Spectral theory: what's it all about and what is its use

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University College London

10 September 2024

https://en.wikipedia.org/wiki/Pendulum

Period of oscillation of a pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}\ \text{seconds}\,,$$

where

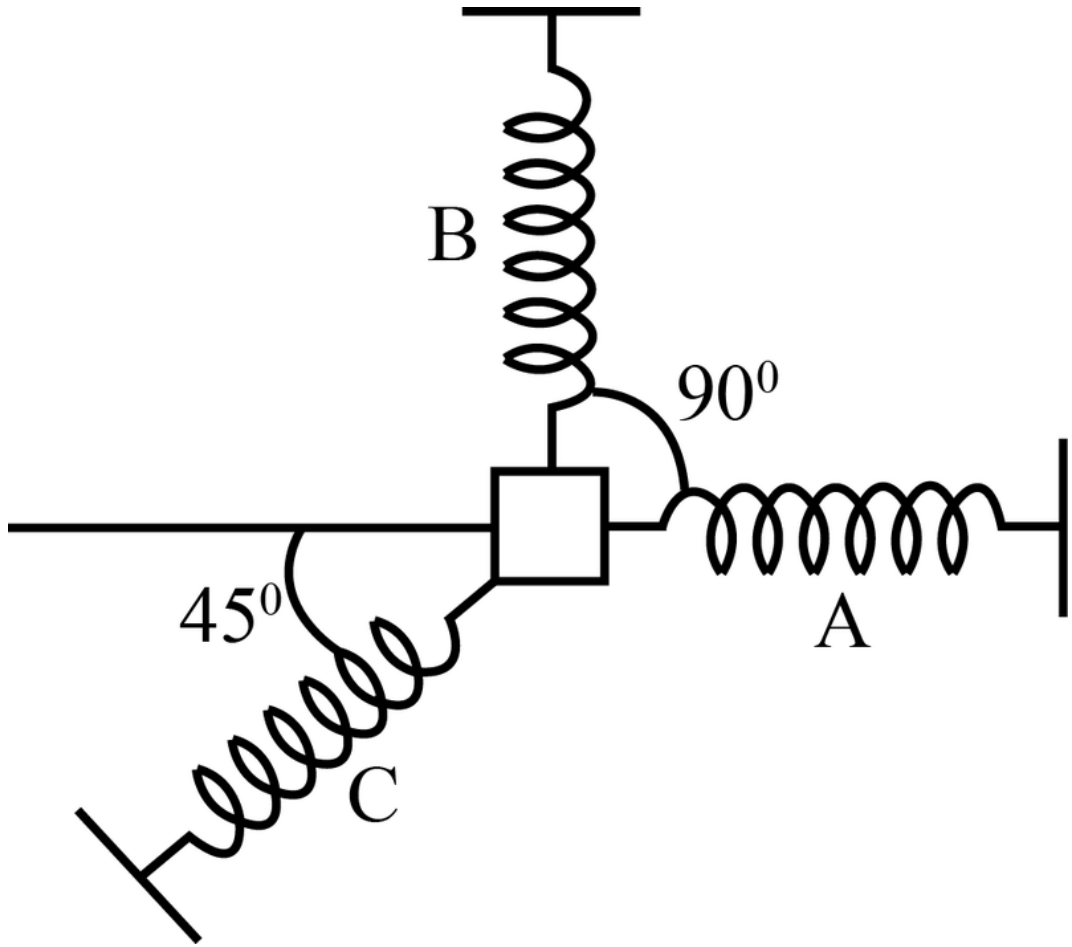
$$g \approx 9.8\frac{m}{s^2}$$

is the acceleration of gravity (gravity of Earth) and L is the length of the pendulum.

Natural frequency of a pendulum:

$$f = \frac{1}{T}\ \text{Hz}\,.$$

A system with 3 degrees of freedom.



It has 3 natural frequencies.

A string: system with infinitely many degrees of freedom.

`https://en.wikipedia.org/wiki/String_vibration`

It has infinitely many natural frequencies:

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}, \quad n = 1, 2, \ldots,$$

where T is the tension of the string, μ is the linear density of the string and L is the length of the string.

The collection of all natural frequencies is called *spectrum*.

Lowest natural frequency is called *fundamental frequency*, whereas higher natural frequencies are called *overtones*.

Vibrations of a membrane. Think of a drum.

Things become more complicated: need to deal with *partial* differential equations. Here *partial* refers to the fact that we have more than one spatial coordinate.

Chladni Plate.

`https://www.youtube.com/watch?v=T04iLhJsars`

Ernst Florens Friedrich Chladni (1756 - 1827) was a German physicist and musician.

Acoustic resonator. Like a membrane, but in 3D.

Things get really tough, finding natural frequencies f_n, $n = 1, 2, \ldots,$ becomes very difficult.

A miracle: the Rayleigh–Jeans law (1905) states that

$$f_n \approx c \left(\frac{3n}{4\pi V} \right)^{1/3} \quad \text{as} \quad n \to +\infty,$$

where c is the speed of sound and V is the volume of the resonator.

