HANDOUT 3: GAUSSIAN ELIMINATION, THE FORMAL ELIMINATION PROCESS

This systematic, orderly method of creating a row echelon form is called **standard Gaussian elimination**. We write our system of equations as an augmented matrix (with row sums). Then we work down the rows and across the columns like this.

(1) Choose element a_{11}.

- If $a_{11} = 0$ but there is a non-zero element somewhere in column 1 (say, row 3 for instance) then swap rows: $r_1 \leftrightarrow r_3$.
- If the whole of column 1 is zero then choose a_{12} as the next element and move on to the next stage.
- If $a_{11} \neq 0$ (maybe after the row swap above) then:
  - Subtract a multiple of row 1 from row 2 to create a zero at the beginning of row 2;
  - Subtract a multiple of row 1 from row 3 to create a zero at the beginning of row 3;
  - Repeat until column 1 is all zeros except for the element in row 1.
  - Now choose element a_{22} as the next element and move on to the next stage.

(2) We have chosen element a_{ij}, i.e. element j of row i. We **ignore all the rows above row** i from now on.

- If $a_{ij} = 0$ but there is a non-zero element somewhere below a_{ij} in column j (say, row 4 for instance) then swap rows: $r_i \leftrightarrow r_4$.
- If the whole of column j from row i down to the bottom is zero then choose $a_{i,j+1}$ as the next element and move on to the next stage.
- If $a_{ij} \neq 0$ (maybe after the row swap above) then:
  - Subtract a multiple of row i from row $i + 1$ to create a zero in place j of row $i + 1$;
  - Subtract a multiple of row i from row $i + 2$ to create a zero in place j of row $i + 2$;
  - Repeat until column j is all zeros below row i.
  - Now choose element $a_{i+1,j+1}$ as the next element and move on to the next stage.

(3) Repeat step 2 until we reach generalised row echelon form.