Dark Matter Spinors With Torsion

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Outline

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- What are Dark Spinors?

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- Cosmological Dynamics

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- Important properties include

  - Double helicity structure compared to Dirac spinors

  - Spin 1/2 but mass dimension 1

  - They satisfy the Klein-Gordon Equation.

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- Dark spinors are an excellent candidate for dark matter as they only couple to the Higgs mechanism.

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- $\tilde{\nabla}_a \lambda = \partial_a \lambda - \frac{1}{4}\Gamma_{abc}\gamma^b\gamma^c\lambda + \frac{1}{4}K_{abc}\gamma^b\gamma^c\lambda\,,$

Contortion

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- Varying with respect to the contortion (tortion) tensor gives us extra Field Equations.

$$\tilde{R}_{ij} - \frac{1}{2}\tilde{R}\eta_{ij} = \kappa\,\Sigma_{ij}\,,$$

$$S^{ij}{}_{k} + \delta^i_k S^j{}_l{}^l - \delta^j_k S^i{}_l{}^l = \kappa\,\tau^{ij}{}_{k}\,.$$

- Each of the Riemann quantities are computed from the full connection, which includes torsion.

Cosmological Principle

- On large the scales the universe is isotropic and homogenous.

- Thus there are certain constraints that need to be implemented on the metric and the torsion tensor.

$$\mathcal{L}_\xi g_{\mu\nu} = 0, \quad \mathcal{L}_\xi S_{\mu\nu}{}^{\lambda} = 0$$

- This leaves only two independent components of the torsion tensor.

$$T_{110} = T_{220} = T_{330} = h(t)$$

$$T_{123} = T_{312} = T_{231} = f(t)$$

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$$\frac{\dot{\phi}(t)}{\phi(t)} = -\frac{\sqrt{\kappa V}}{4\sqrt{3}} \frac{8 + 3\kappa^2\phi^4}{12 - \kappa^2\phi^4} \sqrt{4 - \kappa^2\phi^4}$$

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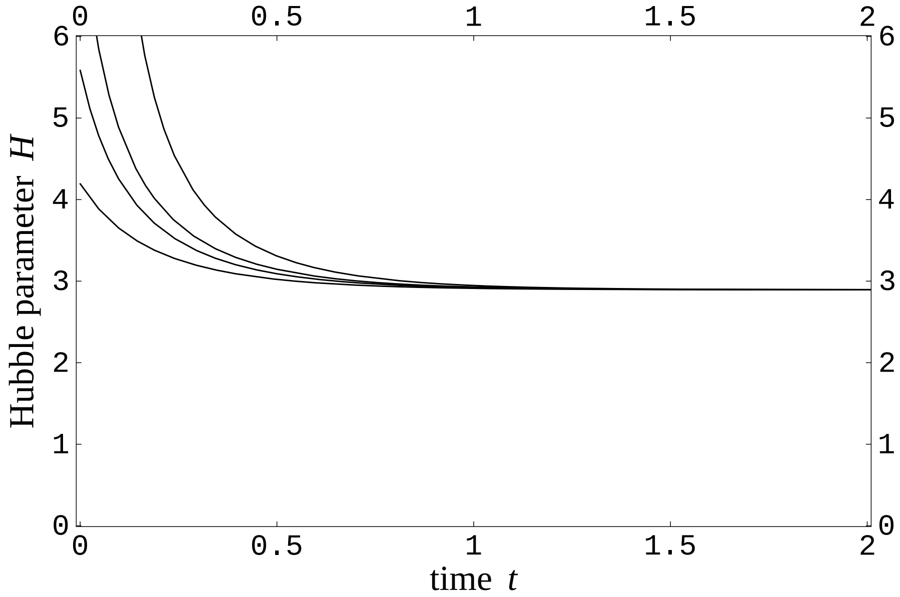
- Finally solving these gives Hubble's parameter.

$$H(t) = \frac{\dot{a}}{a} = \frac{\sqrt{V\kappa}}{2\sqrt{3}} \sqrt{(4 + \kappa^2\phi^4(t))} \sqrt{4 - \kappa^2\phi^4(t)}$$

