of wages as determined by (16) and how this changes over time. An identification problem arises because not all individuals work at the equilibrium wage. If one does not work, Equation (16) defines the wage that one would earn if one did work *and nothing else changed*.

Define a reservation wage $w_{it}^{\mathrm{R}} = w^{\mathrm{R}}(s_{it}, e_{it}, z_{it})$, where e_{it} reflects unobserved tastes for work that may be correlated with a_{it}; z_{it} are taste shifter variables that may include x_{it}. Within this framework, all those with $w_{it} > w_{it}^{\mathrm{R}}$ are observed working: whether selection into work is positive or negative with respect to the unobservables will depend on the nature of the joint distribution of (a_{it}, e_{it}) and, in particular, on whether w_{it} and $w_{it} - w_{it}^{\mathrm{R}}$ are positively associated. However, our approach allows for much more general labor supply determination, including the presence of fixed costs and dynamics. In fact we do not need to specify the way the work decision is determined, as long as the labor market equilibrium implies that identical individuals are paid identical wages: the interpretation of our approach relies on the idea that each individual is associated with a wage, which is his or her opportunity cost of time. Within this context, at each point in time there is a well defined distribution of wages, which is censored by the employment process. This is the source of the identification problem we address.

Ever since the seminal papers on censoring and selection bias by Gronau (1974) and Heckman (1974, 1979) there has been an interest in the question of nonparametric identification of endogenously censored distributions. Heckman and Honoré (1990) provided an in-depth analysis of identification in a Roy model that includes the simple selection model. Heckman (1990), Ahn and Powell (1993), and Das, Newey, and Vella (2003) developed further identification results. Typically semiparametric identification is proved based on the existence of a continuously distributed instrument, excludable from the wage distribution, when observables and unobservables are independent. For nonparametric identification, the conditions are stronger and require unbounded support as in Heckman (1990), for example.

In general it is hard to find instruments that satisfy the foregoing conditions and are derived from economic theory. For example, it has been frequently argued that assets or asset income would be an instrument that satisfies the required conditions. However, other than the fact that most low educated and young individuals have zero asset income, it is very hard to argue that it is independent of unobservables that determine wages and participation: individuals who worked hard in the past and are more productive are likely to have more assets. An alternative source of instruments may be policy changes or other major events that induce a change in participation for a well defined "treatment" group. Such instruments are discrete with just few values and do not satisfy the conditions required for point identification.

Thus, our approach was motivated by the fact that the assumptions needed for point identification are not easy to justify in practice. The worst case bounds do not rely on any assumptions other than reference to a labor market paradigm, as in the preceding text. We then consider stochastic dominance and the weaker median restriction. These assumptions express the notion that workers are likely to be more productive than nonworkers. A direct consequence of the standard labor supply model is that for individuals with identical reservation wages, those with a higher wage will be more likely to work. Even with heterogeneous preferences for leisure, positive selection will persist if the unobservables that determine the reservation wage and the wage are not strongly positively correlated. In fact, as we show empirically, there is strong evidence of positive selection.

Results based on the median or stochastic dominance assumption do not necessarily nest the standard selection model because the latter does not impose positive selection into the labor market. We thus consider an exclusion restriction, namely that out-of-work welfare benefit income is independent of the unobservables that determine wages. The exclusion can be motivated with reference to the simple model already presented, where out-of-work income will affect reservation wages but not wages directly. This approach now nests the standard selection models whose nonparametric identification is discussed in the preceding literature and *does not impose positive selection*. To guarantee point identification, we would need even stronger assumptions than the simple exclusion restriction we consider. An important source of variation for the instrument are the policy reforms in the 1980s as outlined in the Introduction and in a subsequent section. However, in practice we find evidence against the exclusion of our instrument from the distribution of wages. This is because of the way the welfare benefits in the United Kingdom depend positively on housing costs.[11] Because individuals with higher earnings are likely to use more expensive housing, they will also be eligible for higher welfare benefit income if they were to be out of work. This creates a positive relationship between wages and our instrument. We thus relax the exclusion restriction and use the weaker monotonicity assumption that allows for the positive relationship between wages and the instrument. Our approach is designed for transparency, and the information content of our assumptions is easy to see when compared to the worst case bounds.

4. ESTIMATION METHOD

Our main focus will be the bounds to the quantiles. To estimate these, we first estimate the bounds to the distribution of wages. We now describe the nonparametric estimation procedure we have used.

The conditioning vector X includes gender, education, age, and time. Estimation of the worst case bounds and the bounds with monotonicity requires estimation of the employment probability and the distribution of wages observed amongst the workers for each possible set of characteristics X. For tractability, we limit the number of cells as follows. We define three education groups: those who finished full time education at the age of 16 (statutory schooling), those who continued until 18 (high school graduates), and those who completed after 18 (at least some college). We limit the number of ages for which we estimate the bounds to the ages of 25, 30, 35, 40, 45, 50, and 55, smoothing over neighboring age groups using a quartic kernel as described subsequently. We also pool years in pairs from 1978/1979 until 1998/2000.[12]

[11] One of the welfare programs is the afore-mentioned housing benefit. This subsidizes rents, whether in public or private housing, and even mortgage repayments for some time.

[12] Because we only have one quarter of data for the year 2000, we pool these individuals with those sampled in 1998 or 1999.

Thus the probability of employment for an individual with characteristics x_k (age, education, gender, and time period) is estimated by

$$\hat{P}(x_k) = \frac{\sum_{i=1}^{N} I(E_i = 1)\kappa_k(x_i)}{\sum_{i=1}^{N} \kappa_k(x_i)},$$

where $I(A)$ is the indicator function that equals 1 whenever A holds and 0 otherwise, and the weights $\kappa_k(x_i)$ are defined by

$$\kappa_k(x_i) = I(\text{year}_i = \text{year}_k)I(\text{ed}_i = \text{ed}_k)I(\text{gender}_i = \text{gender}_k)\mu_k(\text{age}_i)$$

and

$$\mu_k(\text{age}_i) = \left(\frac{\text{age}_i - \text{age}_k}{3} + 1\right)^2 \left(\frac{\text{age}_i - \text{age}_k}{3} - 1\right)^2 \times I(|\text{age}_i - \text{age}_k| < 3).$$

To estimate the empirical distribution of wages for workers, we found it advantageous to allow for some smoothing. Thus the estimator we use is

$$\hat{F}(w|E_i = 1, x_k) = \frac{\sum_{i=1}^{N} \Phi((w - w_i)/h)I(E_i = 1)\kappa_k(x_i)}{\sum_{i=1}^{N} I(E_i = 1)\kappa_k(x_i)},$$

where h is set at one-fifth of the standard deviation of wages.

The next estimation problem, relevant for computing the bounds with exclusion or monotonicity restrictions, is the estimation of the probability of employment and the distribution of wages conditional on the instrument Z, which in our case is the out-of-work income and can be regarded as continuous.

To reduce the computational burden we use the percentile ranks of-out-of work income Z. We then estimate the bounds to the distribution of wages and the probability of employment only at a subset of the percentile ranks—every five percentiles. The weights for estimation are now given by

$$\kappa_k(x_i, z_i) = I(\text{year}_i = \text{year}_k)I(\text{ed}_i = \text{ed}_k) \times I(\text{gender}_i = \text{gender}_k)\mu_k(\text{age}_i)\phi_k(z_i),$$

where

$$\phi_k(z_i) = \left(\frac{z_i - z_k}{0.2} + 1\right)^2 \left(\frac{z_i - z_k}{0.2} - 1\right)^2 I(|z_i - z_k| \leq 0.2).$$

The estimated bounds are then substituted in Equations (8), (10), and (11) to obtain estimates of the bounds under the exclusion and the monotonicity restriction, respectively. In the latter case, we need to integrate over the distribution of the instrument (see (12)) which we do using the empirical distribution of the instrument Z given $X = x$. The bounds to the quantiles are then estimated by solving the equations analogous to those in (3).

We construct confidence intervals for the parameters of interest, namely the quantiles, the differentials across groups, and the changes in the differentials over time using the bootstrap (see subsequent text) and applying the results of Imbens and Manski (2004). These are narrower than the confidence intervals for the estimated identification region itself.

