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Quantum Phase Transitions
Problem Set 2

☞ *Continuous assessment is based on three homework sets.*
☞ *Please hand in your solutions to problem set 2 on or before Monday, May 20.*
☞ *Please scan your solutions and send them in pdf format to frank.kruger@st-andrews.ac.uk (or upload to MY.SUPA).*

Dynamical Critical Exponent and Upper Critical Dimension [2+2+2+2+2+2=12 points]

Determine the dynamical critical exponent z and the upper critical dimension $d_{\rm u}$ for the quantum phase transitions described by the action

$$S = \int \mathrm{d}\tau \int \mathrm{d}^d\mathbf{r} \left\{ \left(\int \mathrm{d}\tau' \int \mathrm{d}^d\mathbf{r}' \Phi(\mathbf{r},\tau) K_0(\mathbf{r}-\mathbf{r}',\tau-\tau') \Phi(\mathbf{r}',\tau') \right) + a\Phi^2(\mathbf{r},\tau) + b\Phi^4(\mathbf{r},\tau) \right\},$$

where Φ has N real components, $\Phi^2 = \sum_{i=1}^N \phi_i^2$, and the kernel $K_0(\mathbf{r},\tau)$ for small frequency and wavevector is given by:

(a) $K_0(\mathbf{k},\omega) = c|\omega| + dk^2$ (describes the $T = 0$ antiferromagnetic transition for itinerant electrons when $N = 3$),

(b) $K_0(\mathbf{k},\omega) = ic\omega + dk^2$ (superfluid transition when $N = 2$),

(c) $K_0(\mathbf{k},\omega) = c|\omega|/k + dk^2$ (ferromagnetic transition for itinerant electrons when $N = 3$),

(d) $K_0(\mathbf{k},\omega) = c|\omega|/k^2 + dk^2$ (ferromagnetic transition for dirty itinerant electrons when $N = 3$),

(e) $K_0(\mathbf{k},\omega) = c\omega^2 + dk^2$ (superfluid transition for bosons on a lattice and at a commensurate density when $N = 2$), and

(f) $K_0(\mathbf{k},\omega) = c|\omega| + d\omega^2 + ek^2$ (superconducting transition for dirty d-wave superconductor when $N = 2$),

where $k = |\mathbf{k}|$ and a, b, c, d, and e are non-zero constants.