Hubble



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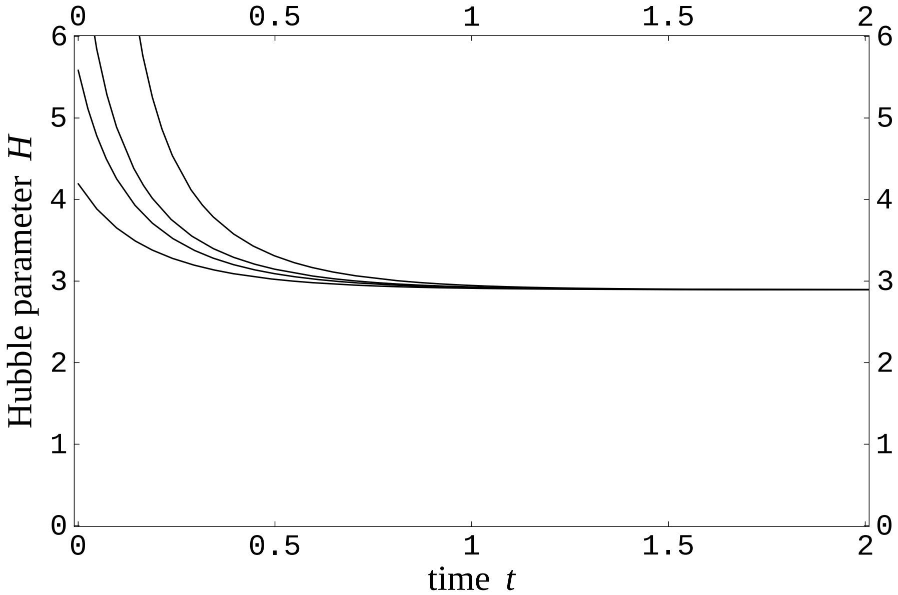
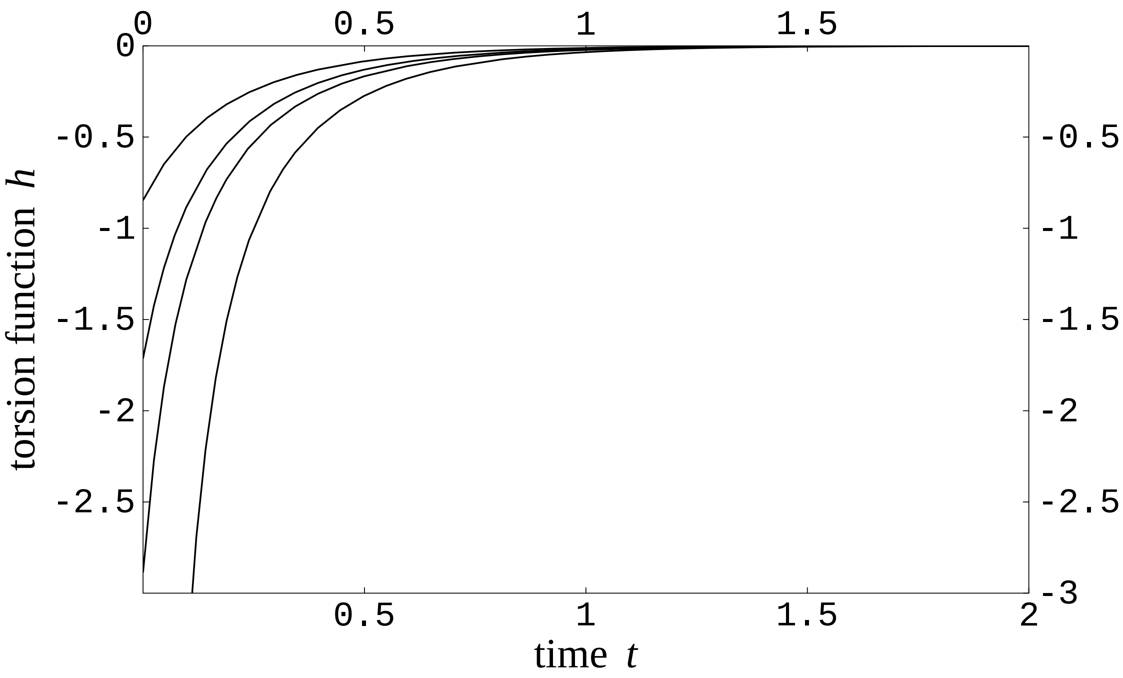
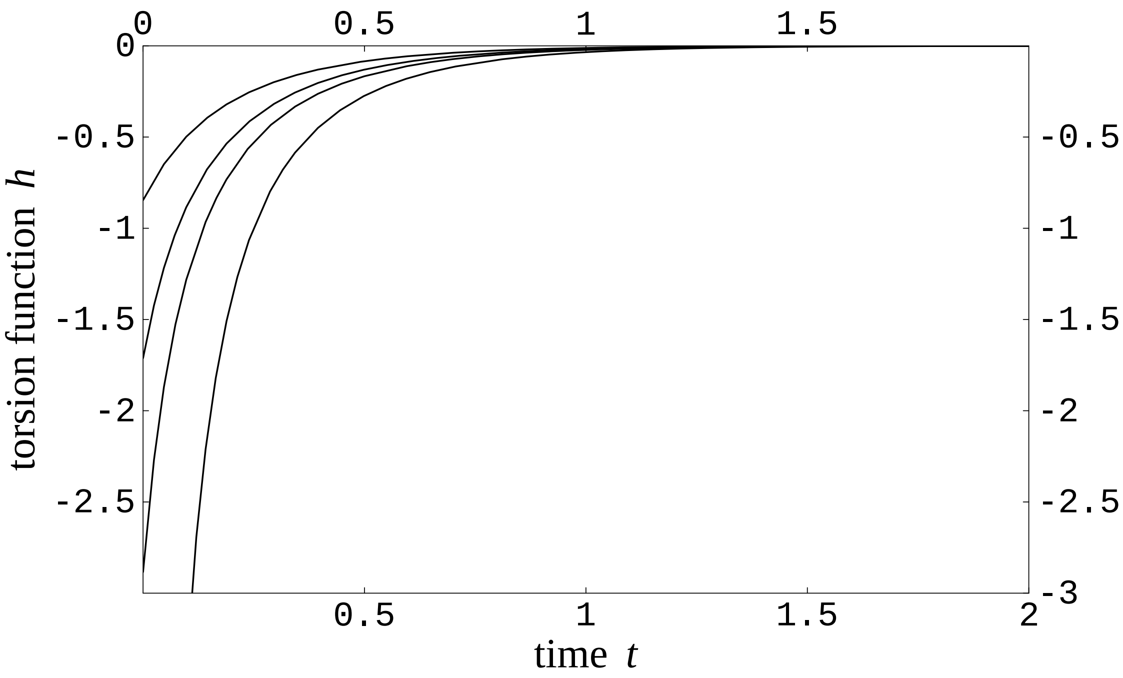


