



**Instabilities of the fractionalized Dirac semimetal in the Kitaev-Kondo model**Jennifer Lin <sup>1</sup> and Frank Krüger <sup>1,2</sup><sup>1</sup>*London Centre for Nanotechnology, University College London, Gordon St., London WC1H 0AH, United Kingdom*<sup>2</sup>*ISIS Facility, Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire OX11 0QX, United Kingdom*

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We study a honeycomb Kondo lattice model in which Dirac conduction electrons are coupled to a spin-1/2 Kitaev quantum spin liquid. For weak Kondo coupling, the spins fractionalize into Majorana fermions comprising a gapless Dirac mode and three gapped visons. In second order perturbation theory, the Kondo coupling gives rise to local Hubbard repulsions and spin-spin interactions between conduction electrons, as well as a vertex coupling electron to gapless Majorana fermions. We analyze the resulting low-energy field theory using a perturbative renormalization group (RG) scheme, accounting for additional density-density interactions generated under RG. At criticality, electrons decouple from Majorana fermions but all three electron interactions acquire positive values. An analysis of susceptibility exponents reveals that the fractionalized Fermi liquid becomes unstable toward antiferromagnetic order and that superconductivity is disfavored.

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Quantum spin liquids (QSLs) represent one of the most striking manifestations of many-body quantum physics. In these states, local magnetic moments fail to order even at zero temperature, remaining in a highly entangled and dynamic quantum state. This phenomenon arises from geometric frustration and strong quantum fluctuations, producing a “liquidlike” magnetic phase in which fractionalized excitations and gauge fields naturally emerge [1,2].

In fractionalized Fermi liquids, dubbed as FL\* phases, conduction electrons coexist with a QSL background. Such exotic states were originally proposed as phases of Kondo lattice models [3,4]. While for strong Kondo coupling the electrons that initially form the local moment spins hybridize with the conduction electrons, resulting in a heavy Fermi liquid with a large Fermi surface, for sufficiently weak Kondo coupling the tendency of spin fractionalization dominates over the Kondo screening. Since the emergent fractionalized quasiparticles don’t carry electrical charge, the resulting FL\* state has a small Fermi surface of the conduction electrons alone, leading to an apparent violation of Luttinger’s theorem [5].

More recently, fractionalized Fermi liquids have been proposed to underlie the pseudogap regime of underdoped cuprate superconductors [6–8], motivated by the experimental observation of small Fermi pockets and Fermi arcs, and reviving the early proposal by Anderson that the cuprates may be understood as doped quantum spin liquids [9].

However, the physics of the cuprates is extremely rich. It is therefore important to first understand fractionalized Fermi liquids and their superconducting pairing instabilities on the

level of simple toy models. One such setting is a Kondo-lattice model in which tight-binding electrons on the honeycomb lattice are coupled to local moment spins that form a Kitaev QSL [10,11].

The spin-1/2 Kitaev model on the honeycomb lattice is one of the very rare examples of an exactly solvable QSL model [12]. The spins are found to fractionalize into a set of Majorana fermions, one spinon mode with a gapless Dirac dispersion and three dispersion-less vison modes that encode local excitations of  $Z_2$  gauge fluxes. Although the exact solvability is broken by the Kondo coupling to conduction electrons, the vison gap guarantees that the QSL, and hence the FL\*, state remain stable against sufficiently weak Kondo coupling.

Utilizing Majorana-fermion mean-field theory, it was found that for a ferromagnetic Kitaev model a nematic triplet superconductor (SC) forms at intermediate Kondo coupling, sandwiched between the FL\* state and the heavy fermion liquid, suggesting that the itinerant Majorana fermions act as “pairing glue” for unconventional superconductivity [10]. The extent of the SC region was found to crucially depend upon the conduction electron filling, being maximum for fillings close to the van-Hove singularity and shrinking almost to zero at half filling where the Fermi level is located at the Dirac points of the conduction electron spectrum. An alternative mean-field treatment based on Abrikosov fermions found a first order instability of the FL\* phase toward the formation of a ferromagnetic topological superconductor [11].

In a very recent investigation [13] the local moment degrees of freedom were integrated out, resulting in a Hubbard repulsion and spin-spin interaction between the conduction electrons to second order in the Kondo coupling. The induced electronic spin-spin interaction has the same sign and bond-directional dependence as the Kitaev exchange. The resulting interacting electron problem was then analyzed using functional renormalization group (fRG). While near the van Hove filling the FL\* was found to become unstable toward the

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formation of a spin-density wave (SDW), at smaller fillings a transition from the FL\* state to a superconductor was found, in the antiferromagnetic case with chiral  $d + id$  symmetry, in the ferromagnetic case of spin-triplet  $p$ -wave type. Unfortunately, the momentum resolution in the fRG was insufficient to investigate the behavior for electron fillings close to the Dirac point.

In this paper we use a complementary method to investigate the instabilities of the fractionalized half-filled Dirac semimetal of the Kitaev-Kondo model. Treating the Kondo coupling in second-order perturbation theory, we derive an effective low-energy continuum field theory of the gapless Dirac Majorana fermions coupled to Dirac conduction electrons. It contains a Hubbard on-site repulsion and a spin-spin interaction between conduction electrons, as well as a four-fermion vertex that couples Majorana fermions and conduction electrons. We use a perturbative parquet RG analysis [14–17] to first obtain the scale dependence of the various interaction parameters, and as a second step determine the susceptibility exponents of different order parameters from the RG flow of the conjugate fields at the critical fixed point.

Our analysis shows that irrespective of the sign of the Kitaev coupling and the ratio of Fermi velocities of Majorana fermions and conduction electrons, the critical behavior is always controlled by the same fixed point at which the conduction electrons decouple from the Majorana fermions and all electron-electron interactions, including an additional density-density repulsion that is generated under the RG, have finite positive values. The renormalization of various fields conjugate to potential order parameters shows that the leading instability is toward an antiferromagnetic SDW state.

Our paper is organized as follows. In Sec. II we introduce the Hamiltonian of the Kitaev-Kondo model on the honeycomb lattice and the action of the corresponding path-integral over Grassmann fields. By integrating out the gapped vison modes, in Sec. III, we derive the effective low-energy continuum field theory. In Sec. IV the RG equations of the rescaled interaction parameters are obtained. With these results, we identify the critical fixed point for the symmetry-breaking instability of the FL\* phase and analyze the RG flow along the critical surface. To understand the type of symmetry breaking, in Sec. V, we compute the susceptibility exponents for different order parameters from the field scaling exponents at the critical fixed point. Finally, in Sec. VI we summarize and discuss our results.

