Least-cost networks

by

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Abstract. Many archaeologists applying GIS software for least-cost studies do not fully understand the functions employed to create least-cost networks, and so they are not aware of the prerequisites and drawbacks of the model implemented by the software. This paper discusses several models for connecting dots on the map resulting in least-cost networks: all pair shortest paths, least-cost network to the builder, network connecting the nearest neighbours, triangulation network, minimum spanning tree, central point network, and main travel route with subsidiary paths. Most of the models are applied to a study region in the Bergisches Land, Germany. This region provides less attractive natural conditions than the area close to the Rhine, and for this reason, only very few settlements existed in this region prior to the year 1000 AD. Detailed historical sources and old maps are available to check whether the models agree with the historical reality.

Key words: least-cost paths, networks, triangulation, minimum spanning tree, Steiner tree

Introduction

Many archaeological least-cost path (LCP) studies deal with networks, i.e. the aim is to reconstruct a network of routes (e.g. van Leusen 2002, 16·12–16·18; Bell et al. 2002). This paper discusses a set of models each minimizing costs in some sense but resulting in a different network topology. Unfortunately, most push-button GIS software packages do not support different LCP network models, and hardly any information can be found in the documentation of the software concerning the models and algorithms used for calculating least-cost networks. Consequently, most archaeologists applying this software are not aware that different models exist and that the software might not be appropriate for the model they have in mind.

The first part of this paper deals with models where all nodes of the network are known. In this case, the aim is to reconstruct links between these nodes so that the network costs are minimized according to some criteria. The second part introduces methods for reconstructing paths when only some important locations are given or a main travel route is known.

A hilly study area east of Cologne, Germany, serves as a test landscape. This area was sparsely populated before Medieval times because the other regions of the Rhineland offered more attractive natural conditions for farming. From the 9th century onward, nearly all the fertile regions of the Rhineland were occupied by farms, and at the same time the climatic conditions for agriculture improved, so that the population increased in the study area (Landschaftsverband Westfalen Lippe, and Landschaftsverband Rheinland 2007, on CD: 282). Therefore, historical data can be used to test the network models. For several parishes in the study area, a list of place names and the year when they were first mentioned was published (Pampus 1998). Nearly all of the place names could be located on maps created in the late 19th century, and this allowed georeferencing the centres of the settlements, most of which were still very small at the end of the 19th century. The parishes with listed place names cover about 448 km² of the study area, and within this area, 513 settlements were georeferenced that were first mentioned before 1500 AD. However, only 15 of these place names were mentioned in documents issued before 1150 AD.
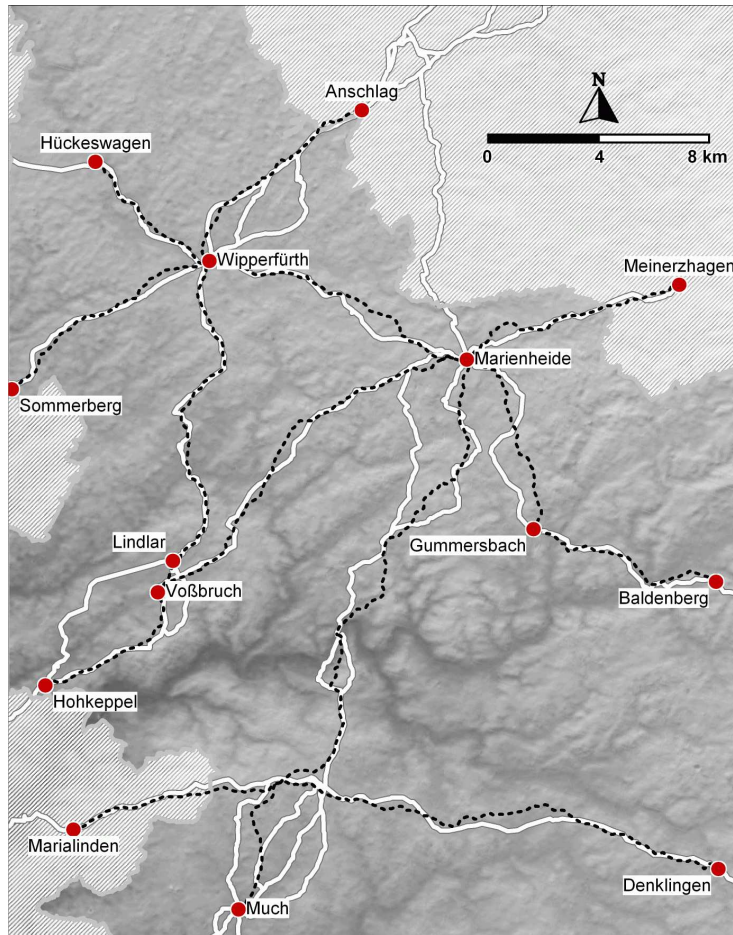
Fig. 1. Hilly study region east of Cologne, Germany. The background map is the ASTER DEM with a resolution of about 30 m, ASTER GDEM is a product of METI and NASA. White lines indicate the ancient trade routes, sometimes with alternative route sections. The black dotted lines are the LCPs agreeing best with the known trade routes. For the areas covered by a white pattern, Pampus (1998) does not provide any information on place names and the year when they were first mentioned.

