Dark Energy and Spinors

James Burnett

Mathematics Department
University College London

3rd December

Dark matter

- Say if you were to measure the rotational speed of a spiral galaxy as a function of the radius.

Dark matter

- Say if you were to measure the rotational speed of a spiral galaxy as a function of the radius.
- The outer stars appear to be going to fast for them to still be "glued" to the galaxy.

Dark matter

- Say if you were to measure the rotational speed of a spiral galaxy as a function of the radius.
- The outer stars appear to be going to fast for them to still be “glued” to the galaxy.
- There are two accepted ways to resolve this, either General Relativity (GR) is wrong, or there is matter that we can’t see, i.e it is dark!

Dark Energy

- If we measure the acceleration of the universe at present time, which is linked to the dominant substance of the universe, it appears to be accelerating at such a rate which only something with an equation of state close to $w = P/\rho = -1$ could cause it. Again this is something we can't see, therefore it is called dark energy.

Dark spinor?

- The Elko spinor is defined as the eigenspinor of the charge conjugation operator C.

Dark spinor?

- The Elko spinor is defined as the eigenspinor of the charge conjugation operator C.
- Written as $\lambda = \begin{pmatrix} \pm\sigma_2\phi_L^* \\ \phi_L \end{pmatrix}$,

Dark spinor?

- The Elko spinor is defined as the eigenspinor of the charge conjugation operator C.
- Written as $\lambda = \begin{pmatrix} \pm\sigma_2\phi_L^* \\ \phi_L \end{pmatrix}$,
- They have some special properties, one being that they their dominant coupling is via the Higgs mechanism and therefore are naturally dark.

Its Lagrangian

- We must just look first at its lagrangian

Its Lagrangian

- We must just look first at its lagrangian
- $\tilde{\mathcal{L}} = \frac{1}{2} g^{ab} \tilde{\nabla}_{(a} \overset{\neg}{\lambda} \tilde{\nabla}_{b)} \lambda - V(\overset{\neg}{\lambda} \lambda),$

Its Lagrangian

- We must just look first at its lagrangian
- $\tilde{\mathcal{L}} = \frac{1}{2} g^{ab} \tilde{\nabla}_{(a} \vec{\lambda} \tilde{\nabla}_{b)} \lambda - V(\vec{\lambda}\lambda)$,
- V can either be quadratic or quartic in λ, due to power counting renormalisation constraints.

Curvature

- When we have a manifold we can define a metric, g, on it.

Curvature

- When we have a manifold we can define a metric, g, on it.
- This contains all the information pertaining to curvature. The trivial metric is the minkowski metric, $\eta = diag\{1, -1, -1, -1\}$

Curvature

- When we have a manifold we can define a metric, g, on it.
- This contains all the information pertaining to curvature. The trivial metric is the minkowski metric, $\eta = diag\{1, -1, -1, -1\}$
- Once we are dealing with a curved space we have to be careful about parallel transporting a vector, therefore we must introduce the covariant derivative, $\nabla_\alpha v_\beta = \partial_\alpha v_\beta + \{\Gamma\}^\gamma_{\alpha\beta} v_\gamma$

Curvature

- When we have a manifold we can define a metric, g, on it.
- This contains all the information pertaining to curvature. The trivial metric is the minkowski metric, $\eta = diag\{1, -1, -1, -1\}$
- Once we are dealing with a curved space we have to be careful about parallel transporting a vector, therefore we must introduce the covariant derivative, $\nabla_\alpha v_\beta = \partial_\alpha v_\beta + \{\Gamma\}^{\gamma}_{\alpha\beta} v_\gamma$
- There is one very important condition with regards to the metric, $\nabla_\alpha g_{\beta\gamma} := 0$

Curvature

- When we have a manifold we can define a metric, g, on it.
- This contains all the information pertaining to curvature. The trivial metric is the minkowski metric, $\eta = diag\{1, -1, -1, -1\}$
- Once we are dealing with a curved space we have to be careful about parallel transporting a vector, therefore we must introduce the covariant derivative, $\nabla_\alpha v_\beta = \partial_\alpha v_\beta + \{\Gamma\}^\gamma_{\alpha\beta} v_\gamma$
- There is one very important condition with regards to the metric, $\nabla_\alpha g_{\beta\gamma} := 0$
- This gives you the relation $\{\Gamma\}^\gamma_{\alpha\beta} = \frac{1}{2} g^{\gamma\mu}(\partial_\alpha g_{\mu\beta} + \partial_\beta g_{\mu\alpha} - \partial_\mu g_{\alpha\beta})$ This is symmetric in α and β

Torsion

- Something which cannot be programmed into the metric is torsion.