## II. MODEL

Our starting model is the  $S = 1/2$  Kitaev model on the honeycomb lattice with Kondo coupling to conduction electrons that could either live on the same lattice or an adjacent honeycomb layer, as illustrated in Fig. 1. The Hamiltonian of this model is given by

$$\begin{aligned} \hat{\mathcal{H}} = & K \sum_{\gamma=x,y,z} \sum_{(i,j)_\gamma} \hat{S}_i^\gamma \hat{S}_j^\gamma - t \sum_{(i,j)} \sum_{v=\uparrow,\downarrow} (\hat{c}_{iv}^\dagger \hat{c}_{jv} + \text{H.c.}) \\ & + J_K \sum_i \sum_\gamma \sum_{v,v'} \hat{S}_i^\gamma \hat{c}_{iv}^\dagger \sigma_{v,v'}^\gamma \hat{c}_{jv'}. \end{aligned} \quad (1)$$

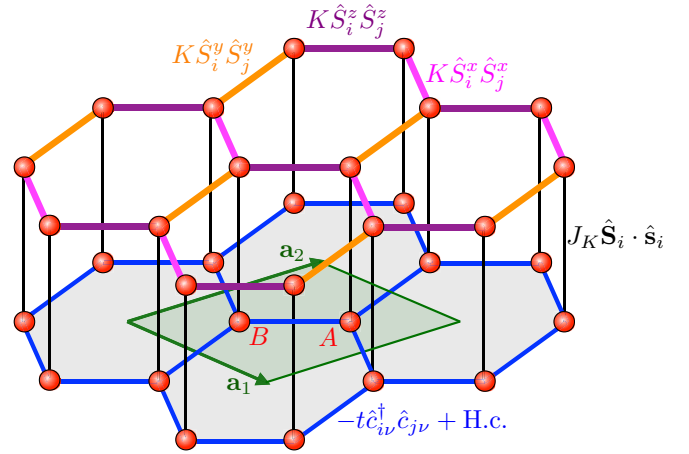


FIG. 1. Illustration of the Kitaev-Kondo model on a honeycomb lattice. The top layer illustrates the  $S = 1/2$  Kitaev QSL model with bond-directional Ising exchanges  $K$ . The bottom layer corresponds to tight-binding electrons with hopping amplitudes  $t$  between neighboring sites of the honeycomb lattice. The local moments of the Kitaev quantum spin liquid are coupled to the conduction electrons via the conventional Kondo interaction  $J_K$ . A possible unit cell spanned by lattice vectors  $\mathbf{a}_{1,2} = (\frac{\sqrt{3}}{2}, \pm \frac{\sqrt{3}}{2})$  is shown in green.

Here  $K$  denotes the Kitaev coupling, which could be antiferromagnetic ( $K > 0$ ) or ferromagnetic ( $K < 0$ ),  $t$  the hopping amplitude of the conduction electrons between neighboring lattice sites, and  $J_K$  the Kondo coupling between local moment spins  $\hat{S}_i$  and conduction electron spins  $\hat{s}_i$  with components  $\hat{s}_i^\gamma = \sum_{v,v'} \hat{c}_{iv}^\dagger \sigma_{v,v'}^\gamma \hat{c}_{jv'}$  (in units of  $\hbar/2$ ), where  $\sigma^\gamma$  denote the standard spin Pauli matrices. In the above Hamiltonian, nearest-neighbor bonds are denoted by  $\langle i, j \rangle$  and distinguished by the subscript  $\gamma = x, y, z$  to express the bond-directional Ising exchange of the Kitaev model (see Fig. 1).

The tight-binding dispersion of the conduction electrons is given by  $\epsilon_\pm(\mathbf{k}) = \pm t |\lambda(\mathbf{k})|$ , where

$$\lambda(\mathbf{k}) = \sum_\gamma e^{i\mathbf{k}\delta_\gamma} \quad (2)$$

with  $\delta_x = \mathbf{a}_1$ ,  $\delta_y = \mathbf{a}_2$ , and  $\delta_z = \mathbf{0}$ . The lattice vectors  $\mathbf{a}_i$  and corresponding unit cell are defined in Fig. 1. In this work we will focus on the case of half-filling, where the Fermi level is located at the Dirac points  $\mathbf{K}_\pm = \frac{2\pi}{3}(1, \pm 1/\sqrt{3})$ . Linearizing the dispersion near the Dirac points,  $\mathbf{k} = \mathbf{K}_\pm + \mathbf{q}$ , we obtain  $\epsilon_\pm(\mathbf{q}) = \pm v |\mathbf{q}|$  with Fermi velocity  $v = \frac{3}{2}t$ .

The Kitaev model is exactly solvable in terms of a set of four Majorana fermions  $\hat{\eta}_i, \hat{\xi}_i^x, \hat{\xi}_i^y, \hat{\xi}_i^z$  [12], which satisfy  $\hat{\eta}_i^\dagger = \hat{\eta}_i$ ,  $(\hat{\xi}_i^\gamma)^\dagger = \hat{\xi}_i^\gamma$ , and the Clifford algebra  $\{\hat{\xi}_i^\gamma, \hat{\xi}_j^{\gamma'}\} = 2\delta_{ij}\epsilon_{\alpha\beta\gamma}\delta_{\gamma\gamma'}$ ,  $\{\hat{\eta}_i, \hat{\xi}_j^\gamma\} = 0$ . In terms of the Majorana fermions, the spin  $1/2$  operators (in units of  $\hbar/2$ ) are expressed as

$$\hat{S}_i^\gamma = i\hat{\eta}_i\hat{\xi}_i^\gamma, \quad (3)$$

where the local constraint  $\hat{\eta}_i\hat{\xi}_i^x\hat{\xi}_i^y\hat{\xi}_i^z = 1$  ensures that the Hilbert space is not artificially enlarged and that the spin-commutation relations  $[\hat{S}_i^\alpha, \hat{S}_j^\beta] = 2\delta_{ij}\epsilon_{\alpha\beta\gamma}\hat{S}_i^\gamma$  follow from the fermion anticommutation relations.

Although the resulting Hamiltonian is initially quartic in terms of the Majorana fermion operators, it can be solved

analytically since the local bond operators  $\hat{A}_{ij}^\gamma = i\hat{\xi}_i^\gamma \hat{\xi}_j^\gamma$  and corresponding plaquette operators, given by the product of bond operators around each hexagon, commute with the Hamiltonian. This results in a free-fermion Hamiltonian with Dirac dispersion  $\epsilon_0(\mathbf{k}) = \pm K|\lambda(\mathbf{k})|$  of the  $\hat{\eta}$  Majorana fermions (spinons) and three flat bands  $\epsilon_\gamma(\mathbf{k}) = \pm\Delta$  for the localized  $\hat{\xi}^\gamma$  Majorana fermions (visons) with an energy gap  $\Delta/|K| \approx 0.525$  [13,18,19]. Note that by definition, the Majorana fermions are always at half-filling.

