Problem Sheet 6 for 6401

Due Thursday 24 Nov 2011, at the Problem Class. You should hand in solutions to all problems, but only some of them will be marked. The deadline for handing in your work is 11.55am.

1. Determine whether the following series converge or diverge. If they converge, find their sums.

(a) $3 + \frac{3}{4} + \frac{3}{4^2} + \dots + \frac{3}{4^n} + \dots$;

(b) $\sum_{n=0}^{\infty} (-1)^n 2^{-n} 3^{n-4}$;

(c) $1 - 1 + 1 - 1 + \dots$;

(d) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$;

(e) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$.

2. Determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} (2n^{-2} + 3n^{-3})$;

(b) $\sum_{n=1}^{\infty} (n^{-1} + 2n^{-2})$;

(c) $\sum_{n=1}^{\infty} ((-1)^n n^{-1} + 2n^{-2})$;

(d) $\sum_{n=1}^{\infty} ((-1)^n n^{-1} + 2(-1)^{n^2+17n} n^{-2})$.

3. [Difficult] By comparing the partial sums of the following series with the carefully chosen integrals, prove that the series

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

diverges, whereas the series

$$\sum_{n=2}^{\infty} \frac{1}{n \ln^2 n}$$

converges.