Torsion

- Something which cannot be programmed into the metric is torsion.
- Therefore we extend our Levi-Civita connection to contain a torsion part, $\Gamma^{\gamma}_{\alpha\beta} = \{\Gamma\}^{\gamma}_{\alpha\beta} - K_{\alpha\beta}{}^{\gamma}$

Torsion

- Something which cannot be programmed into the metric is torsion.
- Therefore we extend our Levi-Civita connection to contain a torsion part, $\Gamma^{\gamma}_{\alpha\beta} = \{\Gamma\}^{\gamma}_{\alpha\beta} - K_{\alpha\beta}{}^{\gamma}$
- Torsion is then defined as the anti-symmetric part, $T_{\alpha\beta}{}^{\gamma} = \Gamma^{\gamma}_{[\alpha\beta]}$.

Field Equations

- The lagrangian we are considering is written
  $S = \int \left(\frac{M_{\rm pl}^2}{2} \tilde{R} + \tilde{\mathcal{L}}_{\rm mat} \right) \sqrt{-g}\, d^4 x,$

Field Equations

- The lagrangian we are considering is written $S = \int \left(\frac{M_{\rm pl}^2}{2} \tilde{R} + \tilde{\mathcal{L}}_{\rm mat} \right) \sqrt{-g}\, d^4x,$
- If we then vary this with respect to the metric and the contorsion tensor we are left with, $\tilde{G}_{ij} = \tilde{R}_{ij} - \frac{1}{2}\tilde{R}g_{ij} = \frac{1}{M_{\rm pl}^2}\Sigma_{ij},$ and $T^{ij}{}_k + \delta^i_k T^j{}_l{}^l - \delta^j_k T^i{}_l{}^l = M_{\rm pl}^2\, \tau^{ij}{}_k,$

The Metric and Connection Coefficients

- We assert the accepted properties of the universe in cosmology as being isotropic and homogeneous which essentially means we only have one variable, time. The metric becomes $ds^2 = dt^2 - a(t)^2\left(dx^2 + dy^2 + dz^2\right)$,

The Metric and Connection Coefficients

- We assert the accepted properties of the universe in cosmology as being isotropic and homogeneous which essentially means we only have one variable, time. The metric becomes $ds^2 = dt^2 - a(t)^2\left(dx^2 + dy^2 + dz^2\right)$,
- The connection coefficients can be written $\Gamma^x_{tx} = \Gamma^y_{ty} = \Gamma^z_{tz} = \frac{\dot{a}}{a}\,, \Gamma^t_{xx} = \Gamma^t_{yy} = \Gamma^t_{zz} = a\dot{a}\,,$

The Metric and Connection Coefficients

- We assert the accepted properties of the universe in cosmology as being isotropic and homogeneous which essentially means we only have one variable, time. The metric becomes $ds^2 = dt^2 - a(t)^2\left(dx^2 + dy^2 + dz^2\right)$,
- The connection coefficients can be written $\Gamma^x_{tx} = \Gamma^y_{ty} = \Gamma^z_{tz} = \frac{\dot{a}}{a}, \Gamma^t_{xx} = \Gamma^t_{yy} = \Gamma^t_{zz} = a\dot{a}$,
- Torsion has only two independent functions, $T_{\hat{1}\hat{1}\hat{0}} = T_{\hat{2}\hat{2}\hat{0}} = T_{\hat{3}\hat{3}\hat{0}} = h(t)$, $T_{\hat{1}\hat{2}\hat{3}} = T_{\hat{3}\hat{1}\hat{2}} = T_{\hat{2}\hat{3}\hat{1}} = f(t)$.

The Metric and Connection Coefficients

- We assert the accepted properties of the universe in cosmology as being isotropic and homogeneous which essentially means we only have one variable, time. The metric becomes $ds^2 = dt^2 - a(t)^2\left(dx^2 + dy^2 + dz^2\right)$,
- The connection coefficients can be written $\Gamma^x_{tx} = \Gamma^y_{ty} = \Gamma^z_{tz} = \frac{\dot{a}}{a}\,, \Gamma^t_{xx} = \Gamma^t_{yy} = \Gamma^t_{zz} = a\dot{a}\,,$
- Torsion has only two independent functions, $T_{\hat{1}\hat{1}\hat{0}} = T_{\hat{2}\hat{2}\hat{0}} = T_{\hat{3}\hat{3}\hat{0}} = h(t)$, $T_{\hat{1}\hat{2}\hat{3}} = T_{\hat{3}\hat{1}\hat{2}} = T_{\hat{2}\hat{3}\hat{1}} = f(t)$.
- Finally our Elko spinor can be written in terms of one variable, $\lambda_{\{-,+\}} = \varphi(t)\,\xi, \lambda_{\{+,-\}} = \varphi(t)\,\zeta,$, where ξ and ζ are constant spinors.