To study the effects of the Kondo coupling  $J_K$  between Kitaev Majorana fermions and Dirac conduction electrons, we employ the coherent state, imaginary-time path integral formalisms. After Fourier transform to frequency  $k_0$  and spatial momenta  $\mathbf{k}$ , the action is given by  $\mathcal{S} = \mathcal{S}_0[\bar{\psi}, \psi] + \mathcal{S}_0[\eta] + \mathcal{S}_0[\xi] + \mathcal{S}_{\text{int}}[\bar{\psi}, \psi, \eta, \xi]$  with

$$\mathcal{S}_0[\bar{\psi}, \psi] = \sum_{v=\uparrow,\downarrow} \int_k \bar{\psi}_v(k) \begin{pmatrix} -ik_0 & -t\lambda^*(\mathbf{k}) \\ -t\lambda(\mathbf{k}) & -ik_0 \end{pmatrix} \psi_v(k), \quad (4)$$

$$\mathcal{S}_0[\eta] = \int_k \eta^\dagger(k) \begin{pmatrix} -ik_0 & -iK\lambda^*(\mathbf{k}) \\ iK\lambda(\mathbf{k}) & -ik_0 \end{pmatrix} \eta(k), \quad (5)$$

$$\mathcal{S}_0[\xi^\gamma] = \int_k (\xi^\gamma)^\dagger(k) \begin{pmatrix} -ik_0 & -i\Delta e^{-ik\delta_\gamma} \\ i\Delta e^{ik\delta_\gamma} & -ik_0 \end{pmatrix} \xi^\gamma(k), \quad (6)$$

$$\mathcal{S}_{\text{int}} = iJ_K \sum_{s=A,B} \sum_\gamma \int_{k_1,\dots,k_4} \delta_{k_1+k_2-k_3+k_4} \times \eta_s(k_1) \xi_s^\gamma(k_2) \bar{\psi}_s(k_3) \sigma^\gamma \psi_s(k_4). \quad (7)$$

Here  $k = (k_0, \mathbf{k})$  and  $\int_k = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \int_{\text{BZ}} \frac{d^2\mathbf{k}}{v_{\text{BZ}}}$ , for brevity. Note that while the conduction electrons are represented by independent Grassmann fields  $\bar{\psi}_{sv}(k)$ ,  $\psi_{sv}(k)$ , the Majorana fermions are represented by a single Grassmann field, and  $\eta_s^\dagger(k) = \eta_s(-k)$  and  $(\xi_s^\gamma)^\dagger(k) = \xi_s^\gamma(-k)$ .

### III. EFFECTIVE LOW-ENERGY THEORY

To derive an effective low-energy field theory, we use second order perturbation theory to obtain the effective interactions  $\sim J_K^2$  between conduction electrons, as well as an interaction between the gapless Majorana fermions and the conduction electrons.

Using that the fermion Green functions are given by the following  $2 \times 2$  matrices in sublattice space,

$$\mathbf{G}_v^\psi(k) = \frac{1}{k_0^2 + t^2|\lambda(\mathbf{k})|^2} \begin{pmatrix} ik_0 & -t\lambda^*(\mathbf{k}) \\ -t\lambda(\mathbf{k}) & ik_0 \end{pmatrix}, \quad (8)$$

$$\mathbf{G}^\eta(k) = \frac{1}{k_0^2 + K^2|\lambda(\mathbf{k})|^2} \begin{pmatrix} ik_0 & -iK\lambda^*(\mathbf{k}) \\ iK\lambda(\mathbf{k}) & ik_0 \end{pmatrix}, \quad (9)$$

$$\mathbf{G}^{\xi^\gamma}(k) = \frac{1}{k_0^2 + \Delta^2} \begin{pmatrix} ik_0 & -i\Delta e^{-ik\delta_\gamma} \\ i\Delta e^{ik\delta_\gamma} & ik_0 \end{pmatrix}, \quad (10)$$

and that the Majorana fermion Green functions satisfy  $G_{s,s'}^\eta(k) = -G_{s',s}^\eta(-k)$  and  $G_{s,s'}^{\xi^\gamma}(k) = -G_{s',s}^{\xi^\gamma}(-k)$ , we obtain an on-site Hubbard repulsion  $U$  and a nearest-neighbor spin-spin interaction  $J$  between conduction electrons,

$$S_U[\bar{\psi}, \psi] = U \sum_{s,v} \int_{k_1,\dots,k_4} \delta_{k_1-k_2+k_3-k_4} \times \bar{\psi}_{sv}(k_1) \psi_{sv}(k_2) \bar{\psi}_{s\bar{v}}(k_3) \psi_{s\bar{v}}(k_4), \quad (11)$$

$$S_J[\bar{\psi}, \psi] = J \sum_{s,\gamma} \int_{k_1,\dots,k_4} \delta_{k_1-k_2+k_3-k_4} \times (\bar{\psi}_s(k_1) \sigma^\gamma \psi_s(k_2)) (\bar{\psi}_{\bar{s}}(k_3) \sigma^\gamma \psi_{\bar{s}}(k_4)), \quad (12)$$

where  $\bar{v} = \downarrow$  if  $v = \uparrow$ ,  $\bar{s} = B$  if  $s = A$ , and vice versa.

To obtain the above long-wavelength expression for  $S_J$  we have performed an expansion around the Dirac points. Note that the original vertex contains additional exponential factors  $e^{i(k_3-k_4)\delta_\gamma}$ . This means that the electron spin-spin interaction inherits the bond-directional dependence from the Kitaev model [13]. However, taking the continuum limit, the directional dependence is lost at zeroth order in the gradient expansion. We drop the next order terms quadratic in  $(\mathbf{k}_3 - \mathbf{k}_4)\delta_\gamma$  since the resulting vertex is irrelevant under the renormalization group.

The strengths of the interactions  $U$  and  $J$  are given by the frequency-momentum integrals

$$U = \frac{3J_K^2}{2} \int_q \frac{q_0^2}{(q_0^2 + K^2|\lambda(\mathbf{q})|^2)(q_0^2 + \Delta^2)} \approx 0.409 \frac{J_K^2}{|K|}, \quad (13)$$

$$J = \frac{J_K^2}{6} \int_q \frac{K\Delta|\lambda(\mathbf{q})|^2}{(q_0^2 + K^2|\lambda(\mathbf{q})|^2)(q_0^2 + \Delta^2)} \approx 0.060 \frac{J_K^2}{K}, \quad (14)$$

where the numerical results are obtained using  $\Delta/|K| \approx 0.525$  [18,19]. While  $U$  is always positive, the sign of  $J$  is set by the sign of the Kitaev coupling  $K$ , and  $|J|/U \approx 0.146$ . This is in perfect agreement with Ref. [13], when taking into account their different definition of  $U$  and  $J$ .