In addition, a publication on Medieval trade routes is used in this study (Nicke 2001). The trade routes described by Nicke were tracked on historical maps from about 1845. Unfortunately, older maps do not cover the study area in total. So the trade routes shown in Fig. 1 rely on the assumption that the Medieval route layout is well preserved on the maps from about 1845. For each trade route two or more locations on the route were selected and LCPs were calculated with different cost components and cost weights. LCPs avoiding wet soils and with a critical slope of 13 % fitted best to the trade routes. Slope dependent costs were modelled by a quadratic function (Herzog in print) and for wet soils a multiplier of 5 was used. The reconstructed paths and the trade routes do not coincide perfectly, and this could be due to (i) inaccuracies and resolution of the elevation model (ASTER data provided by NASA and METI), (ii) natural and man-made landscape modifications (e.g. quarries, meandering of rivers, dam construction for water reservoirs), (iii) additional cost components not considered here or (iv) inaccuracies in locating the trade routes. However, the cost model found is quite appropriate for most trade routes in the study area, and so it is used for all least-cost calculations presented in this paper.

The direction of the paths are not considered in this study, it is assumed that the same route is taken on the way from A to B and back. This requires an isotropic cost function, which can be generated easily by averaging the costs of movement in both directions (Herzog in print). This isotropic cost function is applied to effective slope derived from the digital elevation model as

described by Yu et al. (2003). The software used for this paper is part of a set of programs for least-cost calculations created by the author. Least-cost path concepts implemented in the software were already presented in more detail elsewhere (Herzog in print; Herzog 2013). The programs generate MapInfo Interchange Format (MIF) files that can be imported easily into MapInfo 8.0. The GIS MapInfo 8.0 with the plugin Vertical Mapper was used to create the maps in this contribution. In future it is planned to improve the user-interface of the software so that it can be made available to other people interested in archaeological least-cost analysis.

All pair shortest paths

Within an all pair shortest paths network, each pair of contemporary sites in a region is connected by an LCP. The multitude of paths created by Indians in California may serve as an example. Earle (1991, 11) refers to a historic report describing this network layout, which most likely was a result of the fact that hardly any effort was needed to create a path in fairly flat terrain, and so, whenever desirable a new path was established. On the same page he presents ethnographic evidence that in hunter and gatherer societies, paths are ephemeral, redundant and without centrality if the terrain is fairly flat.
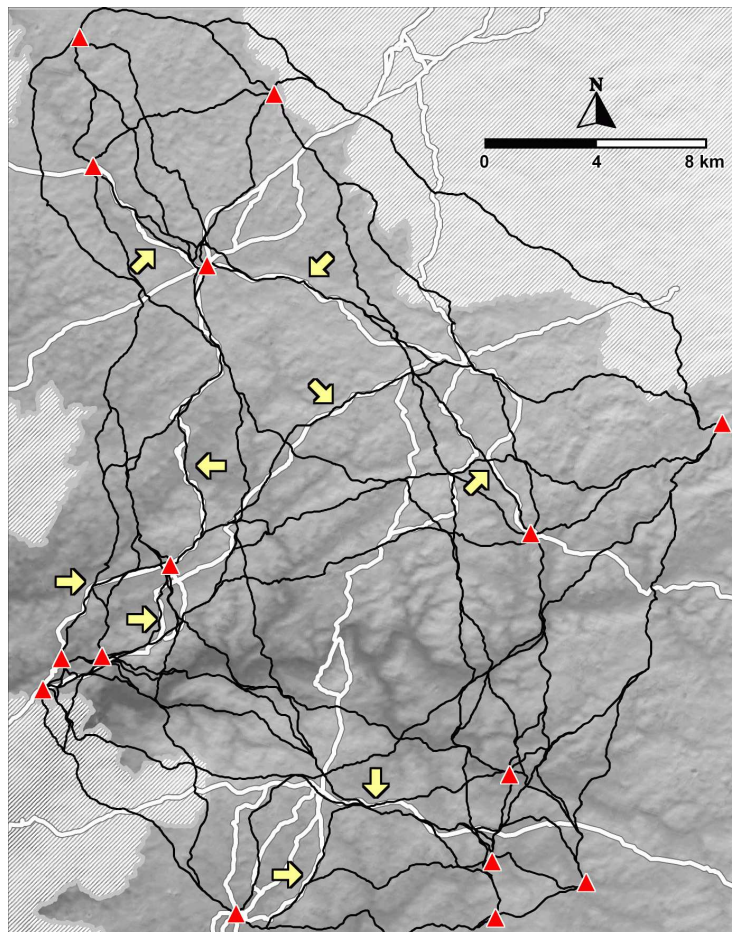

Fig. 2. All pair shortest paths network for the 15 settlements in the study area first mentioned before 1150 AD. The settlements are marked by triangles. The ancient trade routes are depicted as in Fig. 1, the black lines form the all pair shortest paths network consisting of 106 LCPs. Nine arrows indicate the path segments where the LCPs run along the trade routes.

The term *least-cost networks to the user* is used by Waugh (2000, 615) for this network type, because the traveller can move as quickly and directly as possible between any pair of sites in the network. According to this model, whenever a new site was founded within the region considered, this site established shortest path connections to all other sites that existed previously. The number of connections required has a quadratic growth rate, and this is one of the reasons why in general such networks consist of a limited number of sites only. Such networks are highly redundant and without centrality. The relative chronology of the paths cannot be deduced from the path layout. Effective algorithms solving this all-pairs shortest paths problem have been published (Cormen et al. 2001, 620–42).

Due to the quadratic growth rate of the number of connections, only the all pair shortest paths network for the 15 settlements mentioned before 1150 AD was generated. This network consists of 106 LCPs, and nine path segments of this network coincide with the ancient trade routes (Fig. 2).

Least-cost network to the builder

A society living on crops grown in fields near the villages will not tolerate innumerable paths crossing these fields. Instead the route network should cover only a minimum amount of the valuable land used for agrarian purposes. Similarly, if some effort is required for creating paths, connecting all point pairs including those at large distances by LCPs is not effective.

A *least-cost network to the builder* (Waugh 2000, 615) connects all sites but minimizes the total costs of route construction. With respect to modern geography, Waugh notes that such networks are found in areas with sparse population and in places where road construction costs are high. In such a network, long detours are often necessary to reach a neighbouring location.

