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UNIVERSITY COLLEGE LONDON
DEPARTMENT OF PHYSICS AND ASTRONOMY

PHAS0009: MATHEMATICAL METHODS II

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These notes are intended as an aid to revision, but they do not contain all the examples and diagrams from the course and are a supplement, not a substitute for your own notes taken in lectures.

They are also still being revised, added to and corrected.

Contents

1	Differential Vector Operators	4
1.1	Scalar and Vector Fields	4
1.2	Partial Derivatives of Fields	5
1.3	Directional Derivative and Gradient	6
1.4	The Total Differential of Fields	8
1.5	Divergence and Curl of Vector Fields	9
1.6	Product Rules	11
1.7	2nd Order Variations of Fields, Laplace Operator	12
2	Multidimensional Integration	14
2.1	Line Integrals	14
2.1.1	Mathematical Concept	14
2.1.2	Work Done in Moving a Particle Along a Path	15
2.1.3	Practical Evaluation of Line Integrals	15
2.1.4	Conservative Vector Fields	17
2.1.5	Worked Example	19
2.2	Area Integrals	20
2.2.1	Mathematical Concept	20
2.2.2	Area Integrals in Physics	20
2.2.3	Non-Rectangular Regions	22
2.2.4	Worked Example	23
2.2.5	Polar Coordinates	24
2.2.6	Worked Examples	27
2.3	Volume Integrals	28
2.3.1	Cylindrical Coordinates	29
2.3.2	Spherical Coordinates	30
2.3.3	Worked Examples	32
2.4	Surface Integrals	34
2.4.1	Surface Integrals in Spherical Coordinates	36
2.4.2	Surfaces Defined by $z = g(x, y)$	37
2.4.3	Worked Examples	38
2.5	Gauss's Divergence Theorem	43
2.6	Stokes's Theorem	46
2.7	Worked Examples	49

3	Ordinary Differential Equations	51
3.1	Introduction and Simple Examples	51
3.1.1	Terminology	53
3.1.2	A Simple Example	54
3.1.3	Fixing the Arbitrary Constants	55
3.2	Separable First-Order ODE's	56
3.2.1	Worked Examples	57
3.3	Linear First-Order ODE's	60
3.3.1	'Integrating Factor' Method	60
3.3.2	Worked Examples	62
3.4	Perfect Differential Method	64
3.4.1	Worked Example	66
3.5	Second-Order Linear ODE's with Constant Coefficients	67
3.5.1	Homogeneous ODE's	67
3.5.2	Worked Examples	70
3.5.3	Inhomogeneous ODE's	72
3.5.4	Worked Examples	75