Jeans' contribution to the Rayleigh–Jeans law

"It seems to me that Lord Rayleigh has introduced an unnecessary factor 8 by counting negative as well as positive values of his integers".

Lord Rayleigh replied with a paper consisting of one sentence as well: "I agree".

1910: Lorentz visits Göttingen at Hilbert's invitation and delivers a series of lectures "Old and new problems in physics". Lorentz states the Rayleigh–Jeans law as a mathematical conjecture. Hermann Weyl is in the audience.

1912: Weyl publishes a rigorous proof of Rayleigh–Jeans law. Almost incomprehensible.

Comprehensible proof: in R.Courant and D.Hilbert, *Methods of Mathematical Physics* (1924).

Weyl's Conjecture (1913)

One can do better and prove a two-term asymptotic formula for the acoustic resonator with hard walls (Neumann boundary conditions):

$$f_n \approx c \left(\frac{3n}{4\pi V} \right)^{1/3} - \frac{c\, S}{16 V} \quad \text{as} \quad n \to +\infty,$$

where S is the surface area of the boundary of the resonator.

Very difficult problem bordering on *analytic number theory*.

Theorem (Victor Ivrii, 1980) Weyl's conjecture holds if periodic billiard trajectories have measure zero.

Theorem (Vassiliev, 1984) Under certain geometric assumptions on the branching Hamiltonian billiards, Weyl's conjecture holds for higher order operators. Furthermore, I have an explicit algorithm for the evaluation of the second Weyl coefficient.

Yu.Safarov and D.Vassiliev, *The asymptotic distribution of eigenvalues of partial differential operators*, American Mathematical Society, 1997 (hardcover), 1998 (softcover).

"In the reviewer's opinion, this book is indispensable for serious students of spectral asymptotics". Lars Hörmander for the Bulletin of the London Mathematical Society.

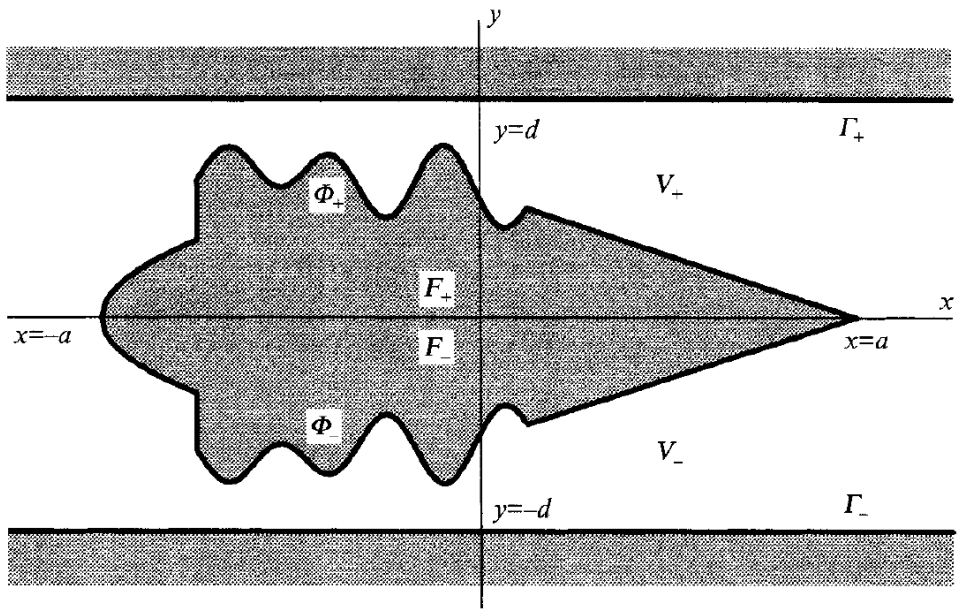
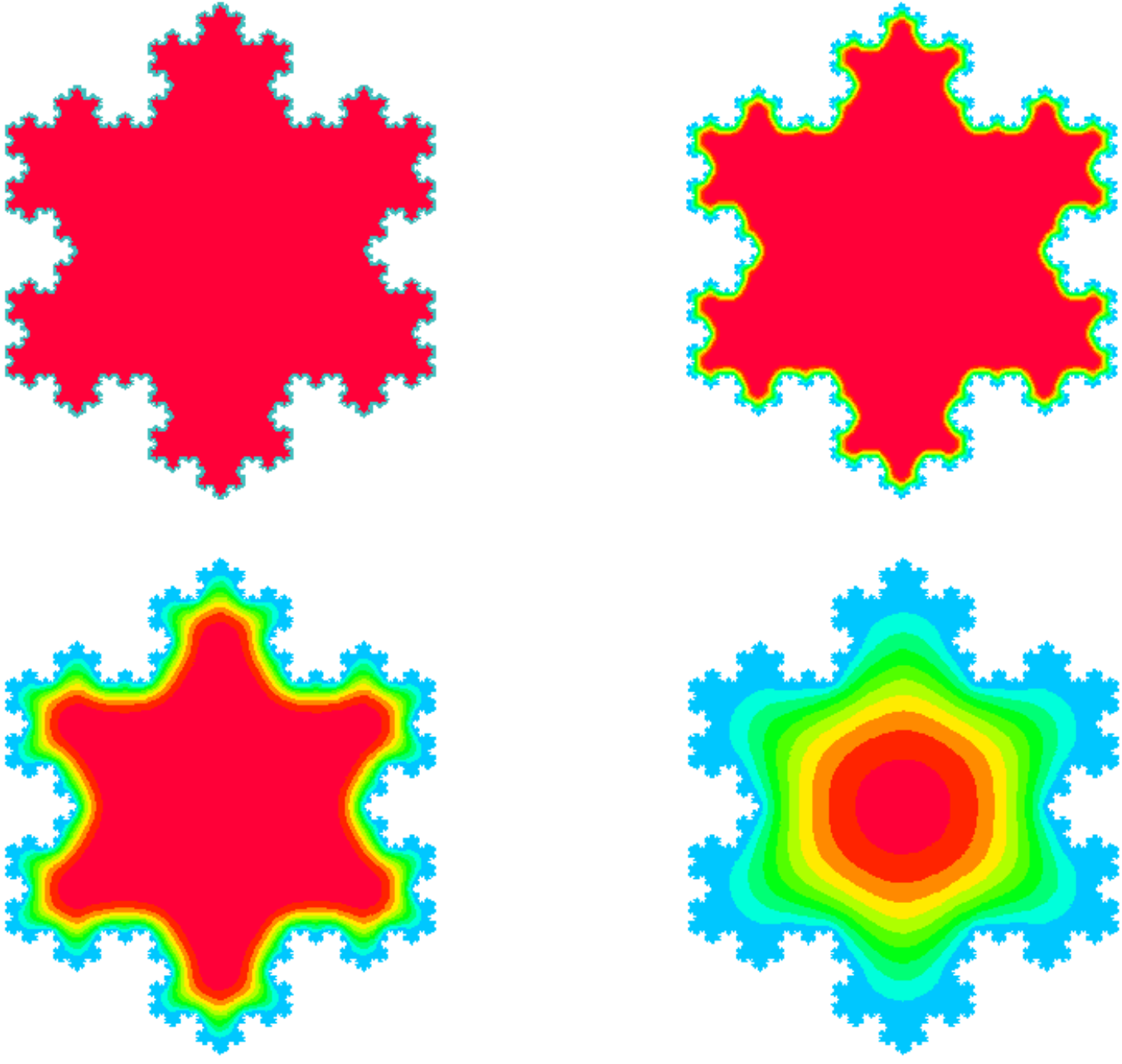