A configuration with extra intermediate points is optimal in terms of total path costs. Such intermediate points are known as Steiner point. In a landscape with constant friction, *Steiner trees* (Smith et al. 2007, 341) are optimal networks to the builder; however, calculation complexity is high. Due to this fact an implementation of this network layout was not attempted, instead readers are referred to the work of Verhagen et al. (2011) on this topic.

All site locations must be known for creating this network. Consequently, a chronological development of this network is not part of the model.

For a given set of site locations, the sum of costs for all paths in the least-cost network to the user may be regarded as an upper bound of the total path costs in the network connecting the sites, whereas the total cost of paths in the least-cost network to the builder serves as a lower bound. Depending on the landscape, the technological skills and the travelling requirements of the society considered, an ancient route network can be ranked somewhere in the range between these extremes. Models for networks that are a compromise between the two extremes are presented in the next sections.

LCPs to the Nearest Neighbours

The model underlying proximal point analysis is based on the assumption that establishing a route to the neighbouring village is more important than the connection to a settlement at a larger distance.

According to Terrel and Welsch (1997), Terrell introduced a simple graph-theoretic technique to model likely interaction spheres, a technique he called proximal point analysis in 1974. This technique was applied by Spennemann (2003) in his attempt to reconstruct the road network on the island of Tongatapu, Kingdom of Tonga. The resulting network connects each settlement to its three nearest neighbours by straight lines.

White (2012) extends this approach by using LCPs instead of straight-line connections and by considering a varying number of neighbours. After analysing the results he comes to the conclusion that up to five neighbours are appropriate for his study area. A disadvantage of this approach is that the resulting network may consist of several connected components, especially if sites are clustered. The examples presented by White exhibit this effect, although the connected components are not necessarily separated by large distances. Within a connected component of a network taking the n nearest neighbours into account, long detours are sometimes necessary to reach a close neighbour with rank n+1. These drawbacks are avoided by triangulation networks.

Least-cost Triangulation Network

Least-cost triangulation networks connect each site with its immediate neighbours by LCPs that form a triangulation. A triangulation LCP network is based on a heuristic approach with the aim of ensuring good connection to neighbouring settlements in all directions; settlements at a distance can only be reached via intermediate stops at neighbouring settlements. With straight-line distances, there are several candidate triangulations for a given set of points. A Delaunay triangulation avoids long thin triangles and acute angles and therefore is preferred in many applications (e.g. Worboys and Duckham 2004, 190–92; Smith 2007, 113–15).

Most GIS software packages support the creation of a straight-line Delaunay triangulation for a given set of irregularly spaced points. On average, for each location the number of neighbours found by the Delaunay triangulation is six. In network analysis, the nodal degree is the number of paths that are connected to a particular seed location in the network (Conolly and Lake 2006, 241). The nodal degree is often considered as a measure of importance or accessibility of a location (e.g. Rodrigue et al. 2009, 29; Llobera et al. 2011). Consequently, the LCP triangulation network model is appropriate only for seed locations of similar importance.

If most interaction takes place between neighbours of roughly the same importance, this network probably is an appropriate compromise between the two extremes – least-cost network to the builder and to the user. Such networks are with hardly any centrality. In a straight-line Delaunay triangulation, adding a new point can change many triangles in the network; however, when adding a new site to an existing road network, such a radical change is highly unlikely.

The following algorithm was used to create least-cost triangulation networks for a given set of seed points:

- Calculate least-cost buffers for the seed cells.

- Assign to each cell the number of the least-cost nearest seed cell, resulting in least-cost Voronoi patches (Fig. 3).

- Neighbours are defined: The seeds of two Voronoi patches are neighbours if the common boundary exceeds a predefined length.

- The quick and dirty least-cost triangulation connects neighbours via LCPs that traverse the common boundary (see below).

The basis of this algorithm are two facts: (1) A cumulative cost surface with several seed points allows to identify maximum cost boundaries that delimit least-cost Voronoi patches (van Leusen 2002, 16·17–16·18; Smith et al. 2007, 116); (2) when considering straight-line distances, a Delaunay triangulation results from connecting all centres of neighbouring Voronoi patches (e.g. Worboys and Duckham 2004, 191, 206). Consequently, a least-cost triangulation can be derived from least-cost Voronoi patches. If the triangulation includes LCPs that are very long and costly in comparison to the average LCPs in the triangulation, it might be useful to delete these LCPs after choosing an appropriate threshold, e.g. by outlier detection (Smith et al. 2007, 181–83).
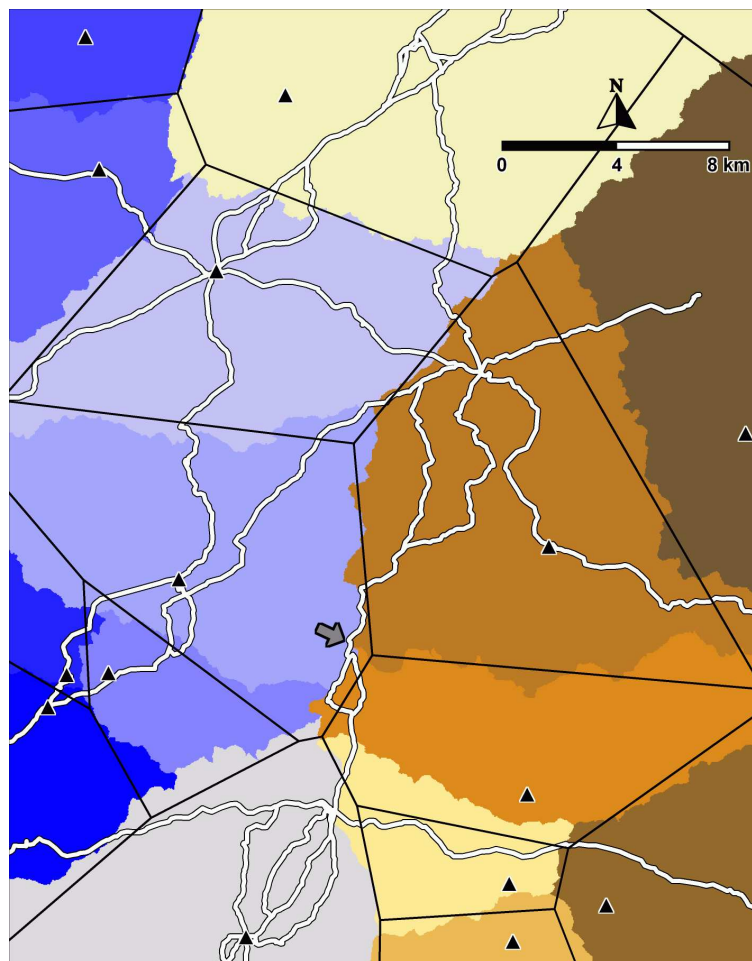