The solutions

- Once you have worked through the equations the solution comes out as

$$h = -\frac{\varphi^4/M_{\rm pl}^4}{4+\varphi^4/M_{\rm pl}^4}\,\frac{\dot{a}}{a}, \qquad (1)$$

$$f = -\frac{2\varphi^2/M_{\rm pl}^2}{4+A_0^4/M_{\rm pl}^4}\,\frac{\dot{a}}{a}, \qquad (2)$$

Some nice graphs.

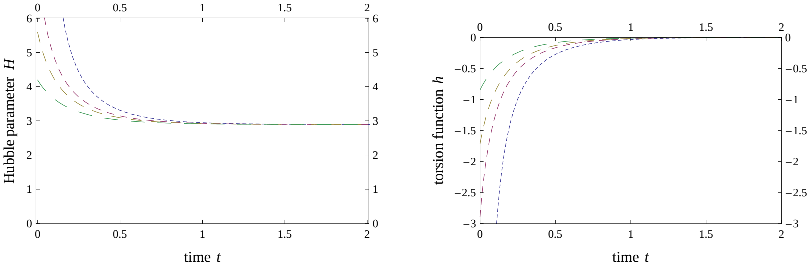

Figure: Left: Hubble parameter and right: torsion function h for $1/M_{\rm pl}^2 = 8\pi$ and $V_0 = 1$. Initial conditions of the matter field are $\varphi_i = \varphi(t=0) = \{0.282, 0.25, 0.23, 0.20\}$, {blue (short dashed), red (dashed), (medium dashed) yellow and green (long dashed) }

Some nice graphs.

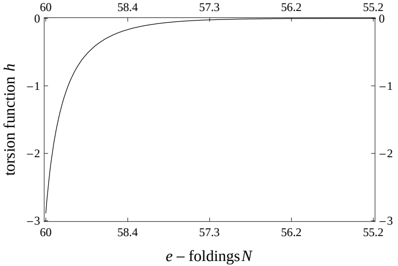

Figure: Torsion function h for $1/M_{\mathrm{pl}}^2 = 8\pi$ and $V_0 = 1$. Initial condition is $\varphi_i = \{0.25\}$.

Equation of state.

- If we say torsion is zero and just look at the standard coupling of curvature to the Elko spinor.

Equation of state.

- If we say torsion is zero and just look at the standard coupling of curvature to the Elko spinor.
- We find that the cosmological field equations are $H^2 = \frac{1}{3M_{\rm pl}^2}\rho, \dot{\rho} + 3H(\rho + P) = 0.$

Equation of state.

- If we say torsion is zero and just look at the standard coupling of curvature to the Elko spinor.
- We find that the cosmological field equations are $H^2 = \frac{1}{3M_{\rm pl}^2}\rho, \dot{\rho} + 3H(\rho + P) = 0$.
- From this we can find out what the exactly the equation of state is and plot it.

Another nice graph

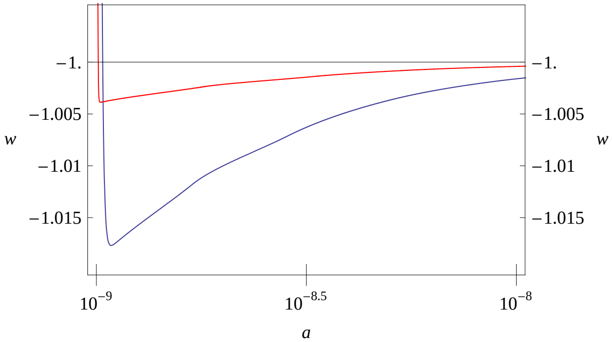

Figure: Equation of state for $V_1(\varphi)$: $M_{\rm pl} = 1$, $\dot{\varphi}(0) = 1$ and $w(0) = -1/3$. With $m_i^2 = \{0.002, 0.001\} = \{\text{red(higher)}, \text{blue(lower)}\}$

Cosmological Perturbation Theory

- Although it is okay to suggest they could be candidate for dark matter/energy, the real test is to work the spectrum via cosmological perturbation theory.

Cosmological Perturbation Theory

- Although it is okay to suggest they could be candidate for dark matter/energy, the real test is to work the spectrum via cosmological perturbation theory.
- This I will hopefully do at some point.

Cosmological Perturbation Theory

- Although it is okay to suggest they could be candidate for dark matter/energy, the real test is to work the spectrum via cosmological perturbation theory.
- This I will hopefully do at some point.
- Cheers!