4.1. *Specification Tests and Confidence Intervals*

4.1.1. *Testing that the bounds do not cross*

If the exclusion and monotonicity restrictions are invalid, they may lead to the restricted upper bound being less than or equal to the restricted lower bound, which implies that they do have a testable implication. However, because the bounds may never cross even when the restrictions are invalid, any specification test will not be an asymptotically uniformly powerful test of the restrictions themselves, but rather that the bounds do not cross. Nevertheless, if we reject the hypothesis that the bounds do not cross, then we must also reject the hypothesis about the assumed relationship between our instrument and the distribution of wages.

Even if, in the population, the bounds are equal, implying a point estimate, in any finite sample they will often cross just because of sampling error. Thus we need a formal test for the hypothesis that the lower bound is less than or equal to the upper bound against the alternative that bounds cross. To achieve this we use the following test statistics: Denote the sum over all discrete values of X by $\sum_X$. Then

$$T_E = \sum_X \Bigl[\sum_W \bigl\{ I(\hat{F}^{\mathrm{ub}}(w|x) - \hat{F}^{\mathrm{lb}}(w|x) < 0)(\hat{F}^{\mathrm{ub}}(w|x) - \hat{F}^{\mathrm{lb}}(w|x))^2 \bigr\} \Bigr] \quad \text{for exclusion,} \tag{17}$$

$$T_M = \sum_X \Bigl[\sum_W \Bigl\{ \sum_Z \bigl[I(\hat{F}^{\mathrm{ub}}(w|x,z) - \hat{F}^{\mathrm{lb}}(w|x,z) < 0) \times (\hat{F}^{\mathrm{ub}}(w|x,z) - \hat{F}^{\mathrm{lb}}(w|x,z))^2 \bigr] \Bigr\} \Bigr] \quad \text{for monotonicity,} \tag{18}$$

where $\hat{F}^{\mathrm{ub}}(w|x)$ and $\hat{F}^{\mathrm{lb}}(w|x,z)$ are the estimates of the upper bound under exclusion and under monotonicity, respectively, and similarly for the lower bounds. Thus the test statistic, when suitably normalized, is the average over the distribution of X (and Z for the monotonicity restriction) and over wages of all the squared violations of the bounds.

Deriving and computing the asymptotic distribution of these test statistics is not straightforward. Moreover, because of the max operator involved in estimating the bounds under the exclusion or the monotonicity restriction, it is unclear whether the nonparametric bootstrap is valid. Thus we have opted to use a combination of the parametric and nonparametric bootstrap in the following way:[13] All our estimates and test statistics are functions of the estimates of $P(X)$ and $F(w|X, E=1)$ or $P(X, Z)$ and $F(w|X, Z, E=1)$ for which the regularity conditions for the bootstrap are satisfied and that are asymptotically normal. We thus start by using the nonparametric bootstrap to compute the covariance matrix of these estimates. We assume the estimates are independent conditional on X and Z, but correlated within these cells. We then draw samples of the employment probability and the observed distribution of wages from their asymptotic normal distribution to compute confidence intervals and values of test statistics that depend on them. In particular, to estimate the critical values of the test statistic for the null hypothesis that the bounds do not cross, we recenter the test statistics so that the null is satisfied and thus simulate values for

$$T_{\mathrm{E}}^* = \sum_X \sum_w \left[I(\Delta\tilde{F}(w|x) - \Delta\hat{F}(w|x) < 0)(\Delta\tilde{F}(w|x) - \Delta\hat{F}(w|x))^2 \right] \quad \text{(exclusion)},$$

$$T_{\mathrm{M}}^* = \sum_X \left[\sum_w \left\{ \sum_Z \left[I(\Delta\tilde{F}(w|x, z) - \Delta\hat{F}(w|x, z) < 0) \times (\Delta\tilde{F}(w|x, z) - \Delta\hat{F}(w|x, z))^2 \right] \right\} \right] \quad \text{(monotonicity)},$$

where the tilde (˜) denotes an estimate based on a bootstrap sample and Δ denotes a difference between the upper and lower bounds; for example, $\Delta\hat{F}(w|x) = \hat{F}^{\mathrm{ub}}(w|x) - \hat{F}^{\mathrm{lb}}(w|x)$ and similarly for all other expressions preceded by Δ. From the simulated values of this centered bootstrap test statistic, we can derive the p values of our test.

In this way, we approach this testing problem analogously to testing for the location of multiple means. We have not proved that the preceding centering procedure provides the appropriate critical values for our test. However, in Appendix A we do provide some Monte Carlo evidence that these test statistics have good size and power properties in our context.

[13]We thank Ariel Pakes for suggesting this.

Deriving point estimates when the bounds cross: It is also necessary to derive consistent estimates of the bounds in the case where they cross. Under the null that the difference between the upper and lower bounds is zero, both the upper and lower bound estimates are consistent estimates of the actual quantile of wages we are interested in. Choosing either the estimate of the upper or of the lower bound would give us consistent estimates of this quantile, but a more efficient approach would be to use a weighted combination of the upper and lower bounds, that is,

$$\hat{w}^q = \alpha \hat{w}^{q(\mathrm{u})} + (1-\alpha)\hat{w}^{q(\mathrm{l})},$$

where α is chosen optimally to minimize the asymptotic variance of $\hat{w}^q$ using the estimated asymptotic distribution of $\hat{w}^{q(\mathrm{u})}$ and $\hat{w}^{q(\mathrm{l})}$. We use the bootstrap to estimate the distribution of the upper and lower bounds.

4.1.2. *Testing for the absence of selection effects*

The worst case bounds cannot be informative about whether there is a nonrandom selection problem; they can just tell us the extent to which selection can bias the results. However, when we impose the monotonicity restriction or the exclusion restriction, the bounds to the distribution may not include the observed distribution, which is evidence that selection does bias the results. However, the difference may not be statistically significant and thus we develop a test statistic for the null hypothesis that the observed distribution is within the bounds.

Under the null hypothesis we have that

$$F^{\mathrm{lb}}(w|x) \le F(w|x, E=1) \le F^{\mathrm{ub}}(w|x).$$

The test statistic we use then is

$$\begin{aligned} T_{\mathrm{S}} = \sum_X \sum_W \Big\{ & \max\big[\hat{F}^{\mathrm{lb}}(w|x) - \hat{F}(w|x, E=1), \\ & \qquad \hat{F}(w|x, E=1) - \hat{F}^{\mathrm{ub}}(w|x)\big]^2 \\ & \times I\big(\max\big[\hat{F}^{\mathrm{lb}}(w|x) - \hat{F}(w|x, E=1), \\ & \qquad \hat{F}(w|x, E=1) - \hat{F}^{\mathrm{ub}}(w|x)\big]\big) > 0\Big\}. \end{aligned}$$

This provides a joint test for all the values of x. To obtain the critical values, we again use the bootstrap, after recentering so that the null is satisfied in the sample.

5. RESULTS

The aim of the empirical analysis is to examine the changes in the distribution of wages to account for the possible effects of changes in worker composition. Although a traditional analysis based on point estimates would offer exact statements about the effects of selection, an analysis based on bounds provides ranges of changes that cannot be explained by composition changes or, alternatively, the maximal effects from selection. We characterize the changes in the overall distribution of wages by considering overall inequality, within-group inequality as well as between-group components based on cohort, gender, and educational differentials. Our inequality measure is the interquartile range, which can be bounded. Across groups we compare medians.

5.1. *Data and Variable Definitions*

The data we used for the analysis are the pooled repeated cross sections of the U.K. Family Expenditure Surveys (FES) from 1978 to the first quarter 2000, which contains the period where most of the important changes took place in the United Kingdom. However, contrary to the earlier U.K. studies, we also include women. Thus the sample consists of all men and women between the ages of 23 and 59 who were not in full time education. This gave us a sample of 187,467 individuals in total. Hourly wages, which are the object of the analysis, are defined as usual weekly earnings divided by usual weekly hours (inclusive of overtime) and are deflated by the consumer all items quarterly retail price index. Deflated wages lower than 50p an hour were also treated as missing at random. We defined individuals to be in "work" (i.e., $E=1$) if they reported themselves as being employed, whether full or part time or self employed, over the last week. We treated the self-employed as employed for estimating employment probabilities. However, because wages and hours of work are not reliably measured for this group, we assumed their wages are missing at random and so exclude them from the calculation of $F(w|x, E=1)$.