In addition to the interactions between conduction electrons, the low-energy theory contains a vertex that couples the conduction electrons to the gapless Majorana fermion,

$$S_\Gamma[\bar{\psi}, \psi, \eta] = i\Gamma \sum_{s,v} \chi_s \int_{k_1,\dots,k_4} \delta_{k_1-k_2+k_3+k_4} \times \eta_s(k_1) \bar{\psi}_{sv}(k_2) \eta_{\bar{s}}(k_3) \psi_{s\bar{v}}(k_4), \quad (15)$$

where  $\chi_A = 1$  and  $\chi_B = -1$  and the coupling constant  $\Gamma$  is given by

$$\Gamma = J_K^2 \int_q \frac{t\Delta|\lambda(\mathbf{q})|^2}{(q_0^2 + t^2|\lambda(\mathbf{q})|^2)(q_0^2 + \Delta^2)}. \quad (16)$$

While the ratio  $|J|/U$  is fixed, regardless of the strength of the Kondo coupling, the ratio  $\Gamma/U$  depends on the ratio  $c/v = |K|/t$  of Fermi velocities of Majorana fermions and conduction electrons. The evolution of the interaction strength  $U$ ,  $|J|$ , and  $\Gamma$  in units of  $J_K^2/|K|$  as a function of  $c/v$  is shown in Fig. 2.

In general, the Kitaev exchange between local moment spins is mediated through a super-exchange mechanism and therefore expected to be weak compared to typical electronic energy scales. For example, ferromagnetic Kitaev exchanges in the range 5–20 meV have been estimated for  $\alpha$ -RuCl<sub>3</sub> [20,21] and Na<sub>2</sub>IrO<sub>3</sub> [22], while certain Co-based honeycomb materials were found to exhibit an antiferromagnetic Kitaev exchange of 2–3 meV [23]. For comparison, the nearest-neighbor tight-binding parameter for graphene is  $t \approx 2.7$  eV [24]. In real materials one should therefore expect a Fermi-velocity ratio  $c/v$  of the order of  $10^{-3}$ – $10^{-2}$ , implying that  $\Gamma \ll U$ . However, we will theoretically explore the full range

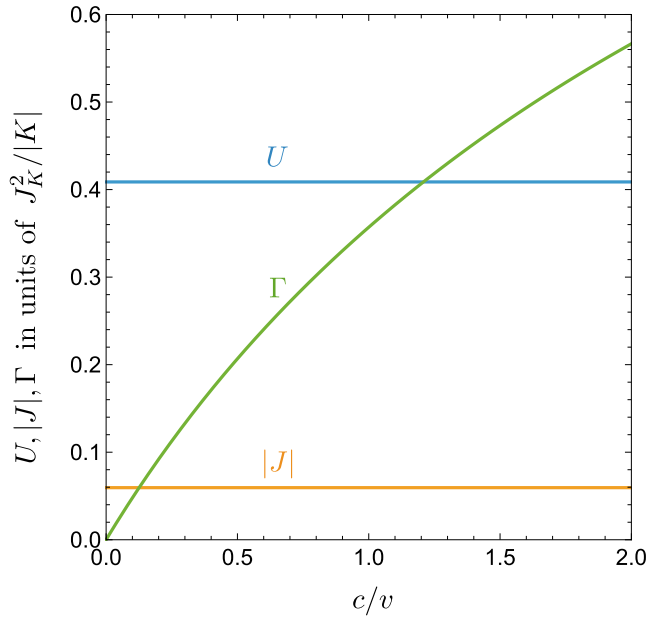


FIG. 2. Interaction  $U$ ,  $|J|$ , and  $\Gamma$  in units of  $J_K^2/|K|$  and as a function of the ratio of the Fermi velocities  $c$  and  $v$  of Majorana fermions and Dirac conduction electrons, respectively.

of  $c/v$  and demonstrate that even in the regime of very strong coupling  $\Gamma$  between Majorana fermions and conduction electrons, the critical behavior will remain the same.

In addition to the interactions  $U$ ,  $J$ , and  $\Gamma$  we will also include a density-density interaction  $\rho$  between conduction electrons on neighboring sites,

$$S_\rho[\bar{\psi}, \psi] = \rho \sum_s \int_{k_1, \dots, k_4} \delta_{k_1 - k_2 + k_3 - k_4} \times (\bar{\psi}_s(k_1) \psi_s(k_2)) (\bar{\psi}_s(k_3) \psi_s(k_4)). \quad (17)$$

Although this interaction is initially zero, it will be generated by the other interactions under the RG.

#### IV. RENORMALIZATION-GROUP ANALYSIS

The quantum criticality of interacting Dirac semimetals is usually analyzed using a Gross-Neveu-Yukawa field theory that describes the coupling of a dynamical order parameter field to gapless Dirac fermions [25,26]. However, in a metallic system with competing interactions it is often unclear what the leading ordering instability is. We therefore use a perturbative parquet RG analysis [14–17] to understand the scale dependence of the interaction parameters  $U$ ,  $J$ ,  $\rho$ , and  $\Gamma$ . Under the RG, interaction parameters can depart significantly from their bare initial values. This can lead to attraction in unconventional superconducting pairing channels, as discussed in the context of iron-based superconductors [15,16].