- Constant Hubble is an attractor.

Torsion

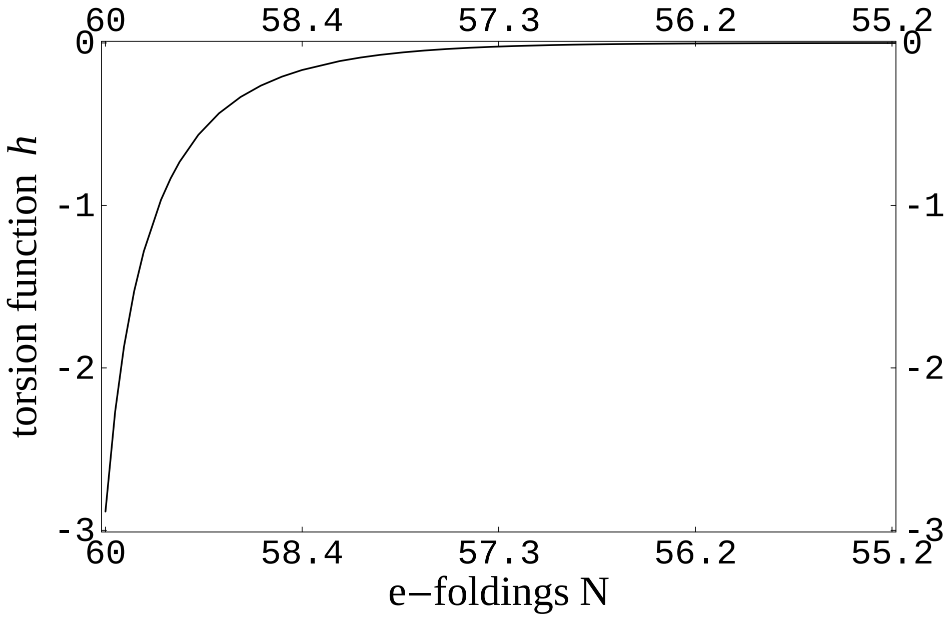


Torsion



- This shows Torsion converging to zero.

Torsion



- This shows more clearly how quickly Torsion decays during inflation.

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- That more attention needs to be given to them.