Constructing out-of-work income: For the models where we use an exclusion or monotonicity restriction, the instrument for employment will be the welfare benefits that the person would be eligible for when out of work. This variable was used before by Blundell, Reed, and Stoker (2003) and is constructed from the IFS[14] tax and welfare-benefit model. More specifically, for singles we use the benefit level for which they would be entitled if they did not work. For married or cohabitating individuals, we take the household benefit level for which they would be eligible if neither worked.

The source of variation for out-of-work income is the demographic composition of the household and the housing costs that the household faces. The latter vary by region and over time due to the numerous policy changes that

[14]IFS, Institute for Fiscal Studies: http://www.ifs.org.uk.

have occurred. These changes include (1) an increase in the allowance for children so that the childless have experienced relative falls in their out-of-work income and (2) a switch in housing policy away from rent ceilings on public housing toward subsidies on the actual rent paid starting in 1983. This policy allowed public housing rents to increase, following market rents, and deregulated market rents finally in 1988. This in turn induced increases in out-of-work benefit income at differential rates across regions, substantially increasing the replacement rates and reducing the incentives to work for those with high levels of housing. Because housing costs and hence housing benefits increased more or less continuously over the period of the data, the replacement rates kept increasing, which can explain at least partly the decline in overall participation, particularly of the unskilled.

We define the instrument as the residual in a regression of out-of-work income on household composition. The resulting variable depends on region of residence and individual household housing costs.[15] Thus, because of the way housing benefit operates, the instrument is likely to be positively associated with wages: Individuals who generally earn more will be in better housing and hence eligible for higher housing benefit when out of work temporarily. Thus the distribution of wages conditional on higher values of benefit income will most likely first-order stochastically dominate the distribution of wages conditional on lower values. Under this assumption, benefit income, although possibly invalid as an exclusion restriction, may satisfy the monotonicity restriction.

The instrument must be related to employment. We test the hypothesis that our measure of out-of-work income does not affect employment conditional on the other observable characteristics (age, time, gender, and education), and we reject both overall and within education and gender cells with a p value of zero in all cases. The distributions of our test statistics under the null are approximated by the bootstrap. As an illustration, in Figure 4 we show the impact of benefit income on the employment probability of 30-year-old women with statutory education. In that case, employment varies from 80% to 50% as benefit income increases from its 8th percentile to its median.

5.2. *The Validity of Our Restrictions*

5.2.1. *The stochastic dominance and median restrictions*

We have argued that the stochastic dominance restriction (4) or the median restriction (6) could be violated, particularly for older individuals, due to asset effects, and for women, due to positive sorting in the labor market.

In Figure 5 we use longitudinal data from the British Household Panel Survey 1991–2001 (BHPS) to show that such effects may not be that important and that positive selection is indeed reasonable. We have regressed log wages for

[15] Removing permanent regional differences does not affect the results.

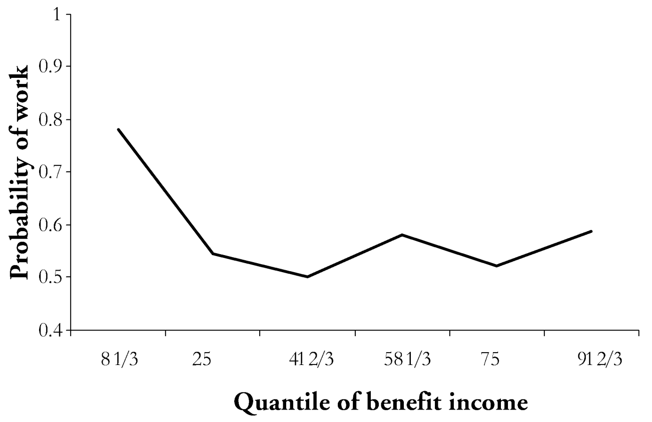


FIGURE 4.—Probability of work and out-of-work income.

each year of the panel separately on age and education, and allocated workers a residual (i.e., actual wage minus predicted wage). We then split the sample

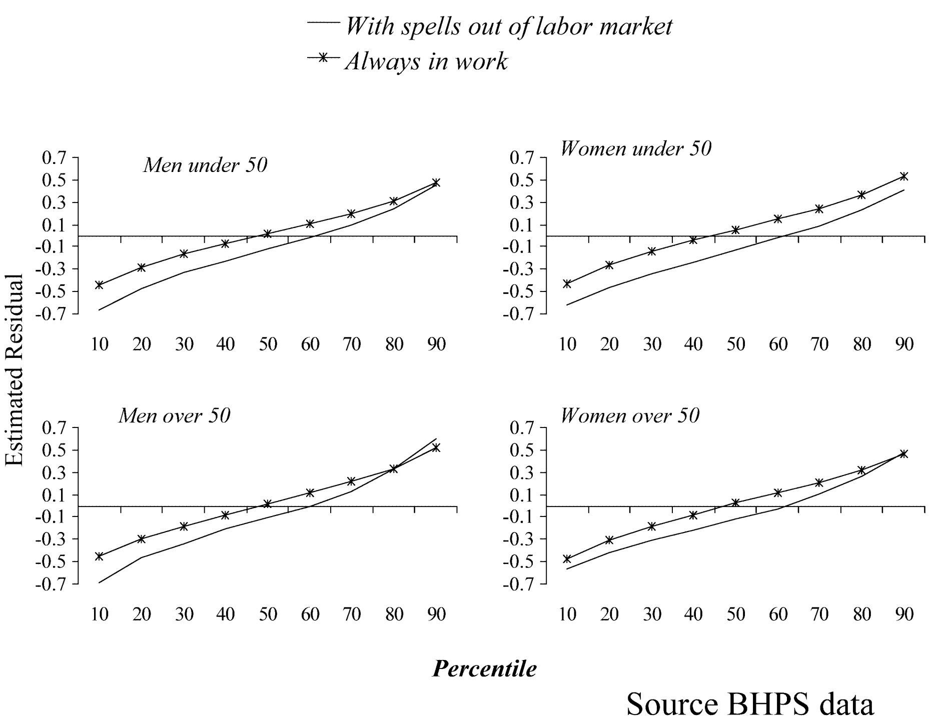


FIGURE 5.—Distributions of residual wages by gender, education, and work histories.

into those who had some employment interruption over the period and those who were continuously employed. Figure 5 shows that the quantiles of the distribution of wages of those continuously in work lie above the quantiles of the distribution for those who have had a work interruption, even controlling for factors such as age and education, which are important determinants of unemployment. The median residual wage for workers is always higher than the median for nonworkers. In fact, this graph provides support for the stochastic dominance assumption. Nevertheless, we are missing individuals who never work, which is approximately 5.63% of men and 11.32% of women.

5.2.2. *The exclusion and monotonicity restriction for out-of-work income*

When we impose the exclusion or the monotonicity restrictions with respect to out-of-work benefit income, the bounds could cross. For women, the bounds never cross and as a result the p values of our tests always imply no restriction can be rejected. This is not surprising given the lower employment rate and the resulting wide worst case bounds of women. For men the results are more conclusive. The exclusion restriction forces the bounds to cross frequently, and this is highly significant both overall and within each education group (in each case the significance levels are always below 2.2%). On the other hand, the monotonicity restriction is never rejected and in each case the p values are larger than 88.5%. When we combine the median with the monotonicity restriction, the lowest p value we obtain is 46%, implying that the combined restrictions do not lead to the bounds crossing in a significant way.

We also tested the independence and additivity restrictions that we consider using to tighten the bounds to the changes in educational and gender wage differentials over time. Independence of education and time is rejected with a p value of zero and we do not present any empirical results based on this. We then tested the hypothesis that the changes in educational differentials for men were independent of age for those below 40 and for those above. The former restriction is acceptable, while the latter is rejected at the 9.5% level. Finally, we repeated these tests for the change in gender wage differentials and the restrictions are accepted easily within both age groups.