The four interactions  $U$ ,  $J$ ,  $\rho$ , and  $\Gamma$  form a basis set for the subspace of interactions explored under the RG since it is not possible to express one of the interactions as a linear combination of the other three. However, by using a Fierz identity one may express the density-density interaction  $\rho$  in terms of the spin-spin interaction  $J$  and an additional pairing vertex. Such ambiguities could lead to biased results

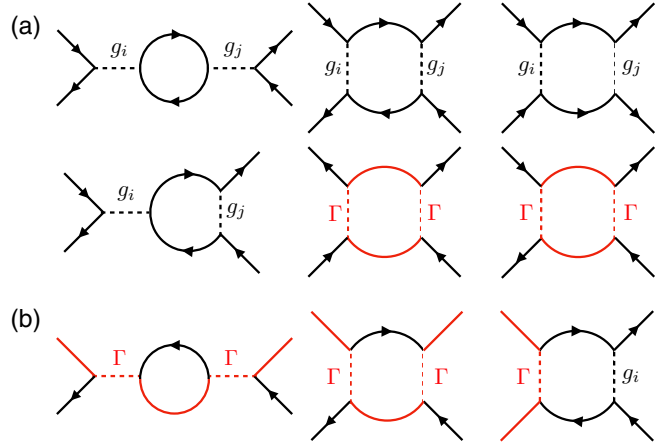


FIG. 3. Second-order, one-loop diagrams that renormalize (a) the electron-electron interactions  $g_i \in \{U, J, \rho\}$  and (b) the coupling vertex  $\Gamma$  between conduction electrons and Majorana fermions. Conduction electron fields  $\bar{\psi}$ ,  $\psi$  correspond to black, Majorana fermion fields  $\eta$  to red lines.

when carrying out a Hubbard-Stratonovich or mean-field decoupling but wouldn't affect our perturbative RG analysis.

To focus on the long-wavelength behavior and impose a momentum cut off,  $|\mathbf{k}| \leq \Lambda$ , where  $\mathbf{k}$  measures the distance for the Dirac points  $\mathbf{K}_\pm$ . Under the perturbative RG scheme we integrate out modes with momenta from the infinitesimal shell  $\Lambda e^{-d\ell} \leq |\mathbf{k}| \leq \Lambda$ , followed by a rescaling of frequency,  $k'_0 = k_0 e^{zd\ell}$ , momenta,  $\mathbf{k}' = \mathbf{k} e^{d\ell}$ , and fields,  $\psi'(k') = \psi(k) e^{(\Delta_\psi/2)d\ell}$ ,  $\eta'(k') = \eta(k) e^{(\Delta_\eta/2)d\ell}$ . Since the inverse fermion propagators are not renormalized by contractions of the interaction vertices we can keep them scale invariant by setting  $z = 1$  and  $\Delta_\psi = \Delta_\eta = -(d+2)$ , where  $d$  is the spatial dimension. The resulting scaling dimensions of the interactions are equal to  $[U] = [V] = [\rho] = [\Gamma] = -d + 1$ .

The above scaling analysis shows that in  $d = 2$  the interactions are irrelevant perturbations at the noninteracting FL\* fixed points. This is expected because of the vanishing density of states at the Dirac points. Strictly speaking, the perturbative RG is controlled in  $d = 1 + \epsilon$  where the interaction strengths at the critical fixed points will be of order  $\epsilon$ .

The one-loop diagrams that renormalize the electron-electron interactions  $g_i \in \{U, J, \rho\}$  and the coupling vertex  $\Gamma$  between conduction electrons and gapless Majorana fermions are shown in Figs. 3(a) and 3(b), respectively. The required shell integrals can be easily calculated in  $d = 2$ ,

$$\int_q^> G_{ss}^\psi(q) G_{ss}^\psi(q) = \frac{1}{8\pi} \frac{\Lambda}{v} d\ell, \quad (18)$$

$$\int_q^> G_{ss}^\psi(q) G_{s's'}^\psi(q) = -\frac{1}{8\pi} \frac{\Lambda}{v} d\ell, \quad (19)$$

$$\int_q^> G_{ss}^\eta(q) G_{ss}^\psi(q) = i\chi_s \frac{1}{8\pi} \frac{2\Lambda}{c+v} d\ell, \quad (20)$$

$$\int_q^> G_{ss}^\eta(q) G_{s's'}^\eta(q) = -\frac{1}{8\pi} \frac{\Lambda}{c} d\ell, \quad (21)$$

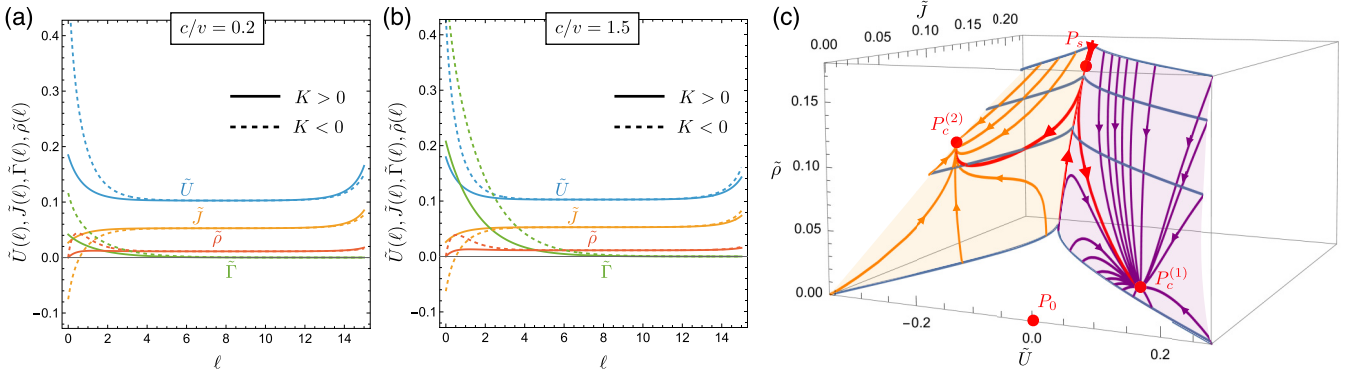


FIG. 4. (a) RG flow of the rescaled interactions  $\tilde{U}$ ,  $\tilde{J}$ ,  $\tilde{\rho}$ , and  $\tilde{\Gamma}$ . The initial interaction strengths are derived from the microscopic Kitaev-Kondo model with Fermi-velocity ratio  $c/v = 0.2$  and for both FM (dashed) and AFM (solid) Kitaev couplings. The Kondo coupling is tuned very slightly above the critical value in both cases. (b) Same as in (a) but for  $c/v = 1.5$ . In all cases the critical behavior is controlled by the same critical fixed point with  $\tilde{\Gamma}_c = 0$  and  $\tilde{U}_c, \tilde{J}_c, \tilde{\rho}_c > 0$ , corresponding to the plateau values. (c) Fixed points and critical surface for  $\tilde{\Gamma} = 0$ . The trajectories show the RG flow within the critical surface. The symmetry-breaking phase transition of the Kitaev-Kondo model is controlled by the critical fixed point  $P_c^{(1)}$ .