Fig. 3. Comparison of least-cost Voronoi patches and straight-line Voronoi polygons for the 15 settlements in the study area first mentioned before 1150 AD. The arrow points to a section of the ancient road Zeitstraße that coincides fairly well with the boundaries between the least-cost patches.

Fig. 3 shows the least-cost Voronoi patches for the 15 settlements mentioned before 1150 AD. The standard Voronoi tessellation is indicated by black straight lines. The two Voronoi diagrams differ in some places so that the adjacent neighbours of a site's territory do not always agree. A section of the Zeitstraße, which could not be reconstructed successfully by the cost model applied in this paper, agrees well with the boundary of a Voronoi patch. Several publications note that Medieval routes sometimes coincide with a boundary (e.g. Gissel 1986). Although the first reconstruction attempt for the section of the Zeitstraße was not successful using the cost model, applying the cost model for generating least-cost Voronoi boundaries results in a fairly good match.
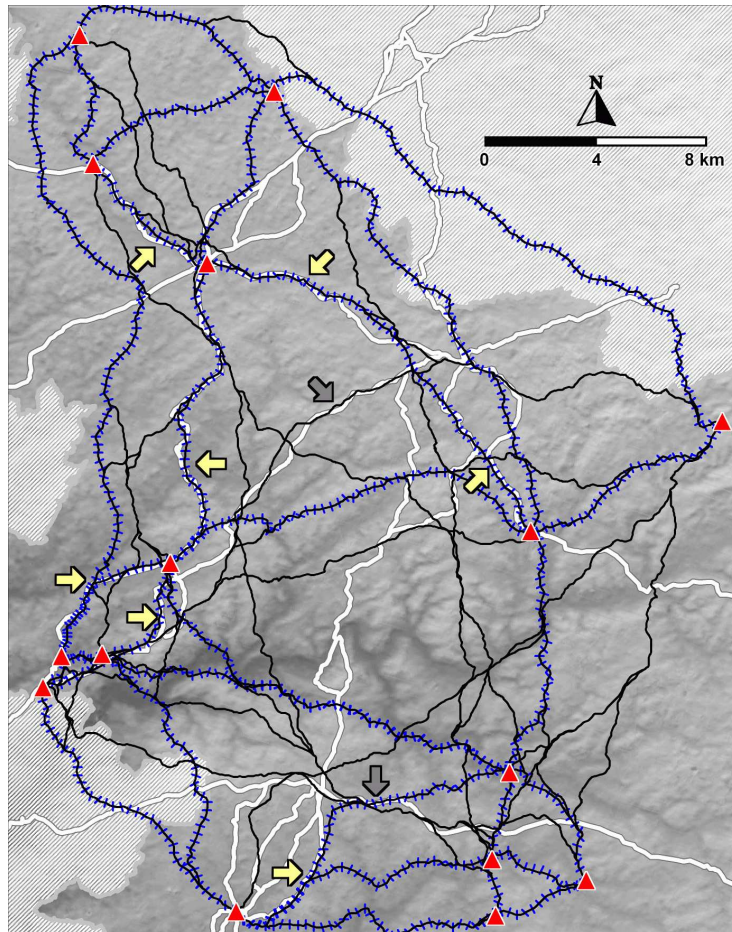

Fig. 4. Same as Fig. 2, in addition the least-cost triangulation network for the 15 settlements is marked by small (blue) bars perpendicular to the route. The colour of two of the arrows shown in Fig. 2 is changed to dark grey because the corresponding path segments are not part of the least-cost triangulation network.

The least-cost triangulation network for the 15 settlements mentioned before 1150 AD consists of 29 LCPs (Fig. 4). Although the number of links is considerably lower than that of the all pair shortest path network, seven out of the nine path segments coinciding with the ancient route are still part of the reconstructed network.

To speed up calculations, a quick and dirty triangulation was implemented that forces the connecting path between two neighbours across the common border. This results in the correct LCP in most but not in all cases. A proper implementation of the least-cost triangulation is planned in near future. Fig. 5 shows the result of this quick triangulation for the 34 settlements in the study area mentioned before 1250 AD. Although there are more paths than in Fig. 4, the number of segments where ancient trade routes and triangulation paths coincide does not increase.

Minimum Spanning Tree

A minimum spanning tree (MST) consists of edges that are part of the Delaunay triangulation (Cormen et al. 2001, 561–79; Smith et al. 2007, 339). Only n-1 connecting lines are in the MST if n is the number of seed locations, and therefore the total costs of all LCPs forming the network are significantly lower than in a least-cost triangulation network. Prim's algorithm (Cormen et al. 2001, 570–73) can be applied to identify the least-cost MST in the least-cost triangulation.