5.2.3. *Selection effects*

We find that for most age and education groups the selectivity test fails to reject the hypothesis that the random selection outcome is included in our restricted bounds (median and monotonicity imposed) with one notable exception: the significance level for men over 40 in the statutory schooling group was 2.2%; thus for this group we can reject the hypothesis of no selection effects. For our other groups, the results are not conclusive about selection.

5.3. *Changes in the Distribution of Wages*

We now use our empirical approach to examine the changing distribution of wages in the United Kingdom from 1978 to 2000. When women are concerned we focus only on how the median wage and the educational and gender wage differentials have evolved. In our graphs, when useful we present 95% confidence intervals for the unidentified parameter as in Imbens and Manski (2004). These intervals are constructed using the bootstrap as described earlier. Following estimates reported in the text, we report a standard error in italicized type in parentheses.

5.3.1. *Trends in inequality*

Figure 6 plots the upper and lower bounds on the interquartile range, our inequality measure, from 1978 to 2000 for the male wage distribution.[16] The central line shows, for comparison, what has happened to wage inequality among workers and the dotted lines give 95% confidence intervals. We can only say for certain that inequality has gone up if the lower bound at the end of the period is higher than the upper bound at the beginning of the period. The worst case lower bound in 1998–2000 is higher than the worst case upper bound in

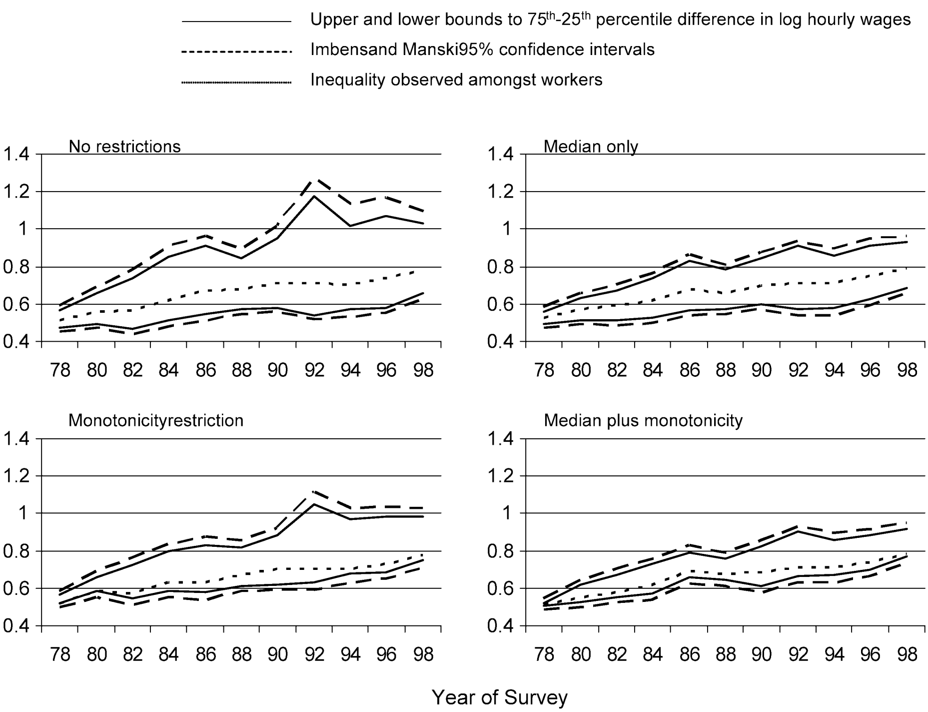


FIGURE 6.—Upper and lower bounds on male wage inequality over time.

[16]Note that we group years in pairs so as to avoid having any empty data cells, particularly for the older cohorts with higher education.

TABLE I

CHANGES TO INEQUALITY OVER TIME[a]

	Lower Bound to the Change in IQR 78–98	95% Confidence Interval
No restrictions	0.089	[0.034, 0.108]
Median restriction	0.127	[0.077, 0.151]
Monotonicity restriction	0.189	[0.162, 0.272]
Median plus monotonicity restrictions	0.252	[0.198, 0.289]
Stochastic dominance	0.185	[0.130, 0.207]
Observed change	0.268	[0.230, 0.294]

[a]IQR denotes interquartile range of log wages. The 95% confidence interval is based on the bootstrap.

1978–1980, suggesting that inequality as measured by the interquartile range must have risen by at least 0.089 log points (*0.02*).[17] This means that selection effects alone cannot explain the rise in inequality observed among workers of 0.252 (*0.022*) points.[18] We then show results using the median restriction, the monotonicity restriction (that does not impose positive selection), and the combination of the median and monotonicity restrictions.

Under the combined median and monotonicity restriction, the interquartile range (IQR) must have risen by at least 0.252 (*0.022*) log points, which is slightly less than the rise of the interquartile range of the distribution of wages for workers (0.268, with a standard error of *0.017*). Thus, once we account for selection, the actual (latent) increase in inequality seems to be at least as large as that observed among workers. If anything, composition changes may have masked some of the increase. Table I summarizes these results and presents p values that demonstrate that inequality must have increased over the sample period.

5.3.2. *Within-group inequality*

A feature of the increase in inequality in Britain (as well as in the United States) has been the large increase in male observed wage dispersion within education and age groups, which authors have attributed to an increase in the importance of unobserved skill (again we focus on men). In Table II we show the lower bound on the increase in within education group inequality together with a 95% confidence interval. The figure presented is the average lower bound to the change in the interquartile range of log wages from 1978

[17]The italicized number presented in parentheses here and subsequently are the standard error of the estimate computed using the bootstrap outlined herein.

[18]As an extra benchmark, we also estimate the increase in the interquartile range while keeping the work force to its original 1978 composition (i.e., controlling for selection on observables). The estimated increase in this case is 0.197.

TABLE II

LOWER BOUND ON THE CHANGE TO THE WITHIN-GROUP INTERQUARTILE RANGE OF LOG WAGES 1978–1998[a]

	All Education Groups	Statutory Schooling	High School Graduates	Some College
No restrictions	0.015	−0.017	0.093	0.112
	[−0.008, 0.051]	[−0.043, 0.031]	[−0.001, 0.173]	[0.030, 0.183]
Median restriction	0.015	−0.017	0.093	0.112
	[−0.002, 0.056]	[−0.037, 0.037]	[0.001, 0.175]	[0.033, 0.185]
Monotonicity restriction	0.088	0.076	0.108	0.140
	[0.081, 0.172]	[0.067, 0.187]	[0.042, 0.220]	[0.069, 0.218]
Median plus monotonicity restriction	0.087	0.076	0.108	0.129
	[0.081, 0.156]	[0.069, 0.160]	[0.034, 0.211]	[0.063, 0.208]
Stochastic dominance	0.071	0.049	0.131	0.142
	[0.054, 0.107]	[0.028, 0.091]	[0.042, 0.206]	[0.070, 0.217]
Observed change	0.169	0.160	0.175	0.203
	[0.143, 0.193]	[0.128, 0.185]	[0.095, 0.253]	[0.132, 0.265]

[a]The 95% confidence intervals for the lower bound of the change in the IQR are given in square brackets.

to 1998. The average is over different age groups and in the first column over education groups as well.[19]

Without imposing any restrictions, we can establish an increase for the entire sample period that is significant for the college group and marginally so for the high school group, but not for the statutory schooling group.[20] However, substantial and significant increases are estimated for all education groups once we impose the monotonicity restriction. The overall picture is shown in Figure 7, where the bounds based on both the monotonicity restriction and the combination of monotonicity and the median restriction are presented. For most periods, imposing positive selection (the median restriction) does not affect these results much relative to using the monotonicity restriction alone. Imposing positive selection seems to matter most toward the end of the sample, where this additional restriction leads to substantially tighter bounds.

[19]When we compare inequality measures across groups, over time, and across restrictions an issue arises because of small cell sizes. For some cells, less than 25% of males are working, implying that the worst case bounds on the top quartile are not defined. Although these are sometimes defined with more restrictive assumptions, for the purposes of comparability, we delete all cells where the inequality measure is undefinded under some model.

[20]The joint test that the lower bound to the IQR did not increase in any age/education group is rejected with a p value of less than 5% even when no restrictions are imposed, despite the fact that the average increase to the lower bound does not seem significant at 5%.

