A New Look For The Neutrino

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Introduction

Traditional model requires deep knowledge of Covariant Differentiation, Spinors and Pauli Matrices

Our model requires only differential forms, wedge products and the exterior derivative.

The Neutrino

Matter Particles can be described mathematically by spinors and are solutions to the Dirac Equation.

The simplest case is when the mass is zero which until recently was thought to be the mass of the Neutrino.

$$i\sigma^{\alpha}{}_{a\dot{b}}\nabla_{\alpha}\xi^{a} = 0. \tag{1}$$

The Traditional Model

The spinor has two complex components.

The Lagrangian for the Neutrino takes the form

$$L_{\mathsf{Weyl}}(\xi) := \frac{i}{2}(\bar{\xi}^{\dot{b}}\sigma^{\alpha}{}_{a\dot{b}}\nabla_{\alpha}\xi^{a} - \xi^{a}\sigma^{\alpha}{}_{a\dot{b}}\nabla_{\alpha}\bar{\xi}^{\dot{b}}) * 1. \tag{2}$$

The New Look

For a moment think in 3D. Attach an orthonormal frame onto each point of a continuum.

Coframe $\{\vartheta^1, \vartheta^2, \vartheta^3\}$: triad of covector fields satisfying metric constraint

$$g = \vartheta^1 \otimes \vartheta^1 + \vartheta^2 \otimes \vartheta^2 + \vartheta^3 \otimes \vartheta^3.$$

Their rotations can be described by a 3x3 matrix.

Rotations

To describe the "deformations" caused by the rotations we use the Torsion field strength tensor.

$$T = \vartheta^1 \otimes d\vartheta^1 + \vartheta^2 \otimes d\vartheta^2 + \vartheta^3 \otimes d\vartheta^3. \qquad (3)$$

One of the irreducible parts of this torsion is Axial Torsion.

$$T^{\text{axial}} = \frac{1}{3}(\vartheta^1 \wedge d\vartheta^1 + \vartheta^2 \wedge d\vartheta^2 + \vartheta^3 \wedge d\vartheta^3). \quad (4)$$

Axial Torsion is a 3 form and can be used as a Lagrangian in 3D.

$$L = T^{\text{axial}} \qquad (5)$$

The Lagrangian is then integrated to give, what physicists call, the Action and can be varied to give the field equations.

Upping The Dimension

Time must be added to the normal 3 spatial dimensions. This means that the coframe must be the same but with an added component.

$\{\vartheta^0, \vartheta^1, \vartheta^2, \vartheta^3\}$

The rotation matrix becomes 4x4 and has 6 independent parts.

Teleparallelism

This rotation matrix becomes our dynamical variable. Axial Torsion is not enough to construct a Lagrangian, we need a 4-form. We can attach a light cone vector $l = \vartheta^0 + \vartheta^3$ to give.

$$L := l \wedge T^{\text{axial}} \qquad (6)$$

It is seriously non-linear (non-quadratic) but much more elegant.

The Transformation

The remarkable fact is that these two models are exactly the same up to a change of variable.

The mapping between the two models is non-linear.

In some cases this is counter-productive but in our case it gives a neater expression.

Both models are invariant under the symmetry group B^2 which is the Lorentz transformation preserving a given non-zero spinor field.

How They Relate

$$L_{\text{tele}}(\vartheta) = \frac{4}{3} L_{\text{Weyl}}(\xi). \tag{7}$$

We no longer need explicitly any Differential Geometric tools and have escaped the convoluted world of Pauli matrices.

Ongoing Project

Wish to derive all of QED from this approach by also adding in the mass m.