The total costs of all LCPs forming the network often exceeds that of the least-cost network to the builder. However, no additional Steiner points need to be constructed, and so generating a least-cost MST is easier. The Steiner ratio can be used to compare the efficiency of the MST with the optimal Steiner tree (Ganley 2004). This key figure is the largest possible ratio between the total length of a MST and the total length of the optimal Steiner tree. With straight-line distances, the ratio is conjectured to be about 1.15. So the total length of the worst-case MST exceeds that of the optimal Steiner tree by 15 %.

Due to the low number of connections, long detours are often necessary in a MST to reach a neighbouring location. Moreover, if a link is broken, network connectivity is lost. Smith et al. (2007, 339) point out that real-world networks normally are implemented with a higher level of connectivity to avoid this problem.
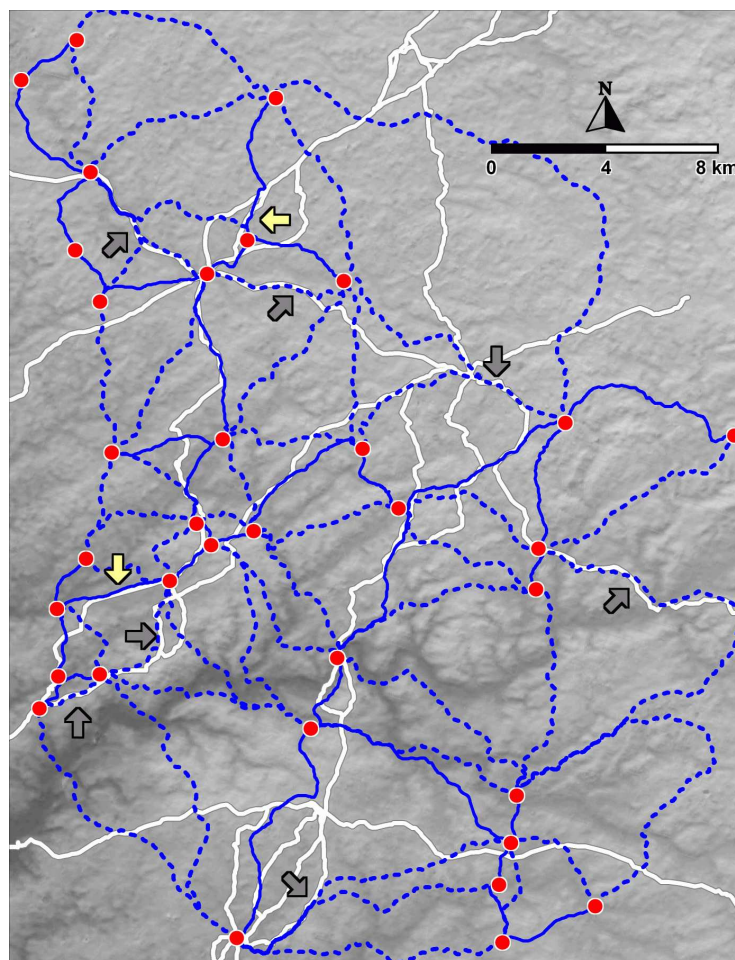

Fig. 5. Dot symbols indicate 34 settlements mentioned before 1250 AD. The 33 LCPs forming the least-cost MST are indicated by solid lines. The quick least-cost triangulation consists both of the solid and dotted (blue) lines. Dark grey arrows mark the path segments where the least-cost triangulation and the trade routes agree quite well. Light (yellow) arrows mark those LCP segments that form the least-cost MST and coincide with a trade route.

The solid (blue) lines in Fig. 5 form the least-cost MST for the 34 settlements in the study area mentioned before 1250 AD. The calculation is based on the quick triangulation. Only two segments of the least-cost MST coincide with the known trade routes.

Spennemann (2003) also applies two variants of the MST that take the importance of the nodes into consideration. In his example, the proxy for importance is the frequency of mentioning a settlement in historic sources. Spennemann uses the terms weighted MST and linked weighted MST in this context, but does not provide any references or a description of the methods. The results shown in the publication indicate that the number of edges of the weighted MST is higher than that of the MST, and with the linked weighted MST still more edges are added. In the example presented by Spennemann, the nodal degree of important sites within the linked weighted MST is higher than that of less important sites. This result indicates that the linked weighted MST approach justifies further investigation. Moreover, in many archaeological least-cost networks, the sites considered differ with respect to their importance, and only very few approaches are known that include importance into the calculation.

Least-cost Basin-Clustering

Rodrigue et al. (2009, 18–19) describe hub-and-spoke networks, where the hub serves as the centre of efficient distribution. Least-cost basin clustering can be applied to identify the hubs and the distribution links in such hub-and-spoke networks. Basin clustering was proposed by Hader and Hamprecht (2003), and a few years ago this approach was applied to Early Iron Age settlements and cemeteries from the Main triangle, Germany (Herzog 2009). At that time, only a straight-line version of the algorithm was implemented by the author, but the need was felt to account for the travel costs in the hilly study area. Finally, after long research into least-cost methods, first results of the least-cost version are shown in the present paper (Fig. 6).

The following algorithm was used to create the least-cost basin clustering for a given set of seed points:

- Calculate the least-cost Kernel Density Estimation (KDE) to derive a density value for each site (Herzog and Yépez in print). The user can control the impact of the sites by choosing the bandwidth parameter, corresponding to the cost distance where the impact of the site ceases. The scale of the clustering depends on the size of the bandwidth parameter. It is possible to refine standard least-cost KDE so that the importance of a site is reflected by the height of the kernel.