FIGURE 1. Waveguide with obstruction ($Y(s) \not\equiv 0$).

not go through the same point twice). For technical reasons we also assume that



Spectral asymmetry.

Single elementary particle living in closed 3-dimensional universe.

Particle is described as a wave governed by the Dirac equation.

Positive natural frequencies are interpreted as energy states of a particle, whereas negative natural frequencies are interpreted as energy states of a antiparticle.

Contributors to this subject area: Sir Michael Atiyah, Isadore Singer, Nigel Hitchin, Peter Gilkey, Lev Pontryagin, Friedrich Hirzebruch, Shiing-Shen Chern, Jim Simons, Bob Seeley.

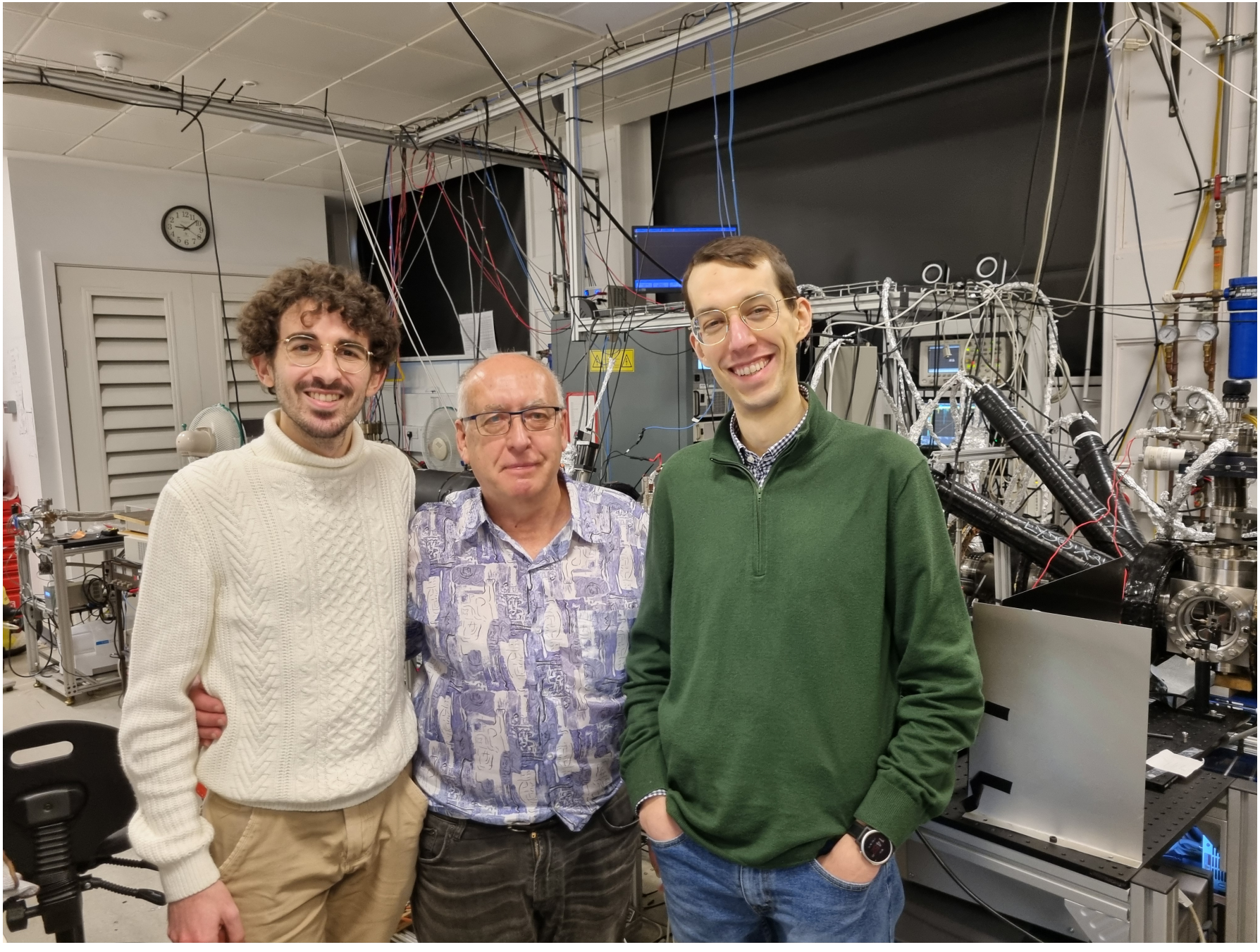
Most important application of spectral theory: atomic physics.

Electrons in the atom are described as waves governed by the Schrödinger equation or the Dirac equation. Natural frequencies are interpreted as energy states of the atom.

Positronium atom. Here the agreement between theory and experiment is 10^{-10}. Triumph of spectral theory.

Problem with positronium: theoretical results are non-rigorous.

Bethe–Salpeter equation.



Ivan Vinogradov - Wikipedia

en.wikipedia.org/wiki/Ivan_Vinogradov

Ivan Vinogradov

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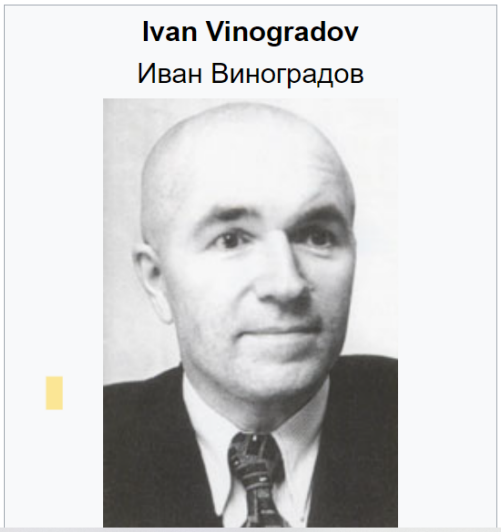
Not to be confused with the Russian mathematicians Alexandre Vinogradov (of Vinogradov sequence) or Askold Vinogradov (of Bombieri–Vinogradov theorem).

In this name that follows Eastern Slavic naming customs, the patronymic is Matveevich *and the family name is* Vinogradov.

Ivan Matveevich Vinogradov ForMemRS[1] (Russian: Ива́н Матве́евич Виногра́дов, IPA: [ɪˈvan mɐtˈvʲejɪvʲɪtɕ vʲɪnəˈgradəf] ; 14 September 1891 – 20 March 1983) was a Soviet mathematician, who was one of the creators of modern analytic number theory, and also a dominant figure in mathematics in the USSR. He was born in the Velikiye Luki district, Pskov Oblast. He graduated from the University of St. Petersburg, where in 1920 he became a Professor. From 1934 he was a Director of the Steklov Institute of Mathematics, a position he held for the rest of his life, except for the five-year period (1941–1946) when the institute was directed by Academician Sergei Sobolev. In 1941 he was awarded the Stalin Prize. He was elected to the American Philosophical Society in 1942.[3] In 1951 he became a foreign member of the Polish Academy of Sciences and Letters in Kraków.