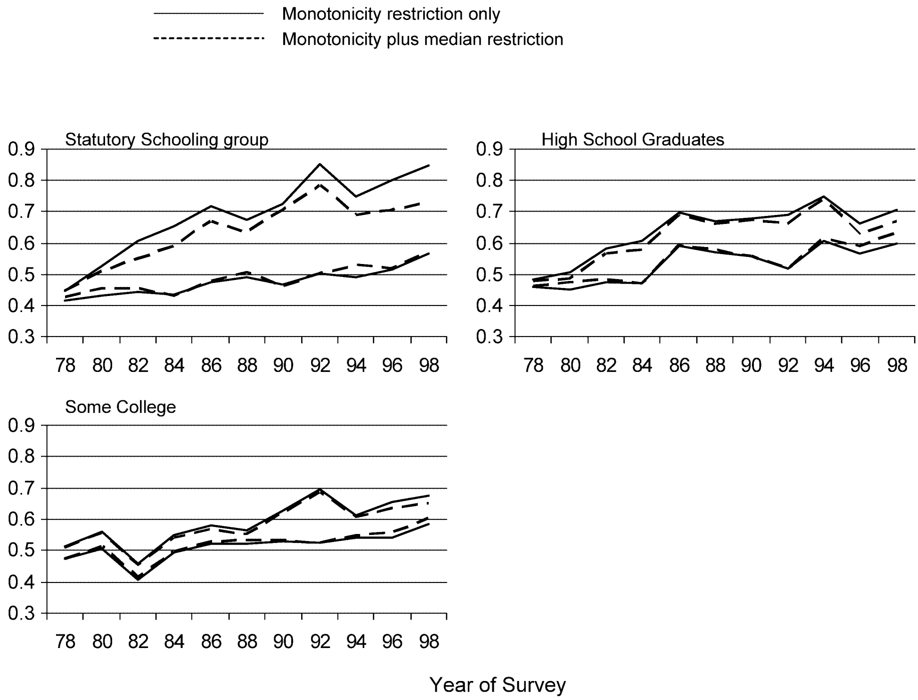


FIGURE 7.—Changes to within-group inequality over time by education group.

5.4. *Life-Cycle Wage Profiles and Intercohort Growth*

Changes in inequality may also be driven by changes in the life-cycle profiles and by the way they relate to intercohort growth of wages. More generally, the study of labor supply, of aging, and of savings and pensions, among other fields, relies on knowledge of wage growth over the life cycle. However, understanding life-cycle growth is fraught with difficulty relative to composition effects induced by selection in and out of work, making bounds particularly useful.

In Figures 8–10 we present bounds to the median wages of each education group by age for five cohorts each born 10 years apart (1925, 1935, 1945, 1955, and 1965). For the oldest cohort only one age point is available. Within each graph we present results based on the worst case bounds, the median restriction, the monotonicity restriction, and the combination of both restrictions. Following these, in Figure 11 we show results for females based only on the monotonicity restriction combined with the median, which proved to be the acceptable restriction that led to the tightest bounds. In these graphs moving from left to right gives the growth of wages by age. Moving from one cohort to the next at the same age gives the cohort/time effect. To establish growth over age we need to compare the upper bound of the median at the lower

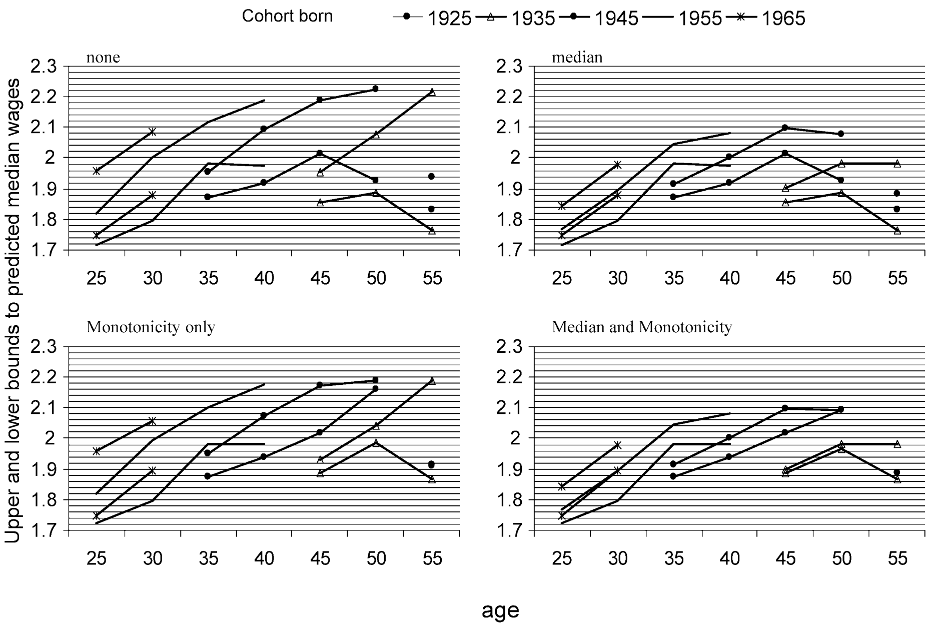


FIGURE 8.—Life-cycle and cohort wage profiles for males who left school at or before 16 (the statutory schooling group).

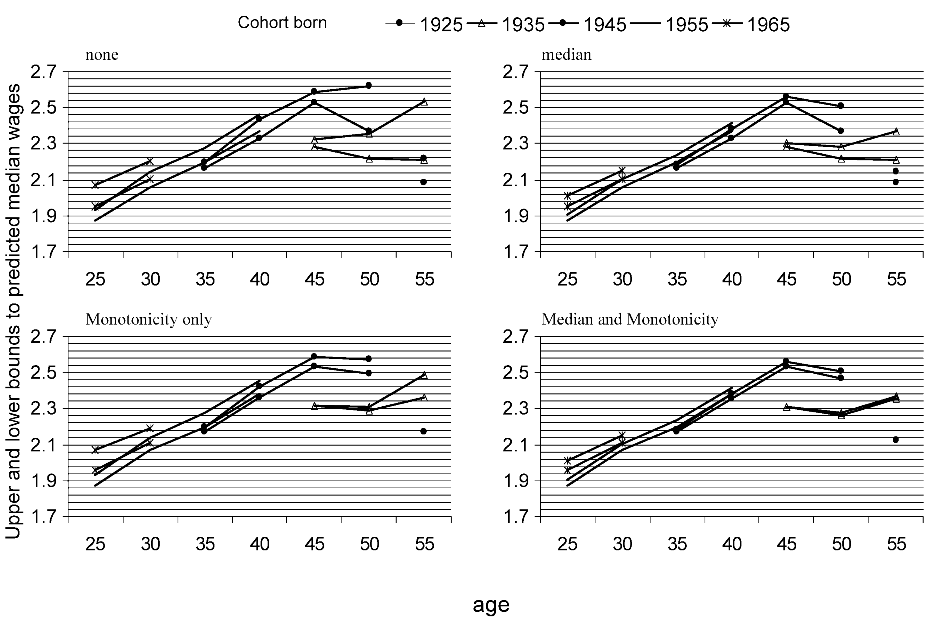


FIGURE 9.—Life-cycle and cohort wage profiles for males who left school at 17 or 18 years of age (high school graduates group).