- Generate a least-cost triangulation to identify the relevant neighbours of each site.

- Establish for each site a single link to a neighbour with higher density. Hader and Hamprecht (2003) propose choosing the neighbour with highest gain, i.e. the neighbour with a higher density where the density difference versus path costs ratio is highest.

- Determine the clusters, i.e. linked groups of sites

- Identify the hubs, i.e. the site of maximum density within each cluster. These are considered as cluster centres.

Fig. 6 shows the result a least-cost basin clustering based on a quick triangulation for the 34 settlements in the study area mentioned before 1250 AD. The basin clustering network consists of six clusters and 27 LCPs. Four segments of these LCPs coincide quite well with

the ancient trade routes. Two of the cluster centres were probably important central places in Medieval times (Lindlar and Gummersbach). But the settlement in the study area that first received town privileges (Wipperfürth) is not a cluster centre.
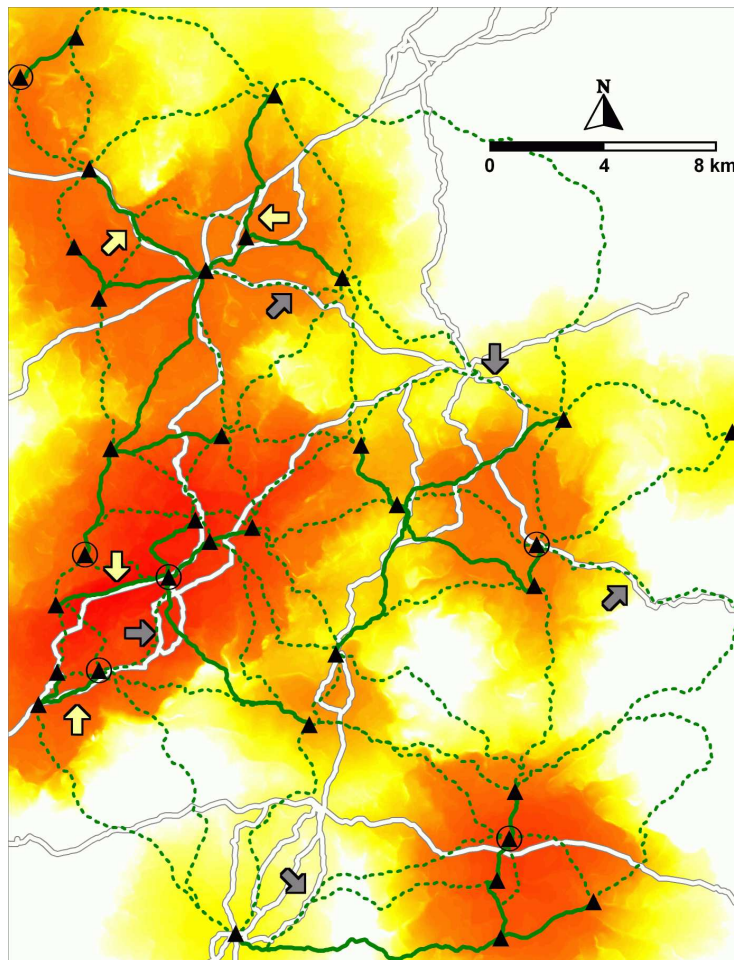

Fig. 6. Black triangles indicate settlements mentioned before 1250 AD. The quick least-cost triangulation consists of the solid and dotted (green) lines. The basin clustering of the settlements is based on the least-cost KDE with a spread parameter of 8 km for level dry ground. The least-cost KDE distribution is shown in the background. The cluster centres are marked by black rings. The LCPs forming the basin clustering are indicated by solid lines. Dark grey arrows mark the path segments where the least-cost triangulation and the trade routes agree quite well. Four light (yellow) arrows mark those LCP segments that are part of the basin clustering and coincide with a trade route.

So the result is far from satisfactory with respect to the cluster centres. This may be due to one of the following reasons: (1) An inappropriate bandwidth parameter was chosen; (2) The model is not appropriate for the Medieval settlements of the study area; (3) The outcomes of this approach suffer from the edge effect because important sites at the border of the study area are not recognized by the method. However, the latter could be fixed by assigning a higher weight to these sites by increasing the height of the kernel.

Central Point Networks

Central point networks spread from a single point, thus exhibiting a high degree of centrality. Van Leusen (2002, 16·12) modelled such a central point network by selecting appropriate end locations and calculating multiple LCPs connecting the central point Wroxeter with these end

locations. However, if probable end locations are not known, it is still possible to model central point networks. A method for generating such central point networks was proposed by Fábrega Álvarez and Parcero Oubiña (2007). They determine probable paths from an origin outward on the basis of the cost-surface: Destination points of these paths are the locations with maximum distance versus cost ratio, i.e. where large distances can be covered at low costs. This procedure is carried out for several origins, and the authors check if a connected route network results.

Another approach to model central point networks was published by Llobera together with Fábrega Álvarez and Parcero Oubiña (2011). A drain procedure is applied to the accumulated cost surface to identify potential paths to the origin. However, the drawbacks of a drain approach are well-known (Smith et al. 2007, 145–46), and it is for this reason that in my view, the initial method of Fábrega Álvarez and Parcero Oubiña is to be preferred.

A historic example of a central point network is given by Hindle (2002, 31–35). He analyses the Gough map of about 1360 covering most of England. He comes to the conclusion that the map reflects the centralisation of the government; a road system radiating from London is evident, despite some inexplicable omissions.