Mathematical contributions [edit]

In analytic number theory, *Vinogradov's method* refers to his main problem-solving technique,

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September 16-18, 2024, Space Research Institute (IKI) (новость размещена: 25.06.2024)
Conference "Cosmic Gas Dynamics"
in memory of Professor V.B. Baranov
dedicated to the 90th anniversary of his birth
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This is the new website of the Institute for Problems in Mechanics of the Russian Academy of Sciences. It was launched on 9th November 2015. The old website is available at http://www1.ipmnet.ru.
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From Wikipedia, the free encyclopedia
Coordinates: 59°49′30″N 30°21′38″E
The Krylov State Research Center (Russian: Крыловский государственный научный центр) is a Russian shipbuilding research and development institute, which operates as a federal state-owned unitary enterprise.
The institute is named after Aleksey Krylov, the Russian naval designer and mathematician who was one of its first superintendents, and is based in Saint Petersburg.[1]
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Industry Shipbuilding
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Key people Andrei Dutov
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A Treatise on the Mathematical | Augustus Edward Hough Love

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Augustus Edward Hough Love

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From Wikipedia, the free encyclopedia

Augustus Edward Hough Love FRS[1] (17 April 1863, Weston-super-Mare – 5 June 1940, Oxford), often known as **A. E. H. Love**, was a mathematician famous for his work on the mathematical theory of elasticity. He also worked on wave propagation and his work on the structure of the Earth in *Some Problems of Geodynamics* won for him the Adams prize in 1911 when he developed a mathematical model of surface waves known as Love waves.[2][3][4] Love also contributed to the theory of tidal locking and introduced the parameters known as Love numbers, used in problems related to Earth tides, the tidal deformation of the solid Earth due to the gravitational attraction of the Moon and Sun.

Life and career [edit]

He was educated at Wolverhampton Grammar School and in 1881 won a scholarship to St John's College, Cambridge, where he was at first undecided whether to study classics or mathematics. His successful progress (he was placed Second Wrangler)[5] vindicated his choice of mathematics, and in 1886 he was elected Fellow of the college. In 1899 he was appointed Sedleian Professor of Natural Philosophy in the University of Oxford, a position which he retained until his death in 1940. He was also a Fellow of Queen's College.

Augustus Edward Hough Love

Born	17 April 1863 Weston-super-Mare
Died	5 June 1940 (aged 77)[1] Oxford, United Kingdom
Nationality	English