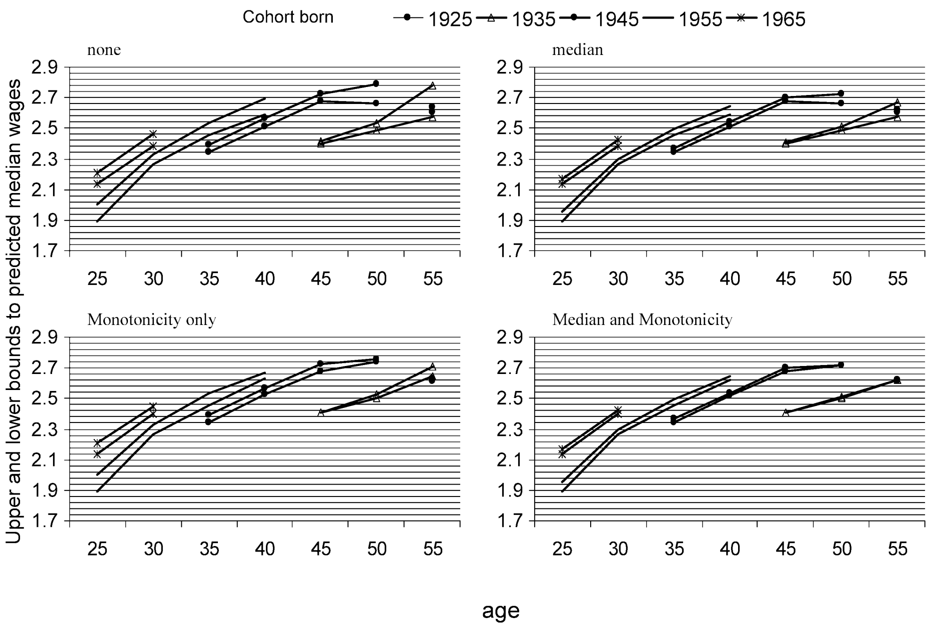


FIGURE 10.—Life-cycle and cohort wage profiles for males who left full time education after 18 years of age (some college group).

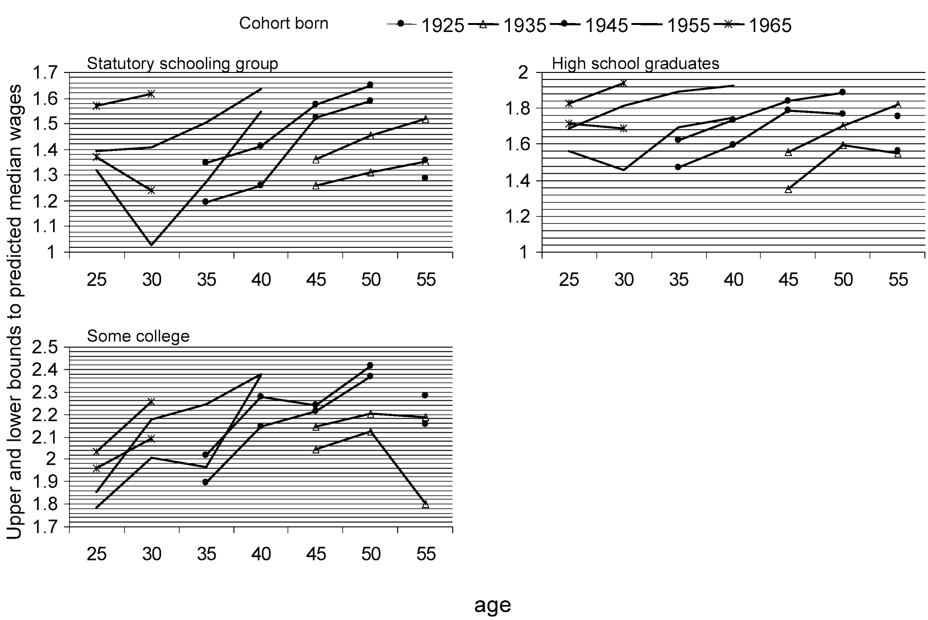


FIGURE 11.—Life-cycle and cohort wage profiles of women by education group obtained by imposing both the median and the monotonicity restrictions.

age to the lower bound at the higher age, and similarly compare intercohort growth.

The group with just statutory schooling[21] (see Figure 8) has the lowest employment rates and therefore the widest bounds. Nevertheless the worst case lower bound to wage growth between the ages of 25 and 35 is 16% (*2.7%*) for the 1955 cohort, and for wage growth between the ages of 35 and 45 it is 6% (*2.7%*) for those born in 1945. The bounds become considerably tighter when we impose restrictions, and almost all ambiguity is eliminated with the combination of median and monotonicity. Combining information across cohorts shows unambiguous growth of wages up until age 45. Beyond that age, when we impose both the median and the monotonicity restrictions, wages are shown to be either flat or declining. If we do not impose positive selection, but just monotonicity, our bounds are uninformative about wage growth beyond 50 for the statutory group.

We can also detect substantial intercohort growth between the 1935 and 1945 cohorts at the age of 45 of at least 6% (*3%*). It is hard to find evidence of growth across more cohorts without making any economic restrictions. With the monotonicity and median restrictions, however, we see across cohort growth of at least 11% (*3%*) between the 1935 and 1945 cohorts, and minimal growth of around 6% (*3%*) at age 35, but the lower bound is below zero at the age of 30. Thus there is evidence that intercohort growth is declining for those who have just statutory schooling, although we are forced to compare these at different ages.

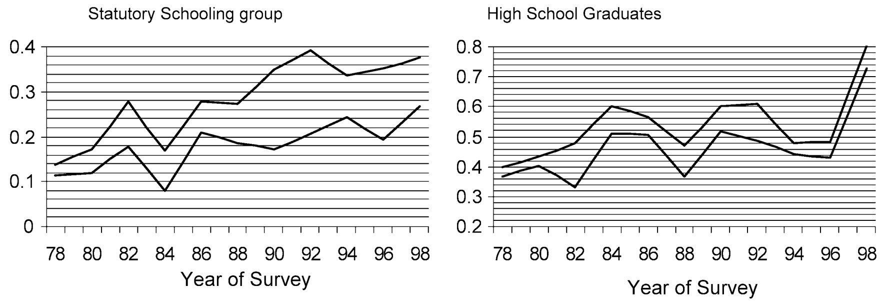
The median and the monotonicity restrictions provide very tight bounds for high school graduates[22] (Figure 9). There is substantial life-cycle wage growth of the median wage between the ages of 25 and 45. Growth becomes flatter beyond that, but wages can be shown to continue to grow once one imposes the monotonicity restriction up until age 55.

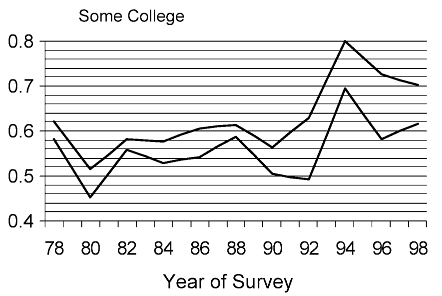
Figure 10 shows that wages of those with some college education are increasing both across cohorts and over the life cycle. Interestingly, the data clearly reject the hypothesis that wages fall at older ages over the life cycle up to the age of 55, because the oldest cohort wages keep growing between the ages of 50 and 55.

The results for women in Figure 11 are based on both the median and the monotonicity restrictions. It is hard to establish much for younger cohorts of women with just statutory schooling (top panel of Figure 11) because employment rates are very low for this group. However, there has been a clear growth of wages with age for the 1945 cohort: growth between the 1945 and 1955 cohort at the age of 40 has been at least 13%. Some life-cycle and intercohort growth is also evident for the two higher education groups (middle and lower panels of Figure 11).

[21]Those who left school by age 16.

[22]Those who left school at age 17 or 18.





	Lower bound to change in 25-45 age differential between 1978-98 (standard error)	p-value for test that Age Differential unchanged over time	p-value for test that age differential in 98 no higher than differential in 78
Statutory Schooling Group	0.131 (0.061)	7.3%	7.3%
High School Graduates	0.330 (0.091)	0.0%	0.0%
Some College	-.0005 (0.096)	6.5%	100.0%
All education groups		0.0%	0.0%

FIGURE 12.—Bounds to the difference in the median log wages of 45 year olds to those of 25 year olds.

Figure 12 shows the bounds to the difference in log wages between 25 and 45 year olds based on the median and the monotonicity restrictions. This confirms the wage growth over the first half of working life for all education groups. For those who have no postsecondary education, we can also see that differentials at the end of the period are indeed higher than those at the beginning: wage differentials associated with cohort/age have risen even after controlling for the possible nonrandom allocation of individuals into work. The table included with in the figure presents the lower bound on the 1978–1998 change and some tests for the hypothesis of no change. It is possible to reject the hypothesis of no change for the high school graduates and, more marginally, for the other two groups (second column). Overall, Figure 12 demonstrates that either cohort effects or an increasing return to experience is an important feature of the changing wage structure over the 1980s and 1990s at least for some education groups.[23]

[23]A test of the restriction that differences have not changed over time for those who have no college education was obtained by exploiting solely that the monotonicity restriction has a p value of 8.5%.