Based on the network structure, it does not seem possible to reconstruct the chronological sequence of the routes in such a network.
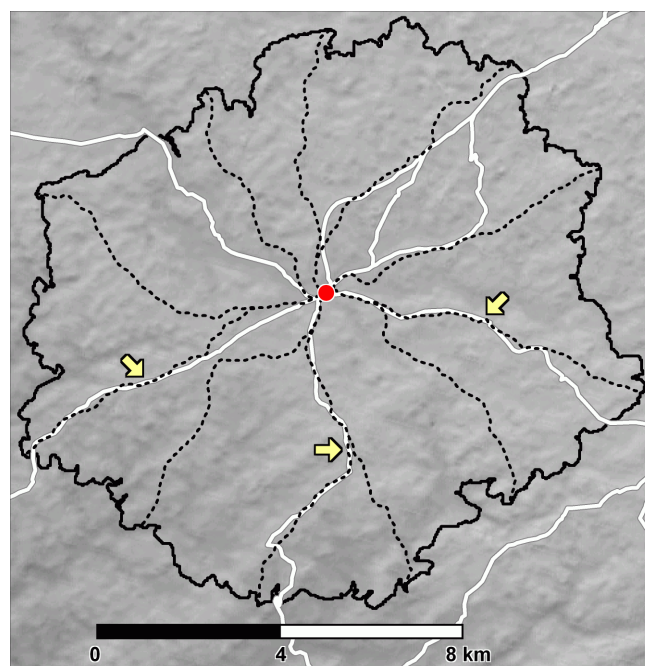

Fig. 7. Central point network (black dotted lines) consisting of 11 paths for the oldest town (Wipperfürth) in the study area. The cost buffer boundary depicted by a black solid line delimits all locations that can be reached by an effort less or equal to covering 9 km on level dry ground. The minimum straight-line distance between any of the reconstructed paths on this boundary was set to 3 km. White lines indicate the known trade routes, sometimes with alternative route sections. Three arrows mark the path segments where the reconstructed network and the trade routes agree quite well.

The following algorithm based on the ideas of Fábrega Álvarez and Parcero Oubiña (2007) is proposed to create a central point network for a given seed point:

- Create the least-cost buffer for the seed point with a user-selected radius (i.e. maximum cost value).

- Identify the location with maximum straight-line distance on the border of the buffer.

- The LCP to this location is the first path in the central point network.

- Successively identify the next best locations on the border of the buffer. Make sure that distinct paths are chosen by keeping a minimum straight-line distance to previously selected paths and by ensuring that the LCPs chosen represent local optima with respect to the neighbouring cells on the border.

Progression on the first paths selected in the process described above is best, and for this reason it seems more likely that these paths correspond to ancient routes than the other paths in the reconstructed network. However, this applies only if the first paths selected cover several directions. An example for an application of the central point network approach given above is depicted in Fig. 7. The centre of the network is Wipperfürth, the oldest town in the study area. Three arrows indicate sections where the reconstructed paths agree well with the known trade routes. These sections are part of the three paths selected first. Both Wipperfürth and Wiehl were first mentioned in 1131. But only one section of the central point network for Wiehl consisting of ten LCPs coincides with an ancient trade route (not shown).

Dendritic Networks

The central feature of a network could also be a line rather than a point, e.g. a river or a main travel route. The least-cost subsidiary paths together with the central line form a dendritic network. The algorithm proposed for generating central point networks can be readily extended for creating the subsidiary paths of a given main travel route. Instead of one seed point, all raster cells traversed by the main travel route form the seeds of the least-cost buffer. So this model assumes that two types of roads existed at a certain period of time, roads of different importance. Several road classes are known from Roman times (Heinz 2003, 22–26): main routes (viae publicae), local routes (viae vicinales) as well as private and military roads.

Bell et al. (2002) describe a central line network in Italy: The Sangro river valley probably was the principal long-distance artery, supplemented by local communications between sites in the same general area of the valley. However, the focus of the study by Bell et al. is only on the local communication routes, and these were reconstructed by generating LCPs connecting sites of a given period.

It is quite obvious that the central line must have existed before the subsidiary paths were created, however, the chronological sequence of the subsidiary paths cannot be deducted from the path layout. As with the central point network, progression on the first paths selected is best. If progression ease was the only criteria for creating subsidiary paths in the past, then we could assume that the longest paths were created first. However, in the example presented in Fig. 8 the longest reconstructed subsidiary path does not agree with an ancient trade route or any other path on a 19th century map. Out of the 19 reconstructed paths, paths number 4 and 10 agree in some sections with the known trade routes. Nearly all reconstructed paths including path no. 19 coincide with roads recorded on the 19th century maps.
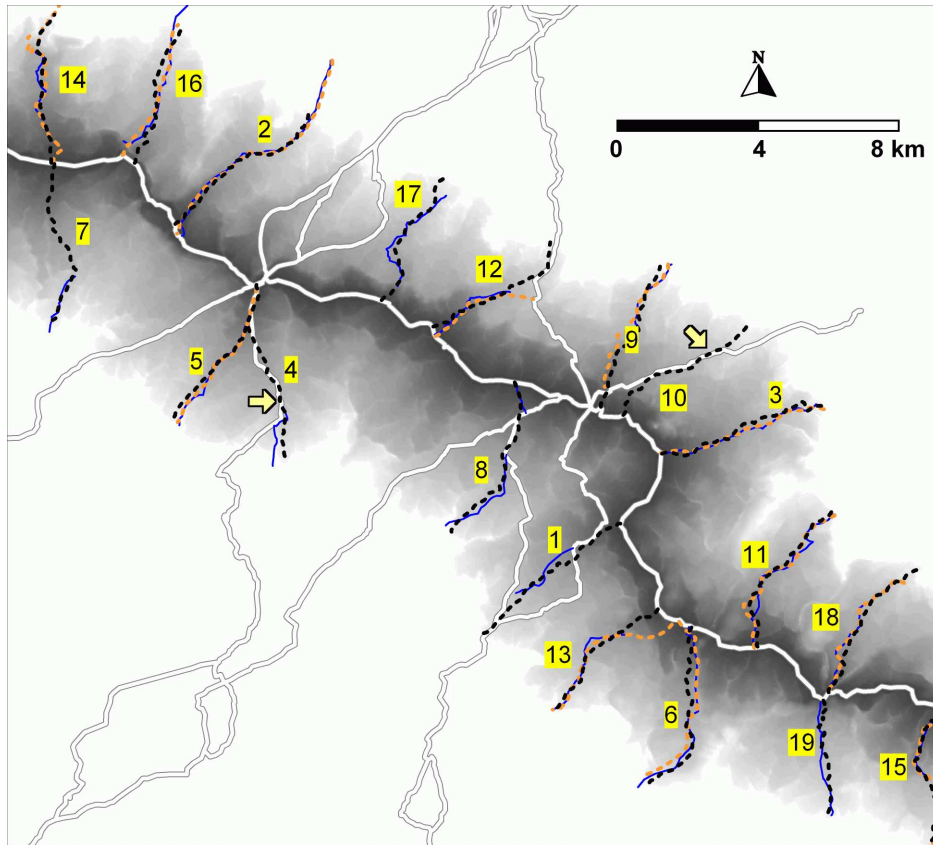
Fig. 8. Reconstructed subsidiary paths (black dotted lines) for the old trade route Eisenstraße. White lines indicate the known trade routes including the Eisenstraße. The least-cost buffer for the Eisenstraße with a limit corresponding to covering 6 km on dry level ground is shown by decreasing colour saturation. The numbers indicate the sequence of selecting the paths, i.e. the reconstructed subsidiary path no. 1 covers the longest distance. Two arrows mark the path sections where the reconstructed network and the trade routes agree quite well. Light (orange) dotted lines are digitised paths from the historic map issued in 1840, dark (blue) solid lines indicate roads on the 1890 map. Only paths coinciding roughly with the reconstructed paths were digitised.