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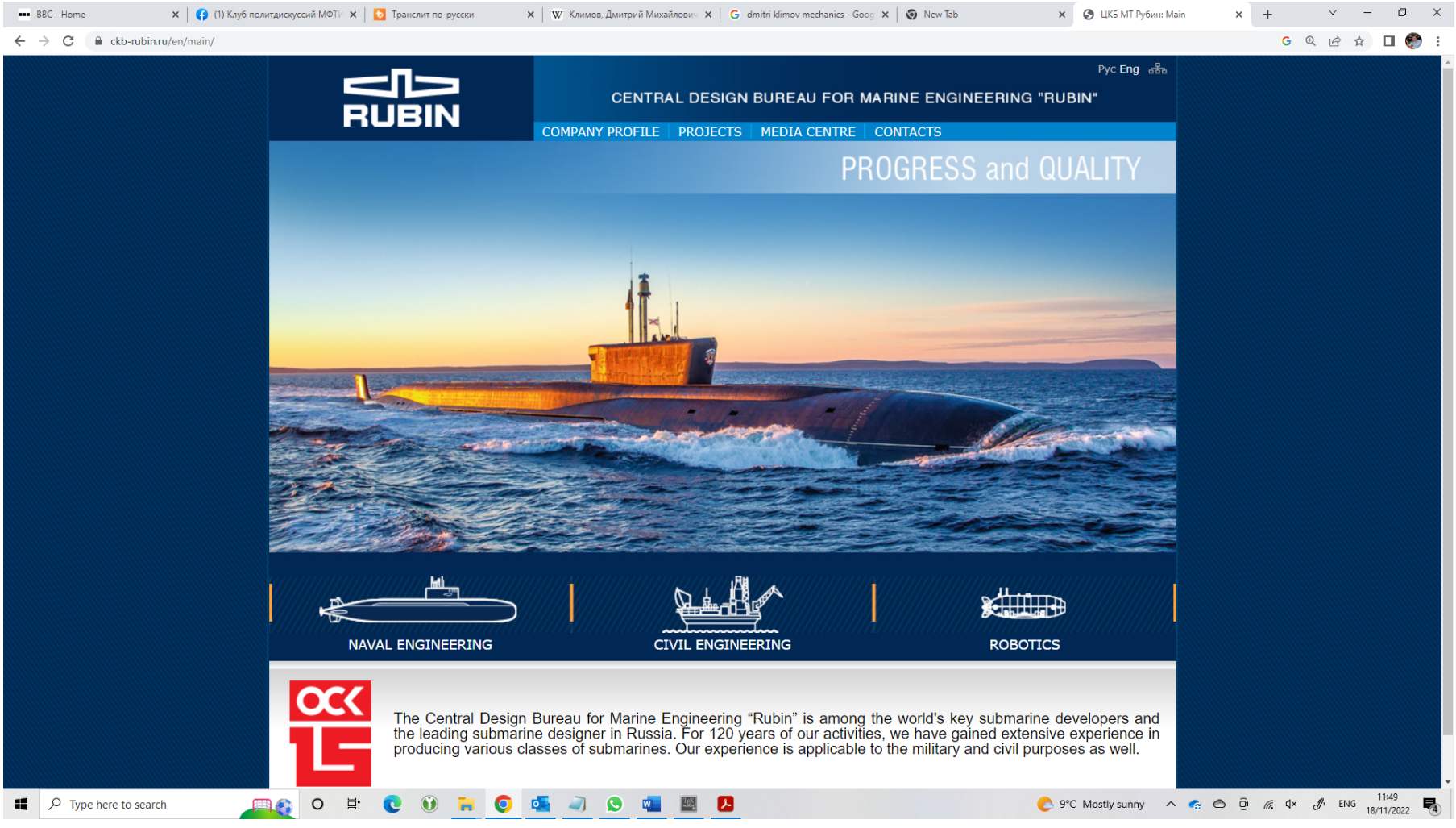
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