Dark Matter Spinors With Torsion

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Outline

• What are Dark Spinors?
• Einstein-Cartan Theory
• Cosmological Field Equations
• Cosmological Dynamics
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Dark Spinors

- Dark spinors are eigenspinors of the Charge Conjugation Operator.
- Important properties include:
  - Double helicity structure compared to Dirac spinors
  - Spin 1/2 but mass dimension 1
  - They satisfy the Klein-Gordon Equation.
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- Need a different definition for the dual in order to have a consistent field theory.
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What Do They Look Like?

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• Need a different definition for the dual in order to have a consistent field theory. 
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• Dark spinors are an excellent candidate for dark matter as they only couple to the Higgs mechanism.
Einstein-Cartan Theory

In Einstein-Cartan theory an additional fundamental tensor has been added namely Torsion. This yields a richer framework and allows extra coupling for the Dark spinors. Working with anholonomic indices:

\[ \tilde{\nabla}^a \lambda = \partial^a \lambda - \frac{1}{4} \Gamma^{abc \gamma}_{b \gamma c \lambda} + \frac{1}{4} K^{abc \gamma}_{b \gamma c \lambda}, \]
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Contortion

\[ \tilde{R}_{ij} - \frac{1}{2} \tilde{R}_{\eta \eta} = \kappa \Sigma_{ij}, \]
\[ S_{ijk} + \delta_{i k} S_{jll} - \delta_{j k} S_{ill} = \kappa \tau_{ijk}. \]

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\[ \tilde{R}_{ij} - \frac{1}{2} \tilde{R} \eta_{ij} = \kappa \Sigma_{ij}, \]

\[ S^{ij}_k + \delta^i_k S^j_l - \delta^j_k S^i_l = \kappa \tau^{ij}_k. \]

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Cosmological Principle

- On large scales the universe is isotropic and homogenous.

- Thus there are certain constraints that need to be implemented on the metric and the torsion tensor.

\[ \mathcal{L}_\xi g_{\mu\nu} = 0, \quad \mathcal{L}_\xi S_{\mu\nu}^\lambda = 0 \]

- This leaves only two independent components of the torsion tensor.

\[ T_{110} = T_{220} = T_{330} = h(t) \]
\[ T_{123} = T_{312} = T_{231} = f(t) \]
Field Equations

We can solve for these unknown functions using the second field equation.

\[
\dot{\phi}(t) = -\sqrt{\kappa V} \sqrt{\frac{4}{3} + \frac{\kappa^2 \phi^4(t)}{4 - \kappa^2 \phi^4(t)}}
\]

Finally solving these gives Hubble's parameter.

\[
H(t) = \frac{\dot{a}}{a} = \sqrt{\frac{V \kappa^2}{\sqrt{3} \sqrt{(4 + \kappa^2 \phi^4(t))}}} \sqrt{\frac{4}{3} - \kappa^2 \phi^4(t)}
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\frac{\dot{\phi}(t)}{\phi(t)} = -\frac{\sqrt{\kappa V}}{4\sqrt{3}} \frac{8 + 3\kappa^2 \phi^4}{12 - \kappa^2 \phi^4} \sqrt{4 - \kappa^2 \phi^4}
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Hubble

• Constant Hubble is an attractor which compares well with observations of Hubble today.
• Constant Hubble is an attractor.
This shows that Torsion falls to zero sufficiently quickly.
• This shows Torsion converging to zero.
Torsion

- This shows more clearly how quickly Torsion decays during inflation.
Conclusion

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