Conclusions and Future Work

Many factors influence the outcome of LCP calculations besides the cost model, including resolution and quality of the topographic data, landscape changes since the period considered, as well as the least-cost algorithm and implementation.

When generating least-cost networks the situation is even more complicated, because additional issues should be considered:

- Did the network evolve, or were the paths constructed after contemporary sites had already been founded? Many studies and methods assume that roughly contemporary sites suddenly popped up out of nowhere. The question how the founders of the sites got to these places is hardly discussed at all. But often the previously existing road connections played an important role for the next period. An example is presented by Hindle (2002, 6, 31). According to Hindle, the Roman road system in England provided a basic network in Medieval times: On the Gough map of about 1360 covering most of England, almost 40 % of the routes shown are along the line of Roman roads. Least-cost Steiner-tree related methods suffer from the drawback that they do not answer the question concerning the

evolution of the network. Triangulations are not quite as problematic because adding a dot may change many triangles in the network; however, this happens only rarely.

- Are all the dots on the map of identical importance? During Medieval times, probably more paths led to a market town than to a single farmstead surrounded by fields. This example shows that the importance of a dot in terms of attracting people plays an important role. The triangulation approach assumes that all dots are of equal importance. According to my knowledge, the MST variants applied by Spennemann (2003) and basin clustering are the only methods taking the importance of the dots into account.

- What is the purpose of the road network considered? The cost factors governing the layout of specific route types and the topology of the networks vary depending on the purpose of the routes. Several road classes are known from Roman times (Heinz 2003, 22–26), and White (2012) presents a classification of four path types in desert areas, ranging from local paths connecting neighbouring settlements to interregional travel paths that often bypass settlement areas. Main routes may evolve from paths connecting neighbouring settlements but may also result from the need for a long-distance connection. Hindle (2002) discusses in some detail English Medieval roads with different purposes: church and corpse roads (pp. 10–12), pilgrim routes (pp. 12–14), monastic routes (p. 14), and drove roads (pp. 14–16). According to Hindle, drove roads tended to avoid villages.

- How important was movement and how costly was it to create paths? If paths are used rarely or if the costs of path construction are high, network layouts similar to optimal networks to the builder are to be expected. However, sometimes a compromise towards a more user-friendly network is needed. Helbing et al. (1997) suggest a method for constructing networks in the range between builder- and user-friendliness.

- Jiménez and Chapman (2002) connect locations by networks called Beta-skeletons. This concept includes both Gabriel networks (see also Smith et al. 2007, 339–341) and Relative Neighbourhood networks. The parameter beta allows the user to identify neighbours on different scales. If beta is set to 2, the resulting graph is a subset of the Delaunay triangulation and a superset of the MST. Implementing and testing this method on a least-cost basis is a future task.

Method	Number of nodes	Number of paths	Coinciding route sections	Depicted in Fig.
All pair shortest paths network	15	106	9	2
Least-cost triangulation network	15	29	7	4
Least-cost triangulation network	34	81	9	5
Least-cost minimum spanning tree	34	33	2	5
Least-cost basin clustering	34	27	4	6

Table 1: Least-cost network methods implemented by the author and results for the trade routes and Medieval settlements in the study area. The focus is on networks connecting a given set of dots. The ratio of the number of coinciding route sections versus the total number of paths in the network is highest for the triangulation network based on settlements mentioned before 1150 AD.

Due to the large variety of factors influencing the outcome of least-cost networks calculations, it is necessary to include ground truthing and checks for equifinality in archaeological studies trying to reconstruct path networks of past times. The examples from the study area in the Bergisches Land show that most of the models discussed in this paper lead to some good reconstruction results (Tab. 1). So a network model reproducing some of the ancient path

sections may still not be the best choice. However, any least-cost network created without validation against archaeological reality is most probably not very helpful.

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