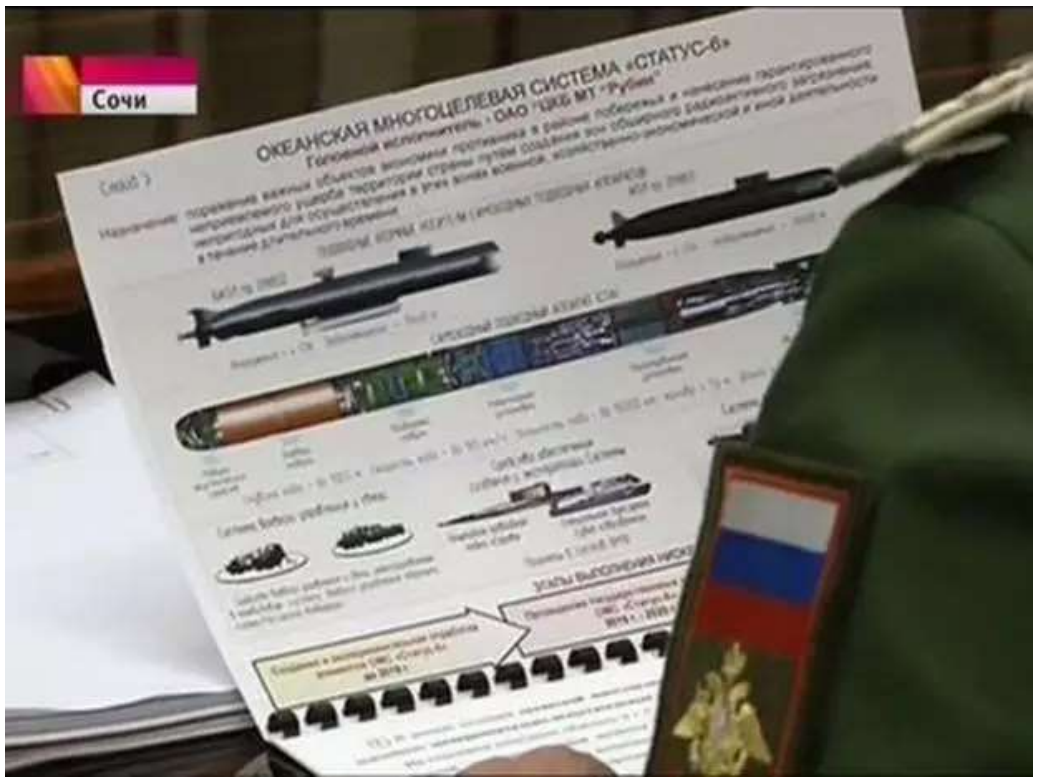
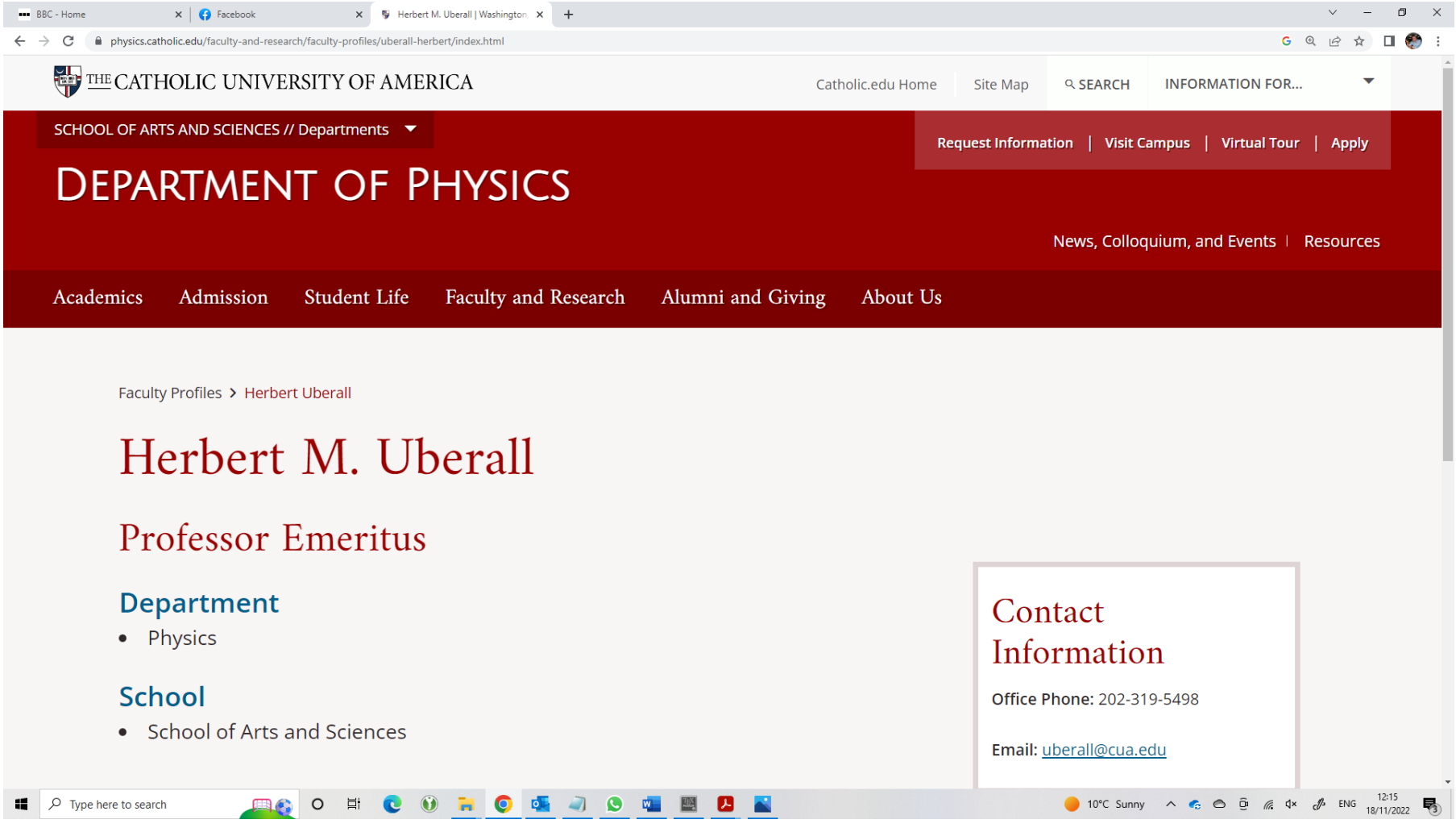
Russian state-controlled TV 'accidentally' broadcasts secret plans for nuclear torpedo system