where  $v$  and  $c$  denote the velocities of Dirac electrons and Majorana fermions, respectively, and we have defined

$$\int_q^> = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} dq_0 \int_{\Lambda e^{-d\ell} \leq |\mathbf{q}| \leq \Lambda} d^2\mathbf{q}, \quad (22)$$

for brevity. The resulting RG equations for the rescaled interactions  $\tilde{g}_i = \frac{1}{8\pi} \frac{\Lambda}{v} g_i$  and  $\tilde{\Gamma} = \frac{1}{8\pi} \frac{\Lambda}{v} \Gamma$  read

$$\frac{d\tilde{U}}{d\ell} = -\tilde{U} - 4\tilde{\rho}^2 + 12\tilde{J}^2 + 4\tilde{U}(\tilde{\rho} + 3\tilde{J}), \quad (23)$$

$$\frac{d\tilde{J}}{d\ell} = -\tilde{J} + \tilde{U}^2 - \tilde{\rho}^2 + 7\tilde{J}^2 + 2\tilde{J}(2\tilde{U} + \tilde{\rho}) + \frac{v}{4c}\tilde{\Gamma}^2, \quad (24)$$

$$\frac{d\tilde{\rho}}{d\ell} = -\tilde{\rho} + \tilde{U}^2 + 3\tilde{\rho}^2 + 3\tilde{J}^2 - 2\tilde{\rho}(3\tilde{J} + 2\tilde{U}) + \frac{v}{4c}\tilde{\Gamma}^2, \quad (25)$$

$$\frac{d\tilde{\Gamma}}{d\ell} = -\tilde{\Gamma} + \frac{2v}{c+v}\tilde{\Gamma}^2 + 2\tilde{\Gamma}(\tilde{\rho} + 3\tilde{J}). \quad (26)$$

As a first step we numerically integrate the RG equations, starting with different initial values  $\tilde{g}_i(0)$  and  $\tilde{\Gamma}(0)$ . As discussed in Sec. III, the ratio  $|\tilde{J}(0)|/\tilde{U}(0) \approx 0.146$  is fixed while the ratio  $\tilde{\Gamma}(0)/\tilde{U}(0)$  is a function of the velocity ratio  $c/v$  (see Fig. 2). The bare value of the density-density interaction is zero,  $\tilde{\rho}(0) = 0$ . The sign of  $\tilde{J}(0)$  can be positive or negative, depending on the sign of the microscopic Kitaev exchange. The overall energy scale of the bare interactions can be tuned by the Kondo coupling  $J_K$ . For small  $J_K$ , all interactions will renormalize to zero, corresponding to the FL\* phase. For  $J_K$  above a critical value at least some interactions will diverge, indicative of symmetry breaking.

In Figs. 4(a) and 4(b) we show the scale dependent interactions for different values of  $c/v$  and both positive and negative signs of  $\tilde{J}(0)$ . In all cases we use bisection to find  $J_K$  slightly above and infinitesimally close to the critical value. This means that over a large range of scales  $\ell$  the trajectories will be stalled very close to a critical fixed point, corresponding to plateaus of  $\tilde{g}_i(\ell), \tilde{\Gamma}(\ell)$ .

We find that  $\tilde{\Gamma}(\ell)$  always renormalizes to zero, even for large  $c/v$  where  $\tilde{\Gamma}$  is initially the largest interaction. This means that the Majorana fermions decouple from the conduction electrons. Moreover, a positive interaction  $\tilde{\rho}$  is generated under the RG. Interestingly, the critical behavior is always controlled by the same fixed point, irrespective of the ratio  $c/v$  and the initial sign of  $\tilde{J}$ . At the critical fixed point we obtain  $\tilde{\Gamma} = 0$ ,  $\tilde{U}_c \approx 0.1027$ ,  $\tilde{J}_c \approx 0.053$ , and  $\tilde{\rho}_c \approx 0.011$  from the plateau values.

To better understand the critical behavior we analyze the RG equations for  $\tilde{\Gamma} = 0$ . In addition to the trivial noninteracting FL\* fixed point  $P_0$ , the coupled RG equations for  $\tilde{U}$ ,  $\tilde{J}$ ,  $\tilde{\rho}$  exhibit three nontrivial fixed points  $P_c^{(1)}$ ,  $P_c^{(2)}$ , and  $P_s$ , which are all in the domain  $\tilde{\rho} > 0$  and  $\tilde{J} \geq 0$  [see Fig. 4(c)]. The nontrivial fixed points are located on the critical surface that separates the FL\* phase at weak interactions from the phases where interactions diverge under the perturbative RG scheme, signaling some form of symmetry breaking.  $P_c^{(1)}$  and  $P_c^{(2)}$  are located on different sheets of the critical surface, shaded in orange and purple in Fig. 4(c), and are stable fixed points along the tangential directions. They are the critical fixed points that correspond to different types of symmetry breaking. The metastable fixed point  $P_s$  is located on the separatrix between the two sheets.

From the plateau values of  $\tilde{g}_i(\ell)$  [see Figs. 4(a) and 4(b)], it is clear that the critical fixed point that controls the symmetry breaking transition of the Kitaev-Kondo model is given by  $P_c^{(1)}$  at

$$\tilde{U}_c = \frac{2\sqrt{3}-1}{24}, \quad \tilde{J}_c = \frac{3-\sqrt{3}}{24}, \quad \text{and} \quad \tilde{\rho}_c = \frac{2-\sqrt{3}}{24}. \quad (27)$$

## V. FIELD RENORMALIZATION AND SUSCEPTIBILITY EXPONENTS

Since all three electron-electron interactions are finite at the critical fixed point  $P_c^{(1)}$  it requires a more careful analysis to identify the type of symmetry breaking that occurs at the phase transition. While sufficiently strong positive  $\tilde{\rho}$  and  $\tilde{U}$

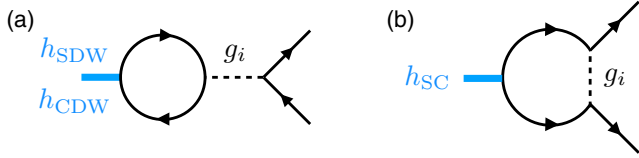


FIG. 5. Diagrams that contribute to the renormalization of the fields (a)  $h_{\text{CDW}}$ ,  $h_{\text{SDW}}$ , and (b)  $h_{\text{SC}}$  to linear order.

favor CDW and SDW orders, respectively, it was found that a positive  $\tilde{J}$  can lead to an instability toward  $d + id$  superconductivity, at least away from half-filling [13].

The susceptibility exponent  $\gamma$  for a given order parameter can be obtained from the field scaling exponent  $y_h$  of the conjugate field  $h$ , using the relation  $\gamma = (2y_h - D)\nu$ , where  $D$  is the dimension and  $\nu$  the correlation-length exponent [27]. For the quantum critical point of the Kitaev-Kondo model,  $D = 2 + 1$ .