5.5. *Educational and Gender Differentials*

5.5.1. *Educational differentials*

In Figure 13 we provide a picture of the educational differentials, and how they change over time and across cohorts based on the monotonicity and the median restrictions.

The difference in the medians of the wage distribution between those who have some college education and the statutory schooling group clearly increases with age. We cannot confirm the presence of cohort or time effects in the returns, although the bounds are consistent with these being quite high. This is a key point, because on the basis of these results we cannot be sure whether returns to education have increased. The changing returns of college relative to high school cohort effects seem clearer: The oldest cohort has the highest returns; the next cohort comes in much lower, followed by an increase between the 1945 and 1955 cohort, and then a zero or high increase between the 1955 and 1965 cohorts. Thus for recent cohorts, the changing returns to education may have contributed to the increase in inequality.

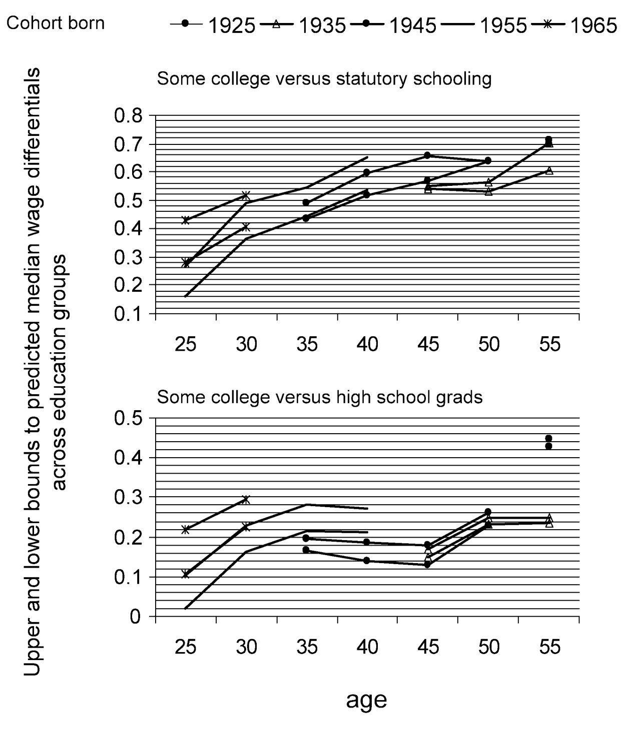


FIGURE 13.—Bounds to the differences in median log wages between education groups obtained by imposing both median and monotonicity restrictions.

To summarize some of these results with numbers, educational differentials between college and high school increased by between 0.04 (*0.069*) and 0.25 (*0.072*) log points for 25 year olds. For 40 year olds the increase lies between 0.095 (*0.08*) and 0.195 (*0.073*) log points.

Further results (not shown on the graph) are obtained by imposing the additivity restriction.[24] For 25 year olds, the increase is estimated to be between 0.140 (*0.039*) and 0.169 (*0.039*) log points and much more precisely estimated. For 40 year olds, we get a point estimate of zero, which is not included in the unrestricted bounds given previously. This suggests that the additivity restriction for 40 year olds is invalid. The p value of the test for this restriction is 9.5%.

For women, all the estimated bounds to the changes span zero, meaning that we cannot reject the possibility that the return to education has remained the same. Note that even the changes among women observed working are small, so this failure to find a clear result with bounds should not be a surprise. Nevertheless when positive selection is imposed, the lower bound to the change in the return to education is high (at least 20%).

5.5.2. *Gender wage differentials*

How do the wages of women compare to those of men? How has this difficult relationship evolved over time? Compositional issues are crucial here, because the selection process into work may be different for women compared to men and may have changed with the large changes in employment rates. This problem is very similar to that faced by people who are interested in understanding the convergence of black and white observed wages in the 1960s and 1970s (see Butler and Heckman (1977)). Brown (1984) and Smith and Welch (1986) adopted different strategies to deal with this that both can be considered as special cases of our approach. More recently, Blau and Kahn (2004) emphasized the potential importance of changes in selectivity as a factor that may explain the observed slowdown in the reduction of the U.S. gender pay gap. (See also Mulligan and Rubinstein (2005).)

The worst case bounds to the change in the differentials are uninformative because of the lower employment rates for women, particularly in the statutory schooling group. This in itself illustrates how important compositional effects can be. To obtain something informative we present results in Figure 14 based on the combination of the monotonicity, the median, and the additivity restriction, which we impose to tighten the bounds and to improve the precision of their estimation. We have also assumed that changes in differentials are the same for all those who have less than college education, thus combining the statutory schooling and the high school group. In the figure the boxes denote

[24]See Equation (15). This restriction is used to see if we can further tighten the estimates with an assumption frequently used in parametric analysis.

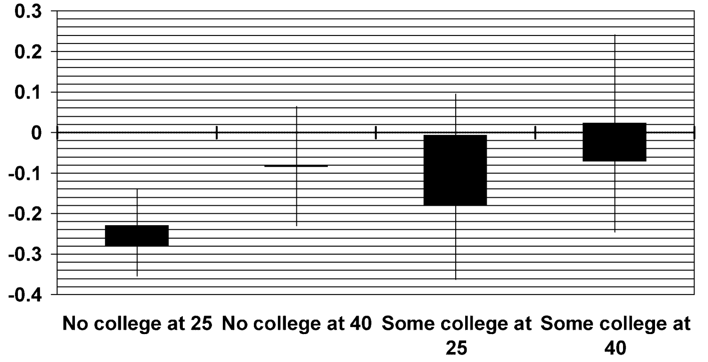


FIGURE 14.—Upper and lower bounds to changes to the difference in median wages between men and women obtained by imposing both restrictions.

the bound to the change in the differential between 1978 and 1998, and the thin lines denote the 95% confidence interval for the unidentified change.

For this combined group the male/female differential declined from 1978 to 1998 by between 0.23 and 0.28 log points, and the confidence interval implies that the difference is significant. We have already argued that the median restriction does not seem to be at odds with the available evidence (see Figure 5 and the discussion around it) generally and even more so for young men and women. However, as a robustness check, we have computed the bounds to the differential by dropping the median restriction, which here imposes positive selection into the labor market for both men and women. We find that the upper bound to the change in the differential is just negative (-0.005) although the upper end of the confidence interval now reaches 0.19. Thus imposing positive selection is quite crucial to obtaining our precise result. However, the point estimate of the bounds do not cross zero even if we do not impose the median restriction. Given also the fact there is little circumstantial evidence against it, we conclude that the differential for this group did indeed decline.

The change observed between working men and women was about 0.21 log points. This suggests that composition effects may conceal part of the improvement in the labor market position of women. The other declines are not significant. Moreover, for the group with some college, the bounds include a zero or even a positive change.

6. CONCLUSIONS

The aim of this paper was to develop an approach to dealing with nonrandom selection into employment when analyzing the wage distribution. The key problem we had to address was that in the presence of censoring there are no obvious identification strategies that will point-identify the wage distribution without making strong assumptions. To deal with this issue, we developed bounds based on theoretically motivated restrictions from economic theory.

In our empirical analysis, we used our approach to examine changes in wage inequality from 1978 to 2000 using the U.K. Family Expenditure Survey, while at the same time accounting for the possible impact of the large changes in employment and its composition. We compared results using worst case bounds that do not impose any restrictions to the ones that do. The restrictions we consider are that the probability of work is higher for those with higher wages, that the wage distribution is independent of out-of-work benefit income, and, alternatively, that higher values of such income are positively associated with wages.

The worst case bounds do establish that inequality has increased. However, the restricted bounds are much tighter, and can lead to stronger and unambiguous conclusions on a number of issues for which the worst case bounds are uninformative. Some of these restrictions have testable implications for which we developed tests. The exclusion of out-of-work benefit income from wages is strongly rejected. A weaker restriction that requires the instrument to be monotonically associated with the wage distribution is never rejected and our empirical results are mainly based on this.