Vladimir Putin had announced the development of a system to defeat Nato defences

Lizzie Dearden @lizziedearden Thursday 12 November 2015 09:18 GMT 11 comments

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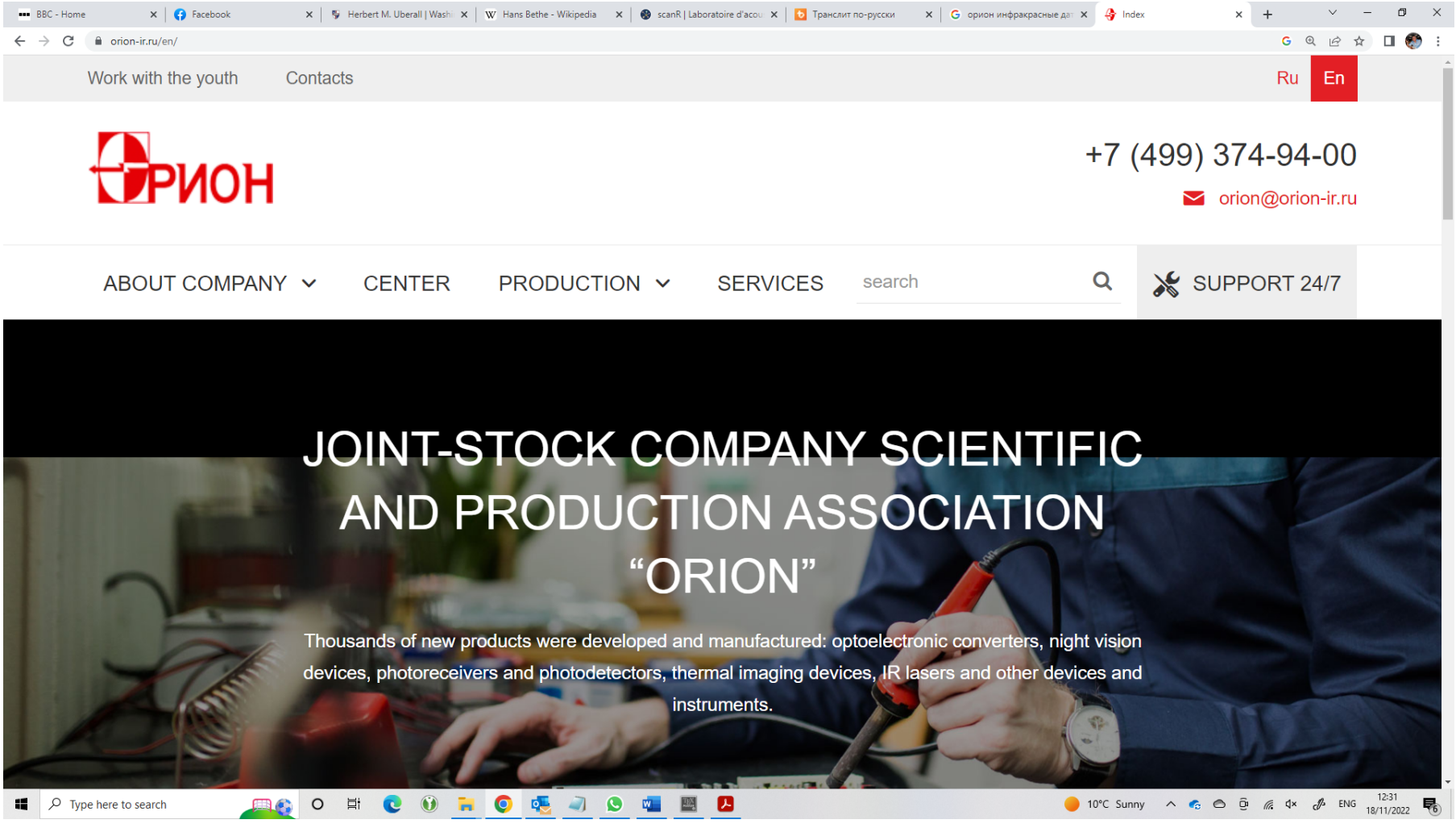
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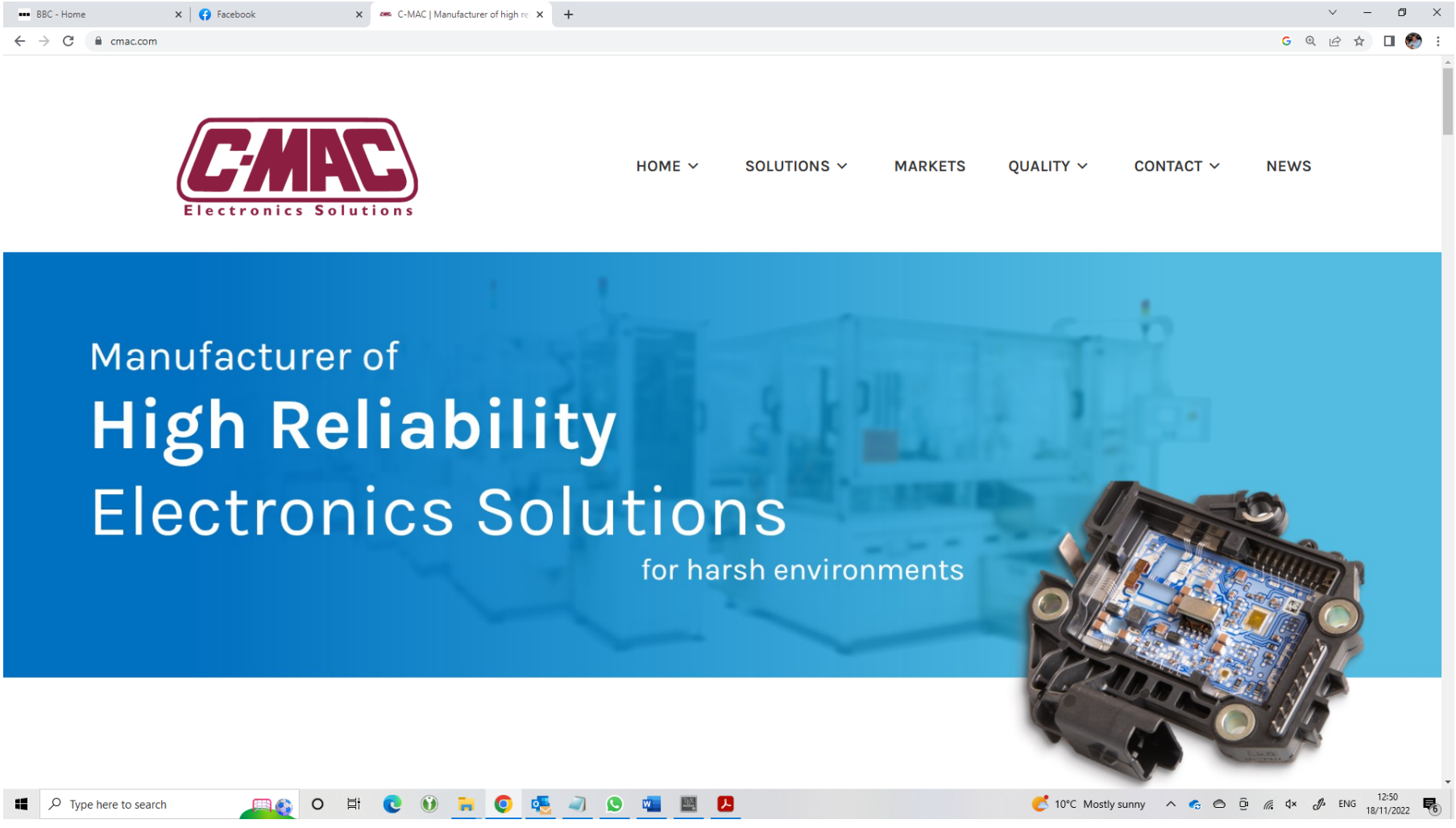
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