The correlation length exponent  $\nu$  can be obtained from the divergence of the interaction parameters. Consider for example a simple situation where all interactions diverge in the same way, corresponding to linearized RG equations  $\delta'_i(\ell) = b\delta_i(\ell)$  for  $\delta_i \in \{\tilde{U}, \tilde{J}, \tilde{\rho}\}$  with some coefficient  $b > 0$ , resulting in  $\delta_i(\ell) = \delta_i(0)e^{b\ell}$ . We can define the correlation length  $\xi \sim e^{\ell^*}$  from the scale  $\ell^*$  where the interactions become of order one,  $\delta_i(\ell^*) \simeq 1$ . This results in  $\xi \sim \delta_i(0)^{-1/b}$  and hence  $\nu = 1/b$ .

In the present situation, linearizing the RG equations (23)–(25) for  $\tilde{g}_i \in \{\tilde{U}, \tilde{J}, \tilde{\rho}\}$  near  $P_c^{(1)}$  leads to coupled linear equations,  $\delta'_i(\ell) = \sum_j A_{ij}\delta_j(\ell)$ , where the matrix  $\mathbf{A}$  is completely determined by the critical values (27). The correlation length  $\nu$  is given by the inverse of the largest eigenvalue of  $\mathbf{A}$ , resulting in  $\nu = 1$ .

We first investigate how the fields conjugate to CDW and SDW order renormalize. The corresponding field terms in the continuum field theory are given by

$$\mathcal{S}_{h,\text{CDW}}[\bar{\psi}, \psi] = -h_{\text{CDW}} \int_k \bar{\psi}_k(\tau_z \otimes \mathbf{1})\psi_k, \quad (28)$$

$$\mathcal{S}_{h,\text{SDW}}[\bar{\psi}, \psi] = -h_{\text{SDW}} \int_k \bar{\psi}_k(\tau_z \otimes \sigma_z)\psi_k, \quad (29)$$

where  $\tau_z$  and  $\sigma_z$  denote Pauli matrices in sublattice and spin space, respectively. The diagrams that contribute to the field renormalization to linear order are shown in Fig. 5(a), resulting in

$$\frac{dh_{\text{CDW}}}{d\ell} = (1 - 4\tilde{U} + 8\tilde{\rho})h_{\text{CDW}}, \quad (30)$$

$$\frac{dh_{\text{SDW}}}{d\ell} = (1 + 4\tilde{U} + 8\tilde{J})h_{\text{SDW}}. \quad (31)$$

It was found that for an antiferromagnetic  $\tilde{J}$ , the leading SC instability of the lattice model away from half filling is in the spin-singlet channel across neighboring sites with a spatial  $d + id$  structure, described by the pairing term [13]

$$\begin{aligned} \hat{\mathcal{H}}_{\text{SC}} &= \sum_{\gamma=x,y,z} \sum_{\langle i,j \rangle_\gamma} \Delta_\gamma \sum_{v,v'} \hat{c}_{iv}^\dagger (i\sigma_y)_{vv'} \hat{c}_{jv'}^\dagger + \text{H.c.} \\ &= \sum_{\mathbf{k}, \gamma} \Delta_\gamma e^{i\mathbf{k}\delta_\gamma} \sum_{v,v'} \hat{c}_{Av}^\dagger(\mathbf{k}) (i\sigma_y)_{vv'} \hat{c}_{Bv'}^\dagger(-\mathbf{k}) + \text{H.c.}, \quad (32) \end{aligned}$$

with SC order parameter  $\vec{\Delta} = \Delta \vec{d}$ ,

$$\vec{d} = \vec{d}_{x^2-y^2} + i\vec{d}_{xy} = \frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}. \quad (33)$$

To obtain the effective pairing term in the low-energy continuum field theory, we expand around the Dirac point  $\mathbf{K}_+$  and keep only the leading term,  $\sum_\gamma \Delta_\gamma e^{i\mathbf{k}\delta_\gamma} \approx \sqrt{6}\Delta$ . The resulting SC field term is given by

$$\begin{aligned} \mathcal{S}_{h,\text{SC}}[\bar{\psi}, \psi] &= -h_{\text{SC}} \int_k \{\bar{\psi}_k[\tau_x \otimes (i\sigma_y)]\bar{\psi}_{-k}^T \\ &\quad + \psi_k^T[\tau_x \otimes (-i\sigma_y)]\psi_{-k}\}, \quad (34) \end{aligned}$$

and the renormalization of  $h_{\text{SC}}$  to linear order obtained from the diagrams in Fig. 5(b), resulting in

$$\frac{dh_{\text{SC}}}{d\ell} = (1 + 6\tilde{J} - 2\tilde{\rho})h_{\text{SC}}. \quad (35)$$

Evaluating the field RG equations (30), (31), and (35) at the critical point  $P_c^{(1)}$  we obtain the field scaling exponents  $y_h$ , and from the relation  $\gamma = 2y_h - 3$  the susceptibility exponents

$$\gamma_{\text{CDW}} = -1 - 8\tilde{U}_c + 16\tilde{\rho}_c = -\frac{2}{3}(2\sqrt{3} - 1), \quad (36)$$

$$\gamma_{\text{SDW}} = -1 + 8\tilde{U}_c + 16\tilde{J}_c = \frac{2}{3}, \quad (37)$$

$$\gamma_{\text{SC}} = -1 + 12\tilde{J}_c - 4\tilde{\rho}_c = -\frac{1}{6}(2\sqrt{3} - 1). \quad (38)$$

Since only  $\gamma_{\text{SDW}}$  is positive, we conclude that the FL\* phase at half filling becomes unstable toward the formation of SDW order.

The Landau free-energy analysis of different SC order parameters of Ref. [13] is fairly general and should also apply to electron fillings close to the Dirac point. One would therefore expect that the chiral  $d \pm id$  state remains the superconducting state with the lowest free energy at half-filling. However, for completeness, we have checked superconducting states with other pairing symmetries. As one might expect, we found that triplet pairing is disfavored by antiferromagnetic  $J$ , while on-site singlet pairing is strongly suppressed by the Hubbard repulsion  $U$ .

## VI. DISCUSSION

We have analyzed the instabilities of the fractionalized Fermi liquid that arises in a Kondo lattice model of Dirac conduction electrons on the half-filled honeycomb lattice coupled to a Kitaev QSL. After fractionalizing the spin-1/2 operators into Majorana fermions, we have used second-order perturbation theory to obtain the interactions between the gapless fermion modes in the effective low-energy, continuum field theory. The bare theory contains three different interaction vertices, a Hubbard repulsion  $U$  and a spin-spin interaction  $J$  between Dirac electrons, as well as a coupling  $\Gamma$  between conduction electrons and Dirac Majorana fermions. We have analyzed the scale dependence of the interactions, using perturbative parquet RG. In addition to the aforementioned interactions, an electronic density-density repulsion  $\rho$  is generated.