Using our analysis, we established that inequality increased both overall and within education groups by more than can be explained by changes in composition. There have also been significant increases in age/cohort differentials. Educational differentials can be shown to have increased for both 25 and 45 year olds, although this result is more significant for the younger men. Gender wage differentials improved for women by at least 23 percentage points in the 25-year-old unskilled group. However, for other groups the upper bound to the change may show an improvement, but it is not significant (40-year-old unskilled group), or the upper bound is zero and even positive (college group).

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APPENDIX A: MONTE CARLO SIMULATIONS OF THE TESTING PROCEDURE

We conducted Monte Carlo simulations to examine the power and size properties of our test. The null hypothesis is that the bounds are equal and the alternative is that they cross. In this appendix, we describe the model used in the simulation.

The model under the null hypothesis is

$$
\begin{aligned}
(19)\qquad & W = \varepsilon_1, \\
& Z = I(\varepsilon_2 > 0), \\
& E = [I(W < 0.75) + I(W \geq 0.75) I(\varepsilon_3 > 0)]Z, \\
& \qquad + [I(W > -0.75) + I(W \leq -0.75) I(\varepsilon_3 > 0)](1 - Z),
\end{aligned}
$$

where ε_1, ε_2, and ε_3 are independent standard normal random variables. Under the null hypothesis, W and Z are independent. As an alternative we consider

$$
(20)\qquad W = \varepsilon_1 - \frac{1}{4}Z.
$$

The structure of the determination of employment shown in Equation (19) will mean that the bounds touch in the population over a positive range of wages, which is our null. To see this note

$$
\begin{aligned}
& \max_z F(w|z, E = 1) \Pr(E = 1|z) \\
& \quad = \min_z [F(w|z, E = 1) \Pr(E = 1) + 1 - \Pr(E = 1|z)].
\end{aligned}
$$

Given that $F(w|z) = F(w)$, this expression is identical to

$$
\begin{aligned}
& \max_z F(w) \Pr(E = 1|z, W < w) \\
& \quad = \min_z [F(w) \Pr(E = 1|z, W < w) + 1 - \Pr(E = 1|z)].
\end{aligned}
$$

This can be rearranged to give

$$
\begin{aligned}
& \max_z F(w) \Pr(E = 1|z, W < w) \\
& \quad = 1 - \max_z \{[1 - F(w)] \Pr(E = 1|z, W \geq w)\}.
\end{aligned}
$$

The foregoing expression can be true if and only if

$$
(21)\qquad \max_z \Pr(E = 1|z, W < w) = \max_z \Pr(E = 1|z, W \geq w) = 1
$$

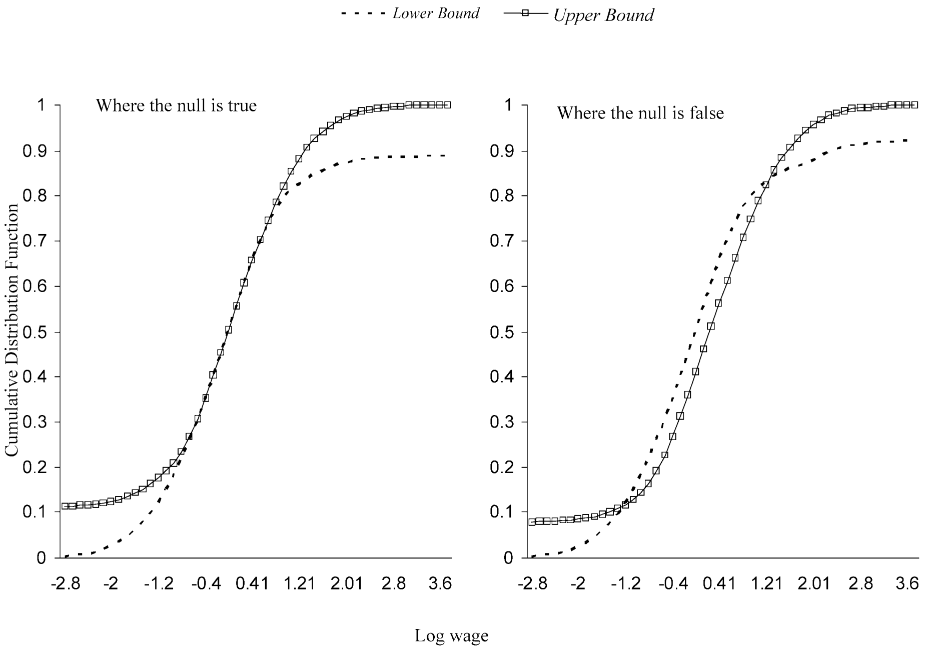

FIGURE 15.—Estimated population distribution functions from simulated data.

is true. The probability $\Pr(E=1|z)=1$ is a sufficient but not a necessary condition for Equation (21) to hold. The bounds will be equal when there is a value of z for which all those with wages *below* w work and a value of z for which all those with wages *above* the same w work. These values of z may not be the same. Both bounds reduce to an unconditional CDF.

Equation (19) and our Monte Carlo simulations force condition (21) to hold for a large range of wages: Consider any value w between -0.75 and 0.75. When $Z=1$ and $W<w$, everyone works. When $Z=0$ and $W>w$, everyone works as well. So the upper and lower bounds for the model coincide for any w between -0.75 and 0.75. Figure 15 shows how the estimated distribution functions obtained by imposing the exclusion restriction look in the population.

We carried out two Monte Carlo simulations: one with 500 observations and one with a 1000. For each of these two simulations we generated 1000 random samples from each of the two models expressed in Equations (20) and (19). Within each replication we used the bootstrap with 114 draws to compute critical values for a nominal size of 5% for the test statistic in (17), as described in the main body of the paper. We also considered a version of the test where the $\sum_w$ is replaced by a $\max_w$ (maximum over wages). The results are shown in Table III.

TABLE III

POWER AND SIZE OF OUR TEST

	H_o Is True				H_o Is False			
	500 Obs		1000 Obs		500 Obs		1000 Obs	
	10%	5%	10%	5%	10%	5%	10%	5%
Max	0.045	0.023	0.049	0.024	0.748	0.602	0.954	0.919
Mean	0.046	0.023	0.045	0.019	0.800	0.667	0.968	0.926

Table III shows first that both tests are a bit undersized, even for the larger sample size of 1000. Both tests have excellent power properties, with the power being a bit better in the version we have adopted at the smaller sample size. Overall the results seem to suggest that our inferential approach is reliable and powerful.

APPENDIX B: USING INDEPENDENCE RESTRICTIONS

Many empirical studies impose independence of the instrument used to correct for selection from the unobservables in wages. This independence assumption is reflected in the fact that the selection model is single index and that the coefficient of the selection correction term(s) do not depend on the instrument. In this section we explore how such an independence assumption, without any other restrictions, can help tighten the bounds to educational differentials or other characteristics X.

Suppose we partition the vector of observables into the subvectors X^1 and X^2, and suppose that the dependent variable can be written as

$$W = m(X^1, X^2) + \varepsilon,$$

where $F(\varepsilon|X^1, X^2) = F(\varepsilon|X^1)$. In this case, none of the quantiles of ε depends on X^2. Hence we can write the qth quantile of W as

$$w^q(X^1, X^2) = m(X^1, X^2) + g(q, X^1).$$

In this context, the impact of changing X^2 (say education) is easily defined. Moreover, the independence restriction can be used to obtain a tight bound for such a return. First note that the impact of changing X^2 from B to A on the dependent variable, defined by $w^q(X^1, A) - w^q(X^1, B) = \Delta_{X_2} m(X^1, X^2)$ does not depend on q.[25] Then under these assumptions the tightest bound on the return $\Delta_{X_2} m(X^1, X^2)$ can be obtained by searching across quantiles. Thus

[25]The analysis could also be carried out in terms of a continuous variable.

the tightest bound takes the form

$$\max_q\{w^{q(\mathrm{l})}(X^1, A) - w^{q(\mathrm{u})}(X^1, B)\}$$
$$\leq \Delta_{X_2} m(X^1, X^2) \leq \min_q\{w^{q(\mathrm{u})}(X^1, A) - w^{q(\mathrm{l})}(X^1, B)\}.$$

If the independence assumption is invalid, the bounds may cross.

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