Regardless of the ratio of Fermi velocities and the sign of the Kitaev coupling, the RG flow is always controlled by the same critical fixed point, at which the Dirac electrons decouple from the Majorana fermions and all three electron-electron interactions have finite positive values. This result suggests that it is justified to integrate out the Majorana fermions entirely, as done in Ref. [13]. However, we stress that the velocity ratio  $c/v$  could potentially renormalize beyond one-loop order, resulting in a different flow of the coupling  $\Gamma$  between conduction electrons and Dirac Majorana fermions. Velocity renormalization is a well known phenomenon of strongly coupled GNY theories, and perturbative RG, even beyond one-loop order, cannot unambiguously rule out the existence of a GNY critical fixed point with finite coupling to Majorana fermions.

To determine the type of symmetry breaking, we have derived the RG equations of the fields conjugate to CDW, SDW, and SC order, from which we determined the field scaling exponents at the critical point and the susceptibility exponents. For the SC order parameter we have focused on singlet pairing across neighboring sites with a spatial  $d + id$  form factor [13]. While the antiferromagnetic spin-spin interaction  $J$  stabilizes antiferromagnetism and SC in almost equal terms, the AFM ordering is strongly enhanced by the Hubbard repulsion  $U$ , whereas SC is destabilized by the repulsive density-density interaction  $\rho$ . Our results show that the only divergent susceptibility is toward antiferromagnetic (SDW) order.

While the perturbative parquet RG is the method of choice to identify the leading symmetry-breaking instability in the presence of multiple competing interactions [14–17], the critical exponents for the correlation length  $\nu = 1$  and SDW susceptibility  $\gamma = 2/3$  obtained this way are only crude estimates. Ultimately, the SDW transition at half-filling should fall into the Gross-Neveu-Heisenberg universality class in 2+1 dimensions. From interpolation between series expansions near lower and upper critical dimensions [28] with previous results from fourth-order  $4 - \epsilon$  [29] and second-order  $1/N$  [30] expansions, functional RG [31,32], as well as quantum Monte Carlo simulations [33–42] estimates of  $1/\nu \approx 0.83$  for the inverse correlation length exponent and  $\eta_\phi \approx 1.01$  for the anomalous dimension of the order parameter field were obtained for  $N = 4$  (valley and spin) two-component Dirac fermion fields [28]. Assuming hyperscaling, this would imply  $\gamma = (2 - \eta_\phi)\nu \approx 1.19$ .

Our analysis is based on an effective continuum field theory and neglects lattice effects. According to the Landau cubic criterion, threefold clock terms, relevant for the honeycomb lattice, would render the transition first order. While such lattice terms are indeed relevant at the Wilson-Fisher fixed point, they turn irrelevant at fermion-induced critical points described by GNY field theories [43,44]. This does not rule out however, that fluctuations from shorter length scales induce first-order behavior before the coarse grained continuum description becomes applicable.

Our results are not in direct contradiction with previous studies of the Kitaev-Kondo model on the honeycomb lattice. Mean field investigations [10,11] found that for a FM Kitaev coupling and electron fillings away from the Dirac point, a  $p$ -wave SC state forms between the FL\* phase and the heavy FL. On approaching half-filling, the FL\*/SC transition moves

toward stronger coupling and the region of superconductivity shrinks to almost zero [10]. At half filling one would expect that fluctuations beyond mean-field theory play a crucial role. The strong renormalization of the interactions parameters indeed confirms this. Most notable are the fluctuation-driven generation of the density-density repulsion  $\rho$ , which disfavors SC, and the sign change of the spin interaction  $J$  from FM to AFM.

In a recent study [13] of the AFM Kitaev-Kondo model, all degrees of freedom associated with the Kitaev sector were integrated out and the resulting electronic Hamiltonian analyzed using functional RG (fRG). The fRG found an instability toward a SDW at electron fillings close to the van Hove singularity and a transition to a  $d + id$  SC state at lower fillings, but still far above the Dirac points. Unfortunately, it was not possible to analyze the behavior at fillings close to the Dirac point, due to finite momentum resolution [13]. It is likely that the AFM state only exists very close to half-filling and that lattice effects, which are included in the fRG, play an essential role in stabilizing exotic superconductivity at larger fillings. On the lattice, the electronic spin interactions inherit the bond-directional dependence of the Kitaev exchange. This frustration is likely to destabilize AFM order.

To better understand the instabilities of different types of fractionalized Fermi liquids toward exotic superconductivity and other broken symmetry states it will be important to investigate other toy models that host fractionalized FL\* phases. In a very recent study [45] it was demonstrated that when the so called “Yao-Lee model” [46], a spin-orbital generalization of the Kitaev model, is Kondo coupled to conduction electrons, the exchange of Majorana spinons can drive a (fractionalized) topological  $d + id$  pairing instability in the weak-coupling regime. In future studies it would be interesting to allow for Kondo coupling not only to the SU(2) symmetric local moment spins but also to the Kitaev orbital pseudospin sector and to systematically investigate the dependence on the electron filling.

A frustrated Kondo lattice model with an FL\* phase that is amenable to sign free auxiliary-field quantum Monte Carlo simulations was constructed [5] by coupling Dirac electrons on the honeycomb lattice to spin-1/2 degrees of freedom on the Kagome lattice. The interactions between the spins are chosen along the lines of the Balents-Fisher-Girvin model [47] which is known to host a  $Z_2$  spin-liquid and a ferromagnetic phase. While the authors characterized the FL\* phase in great detail through spectral functions and mutual information between electrons and spins, they did not investigate possible superconducting instabilities.

Studies of various toy models have shown that fractionalized Fermi liquids naturally arise when quantum spin liquids are weakly coupled to conduction electrons in a Kondo lattice setting. The emergent fermions in such FL\* phases mediate effective electron-electron interactions that can drive instabilities toward exotic superconductivity or other symmetry-broken phases. The emergent fermions therefore act as pairing glue similar to the phonons in conventional BCS superconductivity. Our work shows that at the quantum critical point of the FL\* phase of the half-filled Kitaev-Kondo model, the emergent Majorana fermions decouple from the conduction electrons. It remains an intriguing question whether this

behavior is generic or if certain models can support quantum critical points at which emergent fermions and conduction electrons remain strongly coupled, opening up the possibility for the formation of even more exotic quantum phases.

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### DATA AVAILABILITY

No data were created or analyzed in this study.

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